# CAPTURE OF $\mu^-$ MESONS BY LIGHT NUCLEI

B. L. IOFFE

Institute for Theoretical and Experimental Physics, Academy of Sciences U.S.S.R.

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Different effects in the capture of $\mu^-$ mesons by light nuclei, without neutron or proton emission, are calculated. It is assumed that the interaction is described by the A and V coupling variants. Corrections of first order in nucleon velocity and in $(R/\lambda\!\!\!^-)^2$ are taken into account and turn out to be quite significant. The results of the calculation contain one unknown constant, in addition to the usual matrix element, so that at least two experiments are necessary to verify the theory.

AT the present time there are all grounds for assuming that the interaction of $\mu$ mesons with nucleons is analogous to $\beta$ decay and is described by the axial and vector coupling. We consider within this assumption the capture of $\mu^-$ mesons by light nuclei, in which the nucleus jumps into some excited level without emitting either a neutron or a proton.* If the changes in momentum and parity of the nucleus in such a transition have the form $\Delta j = 0, \pm 1$ (no), then the theory of this process is completely analogous to the theory of $\beta$ decay. The essential difference is that the momentum of the emitted neutrino is of order $m_\mu$ (the mass of the muon), so that the corrections after expanding in powers of $R/\lambda\!\!\!^-$ (R is the nuclear radius and $\lambda\!\!\!^-$ the wavelength of the neutrino), and v/c of the nucleon, which are usually small in $\beta$ decay, are very significant here. At the same time, if one confines oneself to muon capture in nuclei with $A \lesssim 16$, then the magnitude of these corrections is still quite small and it is sufficient (error $\sim 5\%$) to consider terms linear in the nucleon velocity and quadratic in $R/\lambda\!\!\!^-$.

1. We start from the universal theory of weak interactions of Gell-Mann and Feynman[4] in which the $\beta$ current of the nucleons and pions is conserved and the vector interaction is completely analogous to the electrodynamics. As shown by Gell-Mann,[3] this makes possible expressing the corrections with respect to v/c of the nucleons and $R/\lambda\!\!\!^-$ associated with the vector interaction in terms of the characteristics of the nuclei during $\gamma$ transitions. The matrix element of the vector interaction, to an accuracy linear with respect to v/c of the nucleons, can be written

$$G_V \int dV \psi_f^* \Big\{ J_0 e^{-i\mathbf{qr}} - \frac{\mu_V}{2M} \sigma [\nabla \mathbf{J} e^{-i\mathbf{qr}}] + \frac{i}{2M} (\nabla \mathbf{J} e^{-i\mathbf{qr}} + \mathbf{J} e^{-i\mathbf{qr}} \nabla) \Big\} \tau \psi_i,$$

$$J_0 = {}^1/_2 \overline{\psi}_\nu \beta (1+\gamma_5) \psi_\mu, \qquad \mathbf{J} = {}^1/_2 \overline{\psi}_\nu \beta \boldsymbol{\alpha} (1+\gamma_5) \psi_\mu, \qquad (1)$$

where $\psi_i$ and $\psi_f$ are the initial and final wave functions of the nucleus, $\mathbf{q}$ is the momentum of the neutrino, and M is the nucleon mass. The first term in (1) is analogous to an electric interaction with the nuclear charge distribution, the second term to the interaction of the magnetic field with the spin $\mu\boldsymbol{\sigma}\cdot\mathbf{H}$, and the third to the interaction of the electromagnetic field with the current. The quantity $e^{-i\mathbf{q}\cdot\mathbf{r}}$ in (1) is expanded in powers of $\mathbf{q}\cdot\mathbf{r}$ up to the second order. Then the first term in (1) becomes:

$$G_V \int dV \psi_f^* \Big(1 - \tfrac{1}{6} q^2 r^2 - \tfrac{1}{6} q_i q_k Q_{ik}\Big) \tau \psi_i J_0;$$

$$Q_{ik} = 3 r_i r_k - r^2 \delta_{ik}. \qquad (2)$$

Estimates based on a square-well potential[5,6] (and also experimental data on the magnitude of the quadrupole moment) show that the contribution from the quadrupole interaction in (2) does not exceed a few per cent, and, consequently, the term proportional to $Q_{ik}$ can be omitted. An expansion of the second term of (1) in powers $\mathbf{q}\cdot\mathbf{r}$ contains no term linear in $\mathbf{r}$, by virtue of the selection rules, while the quadratic term is of the order of $^1/_6 (m_\mu R)^2 (m_\mu/M) \lesssim 5\%$ and can also be neg-

*Analogous calculations for the capture of $\mu$ mesons by protons was carried out by Wolfenstein[1] and Chou Kuang Chao and Maevskiĭ.[2] The former, however, did not take into account the corrections due to the vector interaction (of the "weak magnetic" type[3]) while the latter, on the contrary, did not take into account the pseudoscalar term, which has considerable practical importance.

lected. Finally, in the third term it is sufficient to keep only the expression in which the operator $\nabla$ acts on the current of light particles

$$i\nabla \mathbf{J} e^{-i\mathbf{qr}} = qJ_0.$$

The remaining expressions correspond either to the contribution of the orbital moment to the total magnetic moment of the transition in the nucleus, i.e., they have the same form as the second term in (1) and can be treated like it, or else they are of the order of $\frac{1}{6}(m_\mu R)^2(m_\mu/M)$. The entire matrix element of (1) takes the form

$$G_V\left\{\int dV\psi_f^*\tau\psi_i\left(1-\frac{1}{6}q^2\langle r^2\rangle_e+\frac{q}{2M}\right)J_0 + i\frac{\mu_V}{2M}\int dV\psi_f^*\boldsymbol{\sigma}\tau\psi_i[\mathbf{q}\times\mathbf{J}]\right\}, \quad (3)$$

where $\mu_V$ can be interpreted as the magnetic moment of the transition, while $\langle r^2\rangle_e$ is the square radius of the nucleus corresponding to the given transition. The quantity $\mu_V$ is determined, according to Gell-Mann,[3] from the width of the nuclear $\gamma$-line belonging to the same isotopic multiplet, while $\langle r^2\rangle_e$ can be calculated from experimental data on the inelastic scattering of electrons on nuclei.

**2.** The matrix element for the axial interaction can be written

$$\int dV\overline{\psi}_f\{G_A\gamma_\lambda\gamma_5 + b(k-q)_\lambda\gamma_5\}\tau\psi_i J_\lambda e^{-i\mathbf{qr}}, \quad (4)$$

where

$$J_\lambda = \{J_0, \mathbf{J}\}, \quad \gamma_\lambda = \{\beta, \beta\boldsymbol{\alpha}\}, \quad q_\lambda J_\lambda = q_0 J_0 - \mathbf{qJ},$$

and k is the 4-momentum of the meson.* The pseudoscalar interaction — the second term in (4) — arises in the form of a correction to the axial interaction, owing to capture of the $\mu$ meson by the virtual $\pi$ mesons.[1] The constant b was determined by Goldberger and Treiman[7] and found to be very large: $bm_\mu = 8G_A$, making allowance for the pseudoscalar term absolutely necessary. By virtue of the identities

$$(k-q)_\lambda J_\lambda = \frac{1}{2}m_\mu\overline{\psi}_\nu(1-\gamma_5)\psi_\mu$$

and $\beta\psi_\mu = \psi_\mu$ ($\mu$ meson at rest) it is possible to write (4) in the form:

$$\int dV\psi_f^*\{G_A\boldsymbol{\sigma}\mathbf{J} + [G_A\gamma_5 + bm_\mu\beta\gamma_5]J_0\}\tau\psi_i e^{-i\mathbf{qr}}. \quad (5)$$

In the nonrelativistic approximation with respect to the nucleon, both matrix elements $\psi_f^*\gamma_5\psi_i$ and $\psi_f^*\beta\gamma_5\psi_i$ must be proportional to the same expression $\psi_f^*\boldsymbol{\sigma}\cdot\psi_i\mathbf{q}$. For free nucleons we have

$$\psi_f^*\gamma_5\psi_i = (1/2M)\psi_f^*\boldsymbol{\sigma}\psi_i\mathbf{q}, \qquad \psi_f^*\beta\gamma_5\psi_i = -(1/2M)\psi_f^*\boldsymbol{\sigma}\mathbf{q}\psi_i.$$

Therefore, the entire expression in the square brackets in (5) can be written

$$\int dV\psi_f^*(G_A\gamma_5 + bm_\mu\beta\gamma_5)\tau\psi_i J_0 e^{-i\mathbf{qr}} = -\rho G_A\int dV\psi_f\boldsymbol{\sigma}\mathbf{q}\tau\psi_i J_0 e^{-i\mathbf{qr}}, \quad (6)$$

where $\rho$ is some constant, whose magnitude depends on the magnitude of the spin-orbit interaction $V\boldsymbol{\sigma}\cdot\boldsymbol{l}$ of the nucleon in the nucleus: $\rho \sim VR^2bm_\mu/G_A \sim (3\text{ to }5)/M$. (For free nucleons $\rho = 7/2M$.) The expansion in powers of $R/\lambda\!\!\!^{-}$ is made in (5) in a manner analogous to that used in the vector interaction. We obtain as a result the following expression for the matrix element for the axial interaction:

$$G_A\int dV\psi_f^*\boldsymbol{\sigma}\tau\psi_i\left\{\left(1-\frac{1}{6}q^2\langle r^2\rangle_A\right)\mathbf{J} - \rho\mathbf{q}J_0\right\}, \quad (7)$$

where $\langle r^2\rangle_A$ is the mean square radius of the nucleus in the transition due to axial interaction. In so far as the matrix element $\psi_f^*\boldsymbol{\sigma}\psi_i$ has the same form as the matrix element of a magnetic dipole transition, then a better approximation is obtained if $\langle r^2\rangle_A$ is taken equal to the square of the magnetic radius in the transition of a nucleus belonging to the same isotopic multiplet.

**3.** Making use of matrix elements (3) and (7), we calculate the total probability for $\mu^-$ capture and the magnitude of the longitudinal polarization of the nucleus in capture of an unpolarized meson, as well as the angular distribution and polarization of the nucleus in the direction of the meson spin, in capture of a polarized meson. It is possible to carry out the calculations by the same technique as described in reference 8. The total capture probability is:

$$W = (1/2\pi^2)a_\mu^{-3}q^2N_0,$$

$$N_0 = G_V^2|M_F|^2\left(1-\frac{1}{3}q^2\langle r^2\rangle_e + q/M\right) + G_A^2|M_{GT}|^2\left[1-\frac{1}{3}q^2\langle r^2\rangle_A + \frac{2}{3}(2a-\rho)q\right], \quad (8)$$

where $a_\mu$ is the radius of the Bohr orbit of the meson, $M_F = (\int 1)$ and $M_{GT} = (\int\boldsymbol{\sigma})$ are the nuclear matrix elements, and $a = \mu_V G_V/2MG_A$. The probability $w(\mu')$, that on capture of an unpolarized meson the nucleus can have a spin $\mu'$ along the direction of the unit vector $\mathbf{n}_j$ is given by the formula

*In our notation $G_V$ and $G_A$ are positive; b is also positive.[7]

$$w(\mu') = 1 + Q_{j'j}\left[\tfrac{1}{3} - (\mathbf{n}_j\boldsymbol{\nu})^2\right](N'/N_0) + \boldsymbol{\nu}\mathbf{n}_j(\mu'/j')(N_\nu/N_0), \qquad Q_{j'j} = \Lambda_{j'j}\left(\frac{3\mu'^2}{j'(j'+1)} - 1\right),$$

$$\Lambda_{j'j} = \frac{j'+1}{2j'-1}\begin{cases} 1, & j = j'-1 \\ -(2j'-1)/(j'+1), & j = j' \\ j'(2j'-1)/(j'+1)(2j'+3), & j = j'+1 \end{cases}. \qquad (9)$$

Here

$$N' = -G_A^2 |M_{GT}|^2 (\rho + a) q,$$

$$N_\nu = G_A^2 |M_{GT}|^2 \lambda_{j'j}\left(1 + 2aq - \tfrac{1}{3} q^2 \langle r^2 \rangle_A\right) + 2\delta_{jj'}\sqrt{j/(j+1)}\, G_A G_V \operatorname{Re} M_F M_{GT}^* \left[1 - \rho q - \tfrac{1}{6} q^2 (\langle r^2\rangle_e + \langle r^2 \rangle_A) + q/2M\right],$$

$$\lambda_{j'j} = [j'(j'+1) - j(j+1) + 2]/2(j'+1), \qquad (10)$$

j and j′ are the spins of the initial and final nuclei, respectively, and $\boldsymbol{\nu} = \mathbf{q}/q$. Upon capture of a polarized meson, the angular distribution of the recoil nuclei has the form

$$w = 1 - \langle \boldsymbol{\sigma} \rangle \boldsymbol{\nu} N_3 / N_0, \qquad (11)$$

and the polarization of the final nucleus, averaged over the directions of the emitted neutrino, is determined by

$$w_\sigma(\mu') = 1 + \mathbf{n}_j \langle \boldsymbol{\sigma} \rangle (\mu'/j')(N_1/N_0), \qquad (12)$$

where $\langle \sigma \rangle$ is the expectation value of the spin of the $\mu$ meson in the K-orbit, and:

$$N_3 = G_V^2 |M_F|^2 [1 - {}^1/_3 q^2 \langle r^2 \rangle_e + q/M] - {}^1/_3 G_A^2 |M_{GT}|^2 [1 - {}^1/_3 q^2 \langle r^2\rangle_A + 2(a+\rho) q],$$

$$N_1 = G_A^2 |M_{GT}|^2 \lambda_{j'j} [1 + {}^2/_3 (2a - \rho) q - {}^1/_3 q^2 \langle r^2 \rangle_A] - 2\delta_{jj'}\sqrt{j/(j+1)}\, G_A G_V \operatorname{Re} M_F M_{GT}^* [1 + q/2M + (q/3)(2a - \rho) - (q^2/6)(\langle r^2 \rangle_e + \langle r^2 \rangle_A)]. \qquad (13)$$

In the calculation of (8) to (13), only terms linear in v/c of the nucleons and in $(qr)^2$ were taken into account. The hyperfine structure of the mesic atom was not taken into account. Therefore, strictly speaking,[9] our results are correct only for the capture of muons by nuclei with zero spin. If we pass in (8) to (13) to the case of $\mu$ capture by protons ($M_F = 1$, $M_{GT} = \sqrt{3}$) and neglect the pseudoscalar term ($\rho = -1/2M$), then the equations agree with the results of reference 2. In neglecting terms $\sim (v/c)_{nuc}$ and $\sim (R/\lambda\!\!\!^-)^2$, relations (8) to (11) become the usual relations for K capture (see for example, reference 10). However, these terms are quite significant.

**4.** We consider the more interesting case of the capture of $\mu^-$ mesons in $C^{12}$, with production of $B^{12}$, which then $\beta$-decays back into $C^{12}$. The quantity a for the transition $C^{12} \rightarrow B^{12}$ (transition $0 \rightarrow 1$, no) was calculated by Gell-Mann:[3] $a = 2.34 \pm 0.25/M$. Taking $\langle r^2 \rangle_A^{1/2}$ to be the electrostatic radius of the nucleus* $C^{12}$, see reference 11 ($\langle r^2 \rangle_A^{1/2} = 2.4 \times 10^{-13}$ cm) and q = 90 Mev, we find $q^2 \langle r^2 \rangle_A/3 = 0.37$, $4aq/3 = 0.3$. The constant $\rho$ cannot be calculated theoretically. It is possible to determine it from experimental data[12] on the lifetime of the $\mu$ mesons relative to capture by $C^{12}$, according to which $N_0/G_A^2 |M_{GT}|^2 = 1.37 \pm 0.08$. Hence $2\rho q/3 = -0.44$, which agrees with the theoretical estimate ($2q\rho/3 = 0.95$ for free protons) and leads one to believe that the pseudoscalar term has the order of magnitude predicted by Goldberger and Treiman.[7]†

Since the theory includes one more constant to be determined from experiment, besides the constants and matrix elements that are known from $\beta$ decay, a verification of the theory, and, in particular, a verification of the hypothesis of renormalizability of the vector interaction can be achieved if at least one more experiment is performed in addition to measuring the capture probability. Measurement of the polarization of the $B^{12}$ nucleus along the direction of the spin of the $\mu$ meson in the capture of a polarized meson in $C^{12}$ is unsuitable for this purpose, since it follows from (13) and (8) that the magnitude of this polarization in the Gamow-Teller transition does not depend on terms containing a or $\rho$. (The absence of such a dependence denotes, on the other hand, that measurement of the polarization of $B^{12}$ may be a good verification of the two-component $\mu$-meson hypothesis in $\mu$ capture, see reference 10). The hypothesis of renormalizability of the vector interaction can be verified in experiments on the measurement of the longitudinal polarization of the nucleus in capture of unpolarized mesons or by measuring the angular distribution of the recoil nuclei in capture of polarized mesons. In both these cases the effect of the terms proportional to a and $\rho$ is sufficiently large,

*It follows from the experimental data of Fregeau[11] that the magnitude of the radius of the $C^{12}$ nucleus changes very little on passing from elastic to inelastic processes.

†It is somewhat strange that $\rho$ appears to be negative for $\mu^-$ capture in $C^{12}$, and positive for the capture by protons. Estimates with relativistic-interaction models indicate that $\rho < 0$ might arise with the same interactions that cause spin-orbit coupling, but should not be as negative as is required by experiment.[12]

reaching 30 to 50%. The polarization of the product nuclei can be determined experimentally from the asymmetry of the electrons of the subsequent $\beta$ decay. The coefficient of asymmetry of the electrons contains also a correction term $\sim (v/c)_{nuc}$. This correction, however, is very small, of the order of 2 or 3%.

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