SOVIET PHYSICS JETP VOLUME 37 (10), NUMBER 1 JANUARY, 1960

INTENSITY RULES FOR BETA TRANSITIONS TO DIFFERENT ROTATIONAL STATES OF EVEN-EVEN DAUGHTER NUCLEI

A. S. DAVYDOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor January 13, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 137-142 (July, 1959)

Relative probabilities are computed for the β decay of an odd-odd nucleus with excitation of different rotational states of the non-axially-symmetrical even-even daughter nucleus. The theory is compared with experiment for the β decay of Re^{186}, Np^{238}, Eu^{154}, Re^{190}, and Ir^{190}.

1. INTRODUCTION

It has been shown by Alaga, Alder, Bohr, and Mottelson[1] that the relative intensities of β transitions from a given initial nuclear state to different rotational states of a daughter nucleus obey rules that are analogous to the intensity rules for the fine structure and superfine structure of atomic spectra. These authors assumed axial symmetry for all nuclei, as a result of which the only rotational levels of even-even nuclei were given by the formula $E_J = AJ(J+1)$, $J = 0, 2, 4, \ldots$. It was further assumed that the wave functions of nuclear states can be represented by simple products of the functions φ_K, representing internal motion, and D^J_{MK}, which depend on the Euler angles that describe the spatial orientation of the nucleus:

$$\psi_{JMK} = \sqrt{(2J+1)/8\pi^2}\, \varphi_K D^J_{MK}.$$

The assumption of axial symmetry for all nuclei does not appear to be justified. It has been shown by the calculations of Geĭlikman,[2] Zaikin,[3] and Davydov and Filippov[4] that the equilibrium shape of a nucleus may not necessarily possess axial symmetry. In references 5 and 6 the energy levels of non-axisymmetric even-even nuclei were calculated for different values of a parameter γ which determines the departure from axial symmetry. It was shown that the rotational states of even-even nuclei include certain excited states which were previously regarded as γ and β vibrational levels. Since the relative energies and wave functions of all rotational levels can be determined unambiguously for each nucleus, when the energy ratio of two levels with spin 2 is known it becomes possible to estimate the relative probabilities of β transitions from a given state of the parent nucleus to different rotational states of a non-axisymmetric nucleus. Such relative probabilities of β transitions will be calculated in the present paper.

2. RATIO OF SQUARED ABSOLUTE VALUES OF MATRIX ELEMENTS FOR β DECAY TO DIFFERENT ROTATIONAL LEVELS

The β decay of an odd-odd parent nucleus with integral spin produces an even-even nucleus. β decay is characterized by the angular momentum L which is carried away by the electron and antineutrino and may be associated with the operator $\mathfrak{M}_{L\mu}$, where μ is the projection of L in some given direction.

To investigate the excitation of rotational states of the daughter nucleus in β decay it is convenient to express $\mathfrak{M}_{L\mu}$ in terms of multipole operators $\mathfrak{M}'_{L\nu}$ defined in the coordinate system fixed in the nucleus; we use the transformation

$$\mathfrak{M}_{L\mu} = \sum_\nu \mathfrak{M}'_{L\nu} D^L_{\nu\mu}(\theta_i). \tag{2.1}$$

Final rotational states of the even-even daughter nucleus can be represented in the adiabatic approximation by the wave functions

$$\psi_{Jmi} = \sum_K A_{Ki} \Phi_{JK}, \tag{2.2}$$

where

$$\Phi_{JK} = \varphi_J(r)\left[\frac{2J+1}{16\pi^2(1+\delta_{0K})}\right]^{1/2} \times (D^J_{mK} + (-1)^J D^J_{m,-K}), \tag{2.3}$$

and $\varphi_J(r)$ is an internal-state wave function of the daughter nucleus. The coefficients A_{Ki} which determine the rotational-state wave functions were calculated in references 5 and 6.

The initial-state wave function of a parent nucleus with spin **I** is represented in the adiabatic approximation by

$$\psi_{IM} = \varphi_I(r)\sum_K a_K D^I_{MK}. \qquad (2.4)$$

The reduced probability of a β transition with angular momentum L carried away by the electron and antineutrino is given by

$$B(L;\ I\to Ji) = \frac{1}{2I+1}\sum_{m\mu M} |(Jmi\,|\,\mathfrak{M}_{L\mu}\,|\,IM)|^2. \qquad (2.5)$$

Substituting (2.1), (2.2), and (2.4) into (2.5) and integrating over the Euler angles, we obtain (2.5) in the form of two factors, one of which depends on $\mathfrak{M}'_{L\nu}$ and the wave functions representing internal motion, while the other factor depends on the parameters a_K and A_{Ki} and the coefficients of vector addition which are dependent on the quantum numbers IKJK′L.

We consider the ratio of reduced probabilities for β transitions from a given initial state of the parent nucleus to different rotational states associated with the same internal state of the daughter nucleus. When

$$L < \begin{cases} \min(K+K'), & \text{for } K\neq 0,\ K'\neq 0,\\ \min(K+K''), & \text{for } K\neq 0,\ K''\neq 0,\end{cases} \qquad (2.6)$$

this ratio will not contain expressions relating to the internal state. Thus

$$B(L;\ I\to J'i')\,/\,B(L;\ I\to J''i'') = \frac{\left|\sum_{KK'} a_K A^{J'}_{K'i'}(ILKK'-K\,|\,J'K')\right|^2}{\left|\sum_{KK''} a_K A^{J''}_{K''i''}(ILKK''-K\,|\,J''K'')\right|^2}. \qquad (2.7)$$

At the present time we unfortunately do not know the coefficients a_K which determine the wave functions (2.4) of odd-odd nuclei. In some instances we must therefore express the initial-state wave function of the parent nucleus by

$$\psi_{IMK} = \Phi(r)\,D^I_{MK}.$$

The probability ratio (2.7) now becomes

$$B(L;\ IK\to J'i')\,/\,B(L;\ IK\to J''i'') = \frac{\left|\sum_{K'} A^{J'}_{K'i'}(ILKK'-K\,|\,J'K')\right|^2}{\left|\sum_{K''} A^{J''}_{K''i''}(ILKK''-K\,|\,J''K'')\right|^2}, \qquad (2.8)$$

if K = 0 or if (2.6) is fulfilled.

Finally, in a still rougher approximation, when definite values of the quantum number K′ are assigned to the rotational states, the branching ratio (2.7) reduces to the following ratio that was derived in reference 1:

$$B(L;\ IK\to J'K')\,/\,B(L;\ IK\to J''K'') = (ILKK'-K\,|\,J'K')^2\,/\,(ILKK'-K\,|\,J''K')^2, \qquad (2.9)$$

when (2.6) is fulfilled or when K = 0 or K′ = 0.

3. COMPARISON WITH EXPERIMENTS

It is well known that the probabilities of allowed β transitions are characterized conveniently by the quantity τf_0, where τ is the half-life and f_0 is the Fermi function which represents the integral of the electron energy distribution function. The Fermi function depends on the transition energy, nuclear charge and charge of the emitted particle.

The product τf_0 is inversely proportional to the square of the absolute value of the matrix element corresponding to an allowed transition (L = 0 or 1 with unchanged parity). In the general case of forbidden β transitions this simple relation does not exist; for these transitions it is evidently essential to use a combination of different types of interaction whose relative contributions are not easily determined. The so-called unique β transitions are exceptions.

In the case of unique n-th forbidden transitions (L = n + 1, with changing parity when n is odd) τf_n is inversely proportional to the square of the transition matrix element, where f_n is the integral of the electron energy distribution function for the given type of decay.

Davidson[7] has shown that, neglecting the influence of the Coulomb field, we may use the approximation

$$f_n = \bar{c}_n f_0, \qquad (3.1)$$

where c_n depends on the transition energy E_0. In a rough approximation

$$\bar{c}_n \approx \{E_0^2 - m^2c^4\}^n, \qquad (3.2)$$

Thus for allowed transitions (n = 0) or unique forbidden transitions it follows from the fact that $(\tau f_n)^{-1}$ is proportional to the square of the absolute value of the transition matrix element that the ratio of the values of τf_n for two β transitions from the same initial state of the parent nucleus to different rotational states of the daughter nucleus will be given by

$$\tau f_n(I\to J'i')\,/\,\tau f_n(I\to J''i'') = B(L;\ I\to J''i'')\,/\,B(L;\ I\to J'i'), \qquad (3.3)$$

where L = n + 1 and the probability ratio must be calculated by means of (2.7) or the approximate formulas (2.8) and (2.9).

In comparing the experimental results with the theory it must be remembered that the experimental values of τf usually represent τf_0, since the Fermi formulas for allowed transitions are used to calculate f.

We shall now apply the foregoing results to the decay of Re^{186}. Figure 1 shows the decay scheme (reference 9, p. 539). The maximum electron energy (in kev) and the value of $\log_{10}(\tau f_0)$ are given near each arrow representing a transition. The spin and energy in kev are given for each rotational level. According to reference 10 the ground state of Re^{186} has zero spin and negative parity, and the transition to the first excited level of Os^{186} corresponds to $L = 2$ with changed parity, i.e., it is a unique singly forbidden transition. This conclusion is supported by the large value of $\log_{10}(\tau f)$ and the shape of the β spectrum, which results in a straight-line Kurie plot when the shape correction factor for a unique singly-forbidden transition is taken into account.

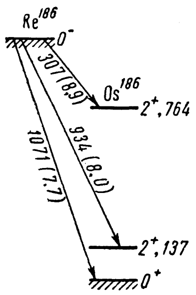

FIG. 1

Since according to the theory of non-axisymmetric nuclei the second excited 2^+ level also represents rotational excitation of Os^{186} the relative probabilities of β decay to these two levels can be calculated. From the energy ratio (5.56) of these two levels it follows that $\gamma = 18°$. Therefore, following reference 5, the wave functions of the rotational levels can be written as

$$\psi_{21} = 0.997\,\Phi_{20} + 0.065\,\Phi_{22},$$
$$\psi_{22} = -0.065\,\Phi_{20} + 0.997\,\Phi_{22}. \qquad (3.4)$$

Using (3.4) and the fact that the Re^{186} ground-state wave function corresponds to $J = K = 0$, we obtain the β-transition branching ratio from (2.7):

$$B(2; 0 \to 21)/B(2; 0 \to 22) = 1.3. \qquad (3.5)$$

The ratio $\tau f_0(0 \to 22)/\tau f_0(0 \to 21) = 7.94$ is obtained experimentally. Using (3.1) and (3.2), we obtain

$$\tau f_1(0 \to 22)/\tau f_1(0 \to 21) = 1.47.$$

According to (3.3) this ratio corresponds to the theoretical branching ratio (3.5).

Our second example is the decay of Np^{238}; the decay scheme is shown in Fig. 2 (see reference 9, p. 730), with the same notation as in Fig. 1. The energy ratio of levels with spin 2 is 23, which corresponds to $\gamma = 8°$. The rotational-state wave functions can be obtained from references 5 and 6.

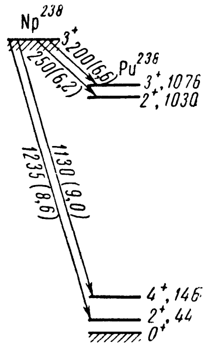

FIG. 2

The ground-state wave function of Np^{238}, with spin 3, is unknown and will be represented by the very simple form

$$\psi_I = \varphi_I\,(D^3_{m3} + \delta D^3_{m0}),$$

where δ is a parameter denoting the fractional share in the ground state of Np^{238} taken by states with $K = 0$ (for simplicity other possible values of K are disregarded). Branching ratios calculated from (2.7) for allowed β transitions (with $L = 1$) to levels of the rotational band of Pu^{238} are given in Table I for two values of δ, together with the inverse ratios of τf_0 for the same transitions.

TABLE I

j	$B(1;\,3\to j)/B(1;\,3\to 20)$ $\delta = 0$	$B(1;\,3\to j)/B(1;\,3\to 20)$ $\delta = 0,047$	$\dfrac{\tau f_0\,(3\to 20)}{\tau f_0\,(3\to j)}$
41	0	2	0.4
22	10^4	800	250
3	10^3	100	100

We shall now consider the β decay of Eu^{154}. According to Juliano and Stephens[8] the decay of Eu^{154} to two excited levels of Gd^{154} having spin 2 and energies 123 and 998 kev, corresponds to the values 12.9 and 11.6, respectively, for $\log_{10}(\tau f_0)$. We obtain $\gamma = 14°$ from the energy ratio of these levels; the wave functions of both rotational states can then be determined from reference 5. The ground state of Eu^{154} has spin 3; assuming that its wave function is $\psi_{3m} = \varphi_3 (D^3_{m3} + \delta D^3_{m0})$, from (2.7) with $\delta = -0.26$ we obtain for β transitions with $L = 1$: $B(1;\ 3 \rightarrow 22) B^{-1}(1;\ 3 \rightarrow 20) = 20$, which agrees with the experimental ratio $\tau f_0 (3 \rightarrow 20)/\tau f_0 (3 \rightarrow 22) \approx 20$. With $\delta = 0$ this ratio would be 2500, and if K is a good quantum number for levels with spin 2 the ratio would become infinite.

We finally consider the excited rotational levels of Os^{190} resulting from the β decay of Re^{190} and K capture in Ir^{190}. The decay scheme is shown in Fig. 3 (see reference 9, p. 550). According to references 5 and 6 the rotational-state wave functions of Os^{190} can be obtained by assuming $\gamma = 21°$.

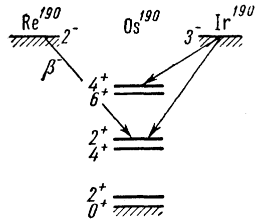

FIG. 3

Taking the wave functions φD^2_{m2} and φD^3_{m3} for the ground states of Re^{190} and Ir^{190}, respectively, (2.7) can be used to calculate the branching ratio for β decay $(L = 1)$ and K capture $(L = 1)$ to different rotational levels of Os^{190}. The results of these calculations are given in Table II, where the unit of measurement is the reduced probability of decay to the second excited level having spin 2.

Table II gives a qualitative explanation of the experimental findings that in β decay of Re^{190} only rotational level 22 is excited and that levels 22 and 42 are excited through K capture in Ir^{190}.

TABLE II

j	$\frac{B(1;\ 2\rightarrow j)}{B(1;\ 2\rightarrow 22)}$, $Re^{190}\rightarrow Os^{190}$	$\frac{B(1;\ 3\rightarrow j)}{B(1;\ 3\rightarrow 22)}$, $Ir^{190}\rightarrow Os^{190}$
41	0	$6\cdot 10^{-4}$
21	$7.6\cdot 10^{-3}$	$8\cdot 10^{-4}$
61	0	0
42	0	$7.4\cdot 10^{-2}$

The foregoing examples serve to illustrate the way in which (2.7) can be used. Ignorance of the wave function (2.4) of the parent nucleus (when $I \neq 0$) prevents a complete comparison of theory and experiment. When experimental values of τf_0 are available for allowed β transitions to different rotational levels of the daughter nucleus, (2.7) can be used to calculate the coefficients a_K, which determine the dependence of the parent-nucleus wave function on the Euler angles.

[1] Alaga, Alder, Bohr, and Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 9 (1955).

[2] B. T. Geĭlikman, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 989 (1958), Soviet Phys. JETP **8**, 690 (1959).

[3] D. A. Zaikin, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 529 (1958), Soviet Phys. JETP **8**, 365 (1959).

[4] A. S. Davydov and G. F. Filippov, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1497 (1959), Soviet Phys. JETP **9**, 1061 (1959).

[5] A. S. Davydov and G. F. Filippov, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 440 (1958), Soviet Phys. JETP **8**, 303 (1959).

[6] A. S. Davydov and V. S. Rostovskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1788 (1959), Soviet Phys. JETP **9**, 1275 (1959).

[7] J. Davidson, Phys. Rev. **82**, 48 (1951).

[8] J. O. Juliano and F. Stephens, Phys. Rev. **108**, 341 (1958).

[9] B. S. Dzhelepov and L. K. Peker, Схемы распада радиоактивных ядер (Decay Schemes of Radioactive Nuclei), U.S.S.R. Acad. Sci. Press, 1958.

[10] Guss, Killion, and Porter, Phys. Rev. **95**, 627 (1954).

Translated by I. Emin
22