SOVIET PHYSICS JETP VOLUME 37 (10), NUMBER 1 JANUARY, 1960

COLLECTIVE LOSSES OF FAST ELECTRONS IN PASSAGE THROUGH MATTER

V. P. SILIN

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor February 23, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 273-282 (July, 1959)

The energy losses of fast electrons passing through thin films are considered from the viewpoint of the theory developed by Frank, Tamm, and Fermi,[1] taking into account spatial dispersion of the dielectric constant.[2] An expression is derived for the longitudinal dielectric constant which takes account of exchange effects in a high density electron gas. The phenomenological theory developed by Landau[3] is used to determine the longitudinal and transverse dielectric constants of a degenerate electron fluid. It is shown that these quantities have singularities corresponding to the propagation of zero sound. The fast-electron losses associated with the excitation of both transverse and longitudinal zero sound are discussed. The dependence of the discrete losses on the scattering angle of fast particles in passage through optically anisotropic bodies is considered.

1. The passage of fast electrons through thin films is characterized by so-called characteristic or discrete energy losses.[4] The electrons lose energy in discrete amounts or quanta; in certain cases this effect results in clearly defined spectral loss lines. The connection pointed out by Bohm and Pines between the characteristic losses and the collective oscillations of electrons in a solid body[5] is important for an understanding of the nature of these losses. The excitation of these collective oscillations corresponds to the radiation of electromagnetic waves in the medium by the charged particle. In absorbing media these waves decay rapidly and there is a transfer of energy from the fast particle to the medium via the collective oscillations. In general, longitudinal electromagnetic fields can be set up in addition to the transverse fields. It is these longitudinal oscillations which are responsible for the collective polarization losses of fast electrons.

A theory for the energy losses of fast particles, based on the excitation of electromagnetic fields in a medium has been developed by Frank and Tamm in connection with the theory of Cerenkov radiation and by Fermi in connection with polarization losses (cf. reference 1). This theory was extended by Fröhlich and Pelzer[6] in order to analyze discrete losses. In this analysis the position of the loss lines is determined by the zeros of the dielectric constant $\epsilon(\omega)$ corresponding to longitudinal oscillations of the electromagnetic field. A more precise formula for the loss line is

$$\operatorname{Im}\frac{1}{\varepsilon(\omega)} = \frac{2n\varkappa}{(n^2+\varkappa^2)^2},$$

where n is the refractive index and κ is the absorption constant.*

A theory for the energy losses of fast charged particles which takes account of spatial dispersion, which is a generalization of the theory developed by Frank, Tamm, and Fermi, has been developed by a number of authors.[2]†

Spatial dispersion means that in an isotropic medium the dielectric constant becomes a tensor, assuming the form

*The macroscopic theory developed by Frank and Tamm, and Fermi for fast charged particles applies only when the impact parameters for the collisions between the fast electrons in the medium are much greater than the interatomic distances. However, this condition no longer holds when the fast-electron scattering angle becomes large compared with the velocity ratio v_0/v (v_0 is the velocity of the electrons in the medium and v is the velocity of the fast electrons).

It has been found experimentally that in scattering at such angles the energy associated with the discrete losses becomes a function of the scattering angle.[7] According to the microscopic theory[8,9] for longitudinal oscillations of the electromagnetic field (i.e., plasma oscillations) this angular dependence is natural since it indicates the dependence of the frequency of the longitudinal oscillations on wavelength. In the language of the macroscopic electrodynamic equations for a medium this corresponds to spatial dispersion of the dielectric permittivity (cf., for example, reference 10).

†This work is very similar to that in which various concrete models are used to represent the medium; however, no connection has been established between particle energy losses and the dielectric constant.[8,11]

$$\varepsilon_{ij}(\omega, \mathbf{k}) = \varepsilon^{tr}(\omega, k)\left\{\delta_{ij} - \frac{k_ik_j}{k^2}\right\} + \frac{k_ik_j}{k^2}\varepsilon^l(\omega, k), \quad (2)$$

where ϵ^{tr} and ϵ^l are the transverse and longitudinal dielectric constants. Using Eq. (2), we obtain the following expression for the energy losses of a fast charged particle in passage through a medium (per unit path length)

$$W = \frac{ie^2Z^2}{\pi v^2}\int_{-\infty}^{+\infty}\omega d\omega\int_0^{\infty} q dq \frac{1}{q^2+\omega^2/v^2}\left\{\frac{1}{\varepsilon^l(\omega, \sqrt{q^2+\omega^2/v^2})} - \frac{v^2}{c^2}\frac{q^2}{q^2+\omega^2[v^{-2}-c^{-2}\varepsilon^{tr}(\omega, \sqrt{q^2+\omega^2/v^2})]}\right\}, \quad (3)$$

where Ze is the charge of the particle and v is its velocity. The first term in the curly brackets is responsible for the radiation of longitudinal waves while the second is responsible for the transverse Cerenkov radiation. Since the usual collective losses for fast electrons are associated with the excitation of longitudinal waves, in what follows we will concentrate our attention on the first term in Eq. (3).

From Eq. (2) we obtain the following expression for the scattering probability of a fast particle through an angle $d\theta$ with the emission of a longitudinal photon (plasmon) in the frequency interval $d\omega$ (per unit path length):

$$\frac{dW^l}{\hbar\omega\, d\Omega\, d\omega} = \frac{2e^2Z^2}{\pi\hbar v^2}\frac{1}{\theta^2+(\hbar\omega/vp)^2}\,\mathrm{Im}\frac{1}{\varepsilon^l(\omega, \sqrt{(p\theta/h)^2+(\omega/v)^2})}, \quad (4)$$

where p is the momentum of the particle and it is assumed that $\theta \ll 1$.*

Since the experimentally measured quantity is $\mathrm{Im}\{1/\epsilon^l(\omega, \mathbf{k})\}$, it is this quantity which must be predicted by a collective-loss theory that uses a model to describe the behavior of the electrons in the medium. Below we obtain expressions for ϵ^l for a high-density electron gas and for an electron liquid and also discuss certain features of the energy losses of fast particles due to the excitation of zero sound.

2. The most detailed studies of the longitudinal dielectric constant of a degenerate electron gas have been carried out in the self-consistent Hartree approximation.[2,8,9,12] In this analysis, in the long wavelength region, where spatial dispersion may be neglected, we have

$$\varepsilon^l(\omega, k) = 1 - \omega_0^2/\omega^2 - 3p_0^2k^2/5m^2\omega^2, \quad (5)$$

where p_0 is the momentum at the Fermi surface.

However, the self-consistent field approximation is valid only for a degenerate electron gas of high density.[12,13] Because real electron densities in solid bodies are not very great, it is convenient to determine the corrections to the work in reference 5 by means of an expansion in inverse powers of the density. The exchange correction can be obtained if we use the Hartree-Fock approximation instead of the Hartree self-consistent field.[8,9] In this case the electron distribution function is given by the equation[13,14]

$$\frac{\partial\delta f}{\partial t} + \frac{\mathbf{p}}{m}\frac{\partial\delta f}{\partial\mathbf{r}} + \frac{1}{2}\int d\mathbf{p}'\frac{4\pi e^2\hbar^2}{|\mathbf{p}-\mathbf{p}'|^2}\left[\frac{\partial f_0}{\partial\mathbf{p}}\frac{\partial}{\partial\mathbf{r}}\delta f(\mathbf{p}', \mathbf{r}) - \frac{\partial f_0}{\partial\mathbf{p}'}\frac{\partial}{\partial\mathbf{r}}\delta f(\mathbf{p}, \mathbf{r})\right] + e\mathbf{E}\frac{\partial f_0}{\partial\mathbf{p}} = 0, \quad (6)$$

where δf is the non-equilibrium correction to the distribution function, f_0 is the equilibrium distribution function for an ideal Fermi gas, and E is the electric field, which is given by the equation

$$\mathrm{div}\,\mathbf{E} = 4\pi e\int d\mathbf{p}\,\delta f. \quad (7)$$

We assume that the wavelengths are large compared with the distances between particles.

In order to determine the longitudinal dielectric constant by means of Eq. (6), it is necessary to express δf in terms of the electric field (the dependence of the field on coordinates and time is of the form $e^{-i\omega t + i\mathbf{kr}}$); then, from Eq. (7) we have

$$4\pi e\int d\mathbf{p}\,\delta f = \{1 - \varepsilon^l(\omega, k)\}\, i\mathbf{kE}, \quad (8)$$

whence ϵ^l can be found. In the long-wavelength region the quantity being sought can be given as an expansion in powers of k. It is not difficult to show that Eqs. (7) and (8) yield:

$$\varepsilon^l(\omega, k) \approx 1 - \frac{\omega_0^2[1 + {}^3/_5(v_0k/\omega^2)]}{\omega^2 + (e^2/3\pi\hbar v_0)v_0^2k^2} \approx 1 - \frac{\omega_0^2}{\omega^2}\left\{1 + \frac{3p_0^2k^2}{5m^2\omega^2}\left[1 - \frac{1}{16}\left(\frac{\hbar\omega_0}{p_0^2/2m}\right)^2\right]\right\}. \quad (9)$$

Here $v_0 = p_0/m$ is the velocity of the electron at the Fermi surface. The small numerical coefficient means that the correction which is found for

*We note that taking account of spatial dispersion makes the usual distinction between remote and near losses unnecessary. Thus, for high momentum transfer (near losses) $\hbar k$ is much greater than the characteristic electron momenta and the latter can be considered free electrons; for ϵ^l we use the expression

$$\varepsilon^l(\omega, k) = 1 - \omega_0^2/[\omega^2 - (\hbar k^2/2m)^2], \quad (2')$$

where $\omega_0^2 = 4\pi e^2N/m$ and N is the number of electrons per unit volume. Correspondingly, for large scattering angles (high ratio of the velocity of the electrons in the medium to the velocity of the fast particle, but much smaller than unity) Eq. (4) yields

$$dW^l/\hbar\omega d\Omega d\omega = \delta(\omega - p^2\theta^2/2m\hbar)\, N d\sigma_{\mathrm{Ruth}}/d\Omega, \quad (4')$$

where $d\sigma_{\mathrm{Ruth}} = (2e^2Z/vp\theta^2)^2 d\Omega$. In other words, the dispersion in the dielectric constant takes account of the usual Rutherford scattering, i.e., near collisions.

the term proportional to k^2 [cf. Eq. (5)] is small even at the densities of valence electrons in real metals; in a favorable case it is 20 − 30%.

The condition for excitation of plasma oscillations corresponds to the vanishing of the quantity ϵ^l. This condition yields the following spectrum for the plasma oscillations:

$$\omega^2 = \omega_0^2 + \frac{3p_0^2k^2}{5m^2} - \omega_0^2 \frac{3}{20}\left(\frac{\hbar k}{p_0}\right)^2. \quad (10)$$

The last relation has been obtained by the author,[13] and later by Nozieres and Pines.[15]

3. We now determine the longitudinal dielectric constant of a degenerate electron liquid, using the theory of a Fermi liquid given by Landau.[3] In accordance with reference 14, we use in place of Eq. (6) the following equation for the non-equilibrium correction to the electron distribution function

$$\frac{\partial}{\partial t}\delta f + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\left\{\delta f - \frac{\partial f_0}{\partial p}\delta\varepsilon_1\right\} + e\mathbf{E}\mathbf{v}\frac{\partial f_0}{\partial \varepsilon} = 0. \quad (11)$$

Here f_0 is the equilibrium distribution function; it differs from the corresponding distribution function for an ideal gas only in that it is a function of the energy of the quasi-particles $\epsilon(\mathbf{p})$ and not the energy of the free electron;

$$\delta\varepsilon_1(\mathbf{p}, \mathbf{r}) = \int \varphi(\mathbf{p}, \mathbf{p}')\,\delta f(\mathbf{p}', \mathbf{r})\,d\mathbf{p}', \quad (12)$$

where $\varphi(\mathbf{p}, \mathbf{p}')$ is in general an unknown function which reflects the correlation between particles. In obtaining a simple qualitative picture we need not take account of the periodic ion fields. Under these conditions the electron velocity is given by the relation

$$\mathbf{v} \equiv \frac{\partial \varepsilon}{\partial \mathbf{p}} = \frac{\mathbf{p}}{m} + \int d\mathbf{p}'\varphi(\mathbf{p}, \mathbf{p}')\frac{\partial f_0}{\partial \mathbf{p}'}. \quad (13)$$

In principle, using Eq. (8) and the relations given above it is possible to determine $\epsilon^l(\omega, \mathbf{k})$. Without making any assumptions as to the form of the function $\varphi(\mathbf{p}, \mathbf{p}')$, however, we can only determine the longitudinal dielectric constant in the long wavelength region (small k):

$$\varepsilon^l(\omega, k) = 1 - \frac{\omega_0^2}{\omega^2}\left[1 + \frac{3}{5}\frac{v_0 p_0}{m\omega^2}k^2\left(1 + \frac{5}{9}A_0 + \frac{4}{45}A_2\right)\right],$$

$$v_0 = (p_0/m)(1 + A_1/3)^{-1}, \qquad p_0 = (3\pi^2)^{1/3}N^{1/3}\hbar. \quad (14)$$

Here p_0 is the electron momentum at the Fermi surface while the coefficients A_l are defined by the expression

$$\frac{2}{(2\pi\hbar)^3}\frac{4\pi p_0^2}{v_0}\varphi(\mathbf{p}_0, \mathbf{p}_0') = \sum_l A_l P_l(\cos\chi), \quad (15)$$

where χ is the angle between the vectors $\mathbf{p}_0$ and $\mathbf{p}_0'$.

Equation (14) for ϵ^l differs from the corresponding expression for an electron gas in the Hartree approximation (5) or the Hartree-Fock approximation (9) by the coefficient for the k^2 term. In the case of an electron liquid the coefficients in the expansion of φ can also be greater than unity. We expect that the region in which the term in Eq. (14) proportional to k^2 may be assumed small will be narrower than the corresponding region for Eqs. (5) and (9). Hence we now consider the longitudinal dielectric constant in the region in which the expansion in powers of k cannot be used.

We assume further that in the expansion in (15) only the first two coefficients A_0 and A_1 are different from zero. In this case Eq. (11) becomes an integral equation with a degenerate kernel and can be solved easily. We obtain

$$\varepsilon^l(\omega, k) = 1 - \frac{3\omega_0^2}{k^2 v_0^2}\frac{\eta(s)}{(1 + {}^1/_3A_1)[1 - A_0\eta(s)] - A_1 s^2\eta(s)}, \quad (16)$$

where

$$\eta(s) = \frac{s}{2}\ln\frac{s+1}{s-1} - 1, \quad s = \frac{\omega}{kv_0}. \quad (17)$$

An important property of the expression which has been obtained for the longitudinal dielectric constant is the singularity which occurs when the denominator in Eq. (16) vanishes

$$(1 + {}^1/_3A_1)[1 - A_0\eta(s)] - A_1 s^2\eta(s) = 0. \quad (18)$$

Equation (18) is the dispersion relation for zero sound.[16] Hence the existence of a singularity in (16) is completely understandable: for the value of ω/kv_0 which satisfies Eq. (18) the electromagnetic field can excite zero sound in this case.

The solution of Eq. (18) becomes especially simple in the case in which A_0 is positive and large compared with unity and A_1. In particular, in this case

$$s = \sqrt{A_0/3} \gg 1, \quad (19)$$

while the expression for ϵ^l can be written in the form

$$\varepsilon^l(\omega, k) = 1 - \omega_0^2/(\omega^2 - A_0 v_0^2 k^2/3). \quad (20)$$

It is interesting to note the similarity between the denominators in (20) and (9) when we do not carry out the expansion in powers of k^2 in the latter. In Eq. (9) the coefficient in front of $v_0^2k^2$ is positive and small compared with unity; the absence of a singularity in Eq. (9) indicates that it is impossible to have zero sound in this case.

The spectrum of plasma oscillations in a degenerate electron liquid has been considered by us earlier.[17,18] Using the results obtained in reference 18 we can obtain an expression for the transverse dielectric constant of the electron liquid. Assuming, as was done in deriving Eq. (16), that all the A_l with $l > 1$ are equal to zero, we have

$$\varepsilon^{tr}(\omega, k) = 1 - \frac{3\omega_0^2}{2\omega^2} \frac{1-(s^2-1)\eta(s)}{1-{}^1/_2 A_1\{({}^1/_3)-(s^2-1)\eta(s)\}}. \quad (21)$$

The singularity in ϵ^{tr} at

$$1 - \frac{1}{2}A_1\left\{\frac{1}{3}-(s^2-1)\eta(s)\right\} = 0 \quad (22)$$

is similar to the corresponding singularity in ϵ^l and provides the possibility of exciting zero sound. The sole difference is that in Eq. (16) we are considering the excitation of longitudinal zero sound while in Eq. (21) we are considering transverse sound (cf. below).

4. For small spatial dispersion the longitudinal dielectric constant assumes the form:

$$\varepsilon^l(\omega, k) = \varepsilon(\omega) - \alpha(k^2/\omega^2). \quad (23)$$

Equation (23) allows us to write the following expression for the scattering probability of a fast electron with the emission of a longitudinal quantum (per unit path length):

$$\frac{dW^l}{\hbar\omega d\Omega d\omega} = \frac{2e^2}{\pi\hbar v^2}\frac{1}{\theta^2+(\hbar\omega/vp)^2}\,\mathrm{Im}\,\frac{1}{\varepsilon(\omega)-\alpha[(p\theta/\hbar\omega)^2+1/v^2]} \quad (24)$$

The case of small absorption is of special interest; in this case ϵ^l may be considered real. Essentially the same situation arises in the case of Eqs. (5), (9), and (14) for frequencies much larger than the electron collision frequency. Assuming that $\alpha = a(p_0/m)^2$ (a is a numerical factor) we can write Eq. (24) in the form*

$$\frac{dW^l}{\hbar\omega d\Omega d\omega} = \frac{2e^2}{\hbar v^2}\frac{1}{\theta^2+(\hbar\omega/vp^2)}\,\delta\left[\varepsilon^l\left(\omega, \sqrt{\left(\frac{p\theta}{\hbar}\right)^2+\left(\frac{\omega}{v}\right)^2}\right)\right]$$
$$= \frac{2e^2}{\hbar v^2}\frac{1}{\theta^2+(\hbar\omega/vp)^2}\,\delta\left[\varepsilon(\omega) - a\left\{\left(\frac{p_0}{p}\right)^2+\left(\frac{p_0 v}{\hbar\omega}\right)^2\theta^2\right\}\right]. \quad (25)$$

The dependence of the discrete energy lines on the scattering angle given by Eqs. (25) — (26) assumes the following form for an electron gas or an electron liquid:

$$\Delta E(\theta) = \hbar\omega \approx \hbar\omega_0 + (av^2p_0^2/2\hbar\omega_0)\,\theta^2, \quad (26)$$

This dependence is apparently observed experimentally.[7] It is clear that the study of this dependence and the precise experimental determination of the coefficient for θ^2 is extremely important in choosing a model to describe electrons in a metal.

*In this we make use of the fact that

$$\lim \mathrm{Im}(\varepsilon^l/|\varepsilon^l|^2) = \pi\delta[\mathrm{Re}\,\varepsilon^l] \text{ for } \mathrm{Im}\,\varepsilon^l \to 0.$$

We now consider certain of the results obtained above and their consequences for Fermi liquids when the spatial dispersion is not weak. We may note first that ϵ^l as given by Eq. (16) becomes complex $\omega < kv_0$. In other words, one can speak of discrete loss lines and use Eq. (25) only for scattering angles which satisfy the inequality

$$\theta < \hbar\omega/v_0 p. \quad (27)$$

For large angles a discrete loss line is impossible.*

The dielectric constant in (16) indicates a relatively complicated dependence for the loss line scattering angle in the angular region $\hbar\omega_0/vp_0$. The picture is simplified considerably for large A_0, in which case Eq. (20) holds. The differential scattering probability becomes

$$\frac{dW^l}{\hbar\omega d\Omega d\omega} = \frac{2e^2}{\hbar v^2}\frac{1}{\theta^2+(\hbar\omega/vp)^2}\frac{\omega_0^2\delta\left(\omega-\sqrt{\omega_0^2+(A_0/3)(v_0p\theta/\hbar)^2}\right)}{\sqrt{\omega_0^2+(A_0/3)(v_0p\theta/\hbar)^2}}. \quad (28)$$

In the angular region

$$\theta \ll \theta_1 = (\hbar\omega_0/v_0p)\sqrt{A_0/3} < \hbar\omega_0/vp_0$$

the results of Eq. (28) are similar to those of Eq. (25). On the other hand, at large values of θ_1 there is an important difference. In this region the energy loss is proportional to the scattering angle $\hbar\omega \approx \sqrt{A_0/3}\, v_0p\theta$, corresponding to the excitation of zero sound photons (19). This means that in the region $\theta > \theta_1$ the angular distribution for scattering (28) also differs considerably from the corresponding distribution for weak spatial dispersion. In particular, whereas in the region $\theta \ll \theta_1$ [as in the case of Eq. (25)] Eq. (28) gives a distribution proportional to $[\theta^2 + (\hbar\omega/vp)^2]^{-1}$, in the region $\theta \gg \theta_1$, Eq. (28) gives a distribution

$$1/\theta[\theta^2+(\hbar\omega/vp)^2].$$

The strong angular dependence of the discrete losses of the scattered electrons means that the lines for the total losses (integrated over the angle) are relatively wide. Hence attempts to observe the zero sound associated with the radiation losses of fast electrons must be directed toward a study of the angular dependence of the wide lines associated with total energy loss. This does not mean that zero sound is not also associi-

*If the condition $\omega < kv_0$ is satisfied the electrons in the liquid can radiate and absorb quanta of zero sound. This condition is analagous to the condition for Cerenkov radiation. For fast-electron scattering angles which satisfy the relation $\theta > \hbar\omega/v_0p$ the "Cerenkov" mechanism becomes important in the energy dissipation of electrons in the medium; this leads to smearing of the loss line even if the usual dissipation characterized by the imaginary part of $\varepsilon(\omega)$ is small.

ated with narrow lines. However, the zero sound effects should be most pronounced for wide lines.

When $A_0 \ll 1$ effects connected with zero sound do not give noticeable broadening of the total-loss lines. For positive A_0 the expression for the zero sound spectrum assumes the form

$$\omega/kv_0 = s = 1 - \exp\{-2/A_0\}. \qquad (29)$$

At small A_0 the excitation of zero sound is possible only in a narrow region close to the angle $\theta = \hbar\omega/pv_0$.

5. We now consider one other possibility which leads to a dependence of the discrete losses on the scattering angle, and which is not a result of spatial dispersion. This possibility arises in optically anisotropic bodies. For example, in the motion of a fast nonrelativistic electron along the axis of symmetry a uniaxial crystal the differential scattering probability per unit path length is*

$$\frac{dW(\theta,\omega)}{\hbar\omega\, d\omega\, d\Omega} = \frac{2e^2}{\pi\hbar v}\,\frac{\operatorname{Im}\varepsilon_\perp(\omega)\,\theta^2 + \operatorname{Im}\varepsilon_\parallel(\omega)\,(\hbar\omega/vp)^2}{[\operatorname{Re}\varepsilon_\perp(\omega)\,\theta^2 + \operatorname{Re}\varepsilon_\parallel(\omega)\,(\hbar\omega/vp)]^2 + [\operatorname{Im}\varepsilon_\perp(\omega)\theta^2 + \operatorname{Im}\varepsilon_\parallel(\omega)(\hbar\omega/vp)]^2}\,. \qquad (30)$$

It is clear that in the case of small absorption, for which we can neglect the imaginary part of ϵ, the discrete loss line is determined by the zeros of the expression

$$\varepsilon_\perp(\omega)\,\theta^2 + \varepsilon_\parallel(\omega)\,(\hbar\omega/vp)^2. \qquad (31)$$

For an optically anisotropic medium the quantities $\epsilon_\perp$ and $\epsilon_\parallel$ vanish simultaneously only by coincidence. Hence the zeros of the expression in Eq. (31) are generally functions of the scattering angle θ. Under these conditions, in the small-angle region $\theta \ll \hbar\omega/vp$ the position of the loss line is determined by the frequency at which $\epsilon_\parallel(\omega)$ vanvanishes. On the other hand, for angles $\theta \gg \hbar\omega/vp$ the loss line is determined by the point at which $\epsilon_\perp(\omega)$ vanishes. In the general case of motion of an electron in an optically anisotropic crystal characterized by the tensor ϵ_{ij}, for arbitrary orientation of the direction of motion with respect to the crystallographic axes, in the denominator of Eq. (30), in place of Eq. (31) we have (cf. references 1 and 2) $\epsilon_{ij}k_ik_j$ [where $k_z = \omega/v$, $k_x = (p\theta/\hbar)\cos\varphi$, $k_y = (p\theta/\hbar)\sin\varphi$]. Hence, in addition to the dependence of energy loss on angle there is an azimuthal dependence on the angle φ. The dependence of the energy loss on scattering angle can lead to a broadening of the total (integrated over scattering angle) loss line.† We consider this problem in somewhat greater detail.

According to Eqs. (30) and (31) the total energy loss in the scattering angle region $\theta < \theta_{max}$ can be written as follows for small absorption:

$$\int\limits_{\theta<\theta_{max}} d\Omega(dW/\hbar\omega d\omega d\Omega) = 2\pi e^2/(\hbar v^2\,|\,\varepsilon_\perp(\omega)\,|), \qquad (32)$$

for the condition

$$0 < -\varepsilon_\parallel(\omega)/\varepsilon_\perp(\omega) < (vp\theta_{max}/\hbar\omega)^2. \qquad (33)$$

This quantity vanishes if this condition is not satisfied. Here $\theta_{max} \sim \hbar\omega/p\bar{v}$, where $\bar{v}$ is the characteristic velocity of electrons in the medium. We assume that $\epsilon_\perp$ and $\epsilon_\parallel$ do not vanish simultaneously.

Thus the line shape is determined by $\epsilon_\perp$ while the line width is given by (33). It is important, as is seen from this condition, that the frequency at which $\epsilon_\perp(\omega) = 0$ lie outside the loss line. If this frequency is far from the edge of the line, the line will be relatively smooth. On the other hand, at high velocities the edge of the loss line may be close to the point at which $\epsilon_\perp(\omega)$ vanishes. In this case the edge of the line will exhibit a sharp narrow peak which will practically obliterate the line. The latter is due to the fact that for angles $\theta \gg \hbar\omega/vp$ the loss line is determined by the point at which $\epsilon_\perp(\omega)$ vanishes. A similar picture holds for the arbitrary orientation of the direction of motion of the fast particle.

6. In considering the losses of relativistic electrons it is necessary to consider the role of transverse quanta, in particular, Cerenkov radiation. In the case of an electron gas the Cerenkov radiation is impossible since the dielectric constant (real part) is less than unity. However the situation is different in an electron liquid. According to Eq. (21) the transverse dielectric constant becomes positive in the region of frequencies and wave vectors which satisfy the condition for the propagation of transverse zero sound (22). In this connection we now consider the discrete transverse losses of relativistic electrons.

Limiting ourselves to small angles ($\theta \ll 1$), from Eq. (2) we have the following expression for the scattering probability of a fast particle per unit length into an angle $d\theta$ with the emission of

*Strictly speaking, for optically anisotropic bodies the radiation of plasmons is nothing more than the Cerenkov radiation.[10]
†The possibility of broadening of this kind has already been indicated in references 10 and 18.

a transverse quantum in the frequency range $d\omega$:

$$\frac{dW^{tr}}{\hbar\omega d\omega d\Omega}=\frac{2e^2Z^2}{\pi\hbar c^2}\frac{1}{\theta^2+(\hbar\omega/vp)^2}$$

$$\times\operatorname{Im}\left\{\left[\frac{v^2}{c^2}\varepsilon^{tr}\left(\omega,\sqrt{\left(\frac{p\theta}{\hbar}\right)^2+\left(\frac{\omega}{v}\right)^2}\right)-1\right]\left(\frac{\hbar\omega}{pv}\right)^2-\theta^2\right\}^{-1}. \quad (34)$$

Neglecting absorption we have

$$\frac{dW^{tr}}{\hbar\omega d\omega d\Omega}=\frac{2e^2Z^2}{\hbar c^2}\frac{1}{\theta^2+(\hbar\omega/vp)^2}$$

$$\times\delta\left\{\left[\frac{v^2}{c^2}\varepsilon^{tr}\left(\omega,\sqrt{\left(\frac{p\theta}{\hbar}\right)^2+\left(\frac{\omega}{v}\right)^2}\right)-1\right]\left(\frac{\hbar\omega}{vp}\right)^2-\theta^2\right\}. \quad (35)$$

The zeros of the argument of the δ-function determine the discrete energy of the Cerenkov photon lost by a fast particle in being scattered through an angle θ.

In the region of the singularity determined by Eq. (22), the transverse dielectric constant (21) assumes the form

$$\varepsilon^{tr}(\omega,k)\approx 1+\frac{\omega_0^2}{\omega^2}\frac{\gamma}{s-s_0}, \quad (36)$$

where $s=\omega/kv_0$, s_0 is the solution of Eq. (22), and the factor γ is

$$\gamma=\left(1+\frac{3}{A_1}\right)\frac{s_0(s_0^2-1)}{3s_0^2-1-A_1/3}.$$

The argument of the δ-function in Eq. (35) vanishes if we substitute ϵ^{tr} as given by Eq. (36) when

$$\hbar\omega=v_0p\,[s_0\theta+\gamma(\hbar\omega_0/cp)^2/\theta]. \quad (37)$$

Here is assumed that $v\sim c$. In order for the formula to apply the condition $\theta\gg(\hbar\omega_0/cp)$ must be satisfied. Hence the frequency determined by Eq. (37) is only slightly different from the frequency of transverse zero sound. It should be noted that in order to neglect the imaginary part of ϵ^{tr} it is necessary that ω be considerably greater than the collision frequency which determines the dissipation in the electron liquid. The latter condition can be satisfied for scattering angles which are not too small.

Finally we may note that in real solid bodies in addition to the effects of electron correlation an important role is played by the lattice field which can complicate the interpretation of the problems considered above. Hence, from our point of view, most interest attaches to attempts to find experimentally qualitative features connected with zero sound and those derived from the theory of a Fermi liquid.

7. In conclusion we may make general remarks relating to the excitation of zero sound and the more general problem of interpretation of discrete losses. To what degree can one speak of the excitation of a freedom in a solid body — in our case, zero sound, and in the more widely discussed case,[4-7] the levels of individual electrons? In the latter case one considers the transitions of electrons from one level to another and the lines which are thus determined, in the opinion of a number of authors, are in fair agreement with this interpretation. For an answer to this problem we consider as an example the dielectric constant corresponding to one level ω_r:

$$\varepsilon=1-\omega_0^2/(\omega^2-\omega_r^2).$$

The frequency of the longitudinal oscillation determined from the condition $\epsilon=0$ is $\omega^2=\omega_0^2+\omega_r^2$. It is clear that for the level $\omega_r\ll\omega_0$ the existence of the level is unimportant for the determination of the frequency of the longitudinal photon. On the other hand, in the case $\omega_r\gg\omega_0$ the frequency of the longitudinal photon (plasmon) is essentially equal to the frequency of the level. Hence the energy of discrete losses may be close to the corresponding energy of the one-electron transitions.

Similarly, in our case it is possible to speak of the excitation of the longitudinal zero sound only under conditions in which the frequency of the zero sound is considerably greater than the frequency $\omega_0=\sqrt{4\pi e^2N/m}$. If this is not the case the frequency of the longitudinal oscillations excited in the medium is considerably different from the frequency of zero sound.

In conclusion I wish to express my gratitude to V. L. Ginzburg and L. D. Landau for a number of useful discussions of the problems considered in the present paper.

[1] I. E. Tamm and I. M. Frank, Dokl. Akad. Nauk SSSR, **14**, 107 (1937); E. Fermi, Phys. Rev., **57**, 485 (1940); L. D. Landau and E. M. Lifshitz, Электродинамика сплошных сред (Electrodynamics of Continuous Media), Gostekhizdat, 1957; B. M. Bolotovskiĭ, Usp. Fiz. Nauk **62**, 201 (1957).

[2] J. Lindhard, Det Kong. Dan. Vid. Salsk, Dan. Mat.-Fys. Medd. **28**, 8 (1954); J. Hubbard, Proc. Phys. Soc. **A68**, 441, 977 (1955); J. Neufeld and R. H. Ritchie, Phys. Rev. **98**, 1632 (1955); R. H. Ritchie, Phys. Rev. **106**, 874 (1957); V. M. Agranovich and A. A. Rukhadze, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 982 (1958), Soviet Phys. JETP **8**, 685 (1959). P. Nozieres and D. Pines, Nuovo cimento **9**, 470 (1958).

[3] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 1058 (1956), Soviet Phys. JETP **3**, 920 (1956).

[4] Marton, Leder, and Mendlowitz, Advances in Electronics and Electron Physics **7**, N.Y. Academic Press, Inc., 1955; G. Fridman, Usp. Fiz. Nauk **62**, 427 (1957); Fortsch. Physik **5**, 51 (1957).

[5] D. Pines and D. Bohm, Phys. Rev. **85**, 338 (1952); D. Pines, Revs. Modern Phys. **28**, 184 (1956); Usp. Fiz. Nauk **62**, 399 (1957).

[6] H. Fröhlich and H. Pelzer, Proc. Phys. Soc. **A68**, 525 (1955).

[7] H. Watanabe, J. Phys. Soc. Japan **11**, 112 (1956); C. Fert and F. Pradal, Compt. rend. **248**, 666 (1959).

[8] A. A. Vlasov, J. Exptl. Theoret. Phys. (U.S.S.R.) 8, 291 (1938); Уч. зап. МГУ (Sci. Notes, Moscow State University) No. 75, Vol. 2, #1 (1945).

[9] I. I. Gol'dman, J. Exptl. Theoret. Phys. (U.S.S.R.) **17**, 681 (1947); Yu. L. Klimontovich, and V. P. Silin, Dokl. Akad. Nauk SSSR **82**, 361 (1952); J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 151 (1952); D. Bohm and D. Pines, Phys. Rev. **92**, 609 (1953); P. S. Zyryanov and V. M. Eleonskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 592 (1956), Soviet Phys. JETP **3**, 620 (1956); R. A. Ferrell, Phys. Rev. **107**, 450 (1957).

[10] V. L. Ginzburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1593 (1958), Soviet Phys. JETP **7**, 1096 (1958).

[11] R. Kronig and K. Korringa, Physica, **10**, 406, 800 (1943); A. I. Akhiezer and A. G. Sitenko, J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 161 (1952); E. L. Feĭnberg, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1125 (1958), Soviet Phys. JETP **7**, 780 (1958); A. Ya. Vyatskin, J. Tech. Phys. (U.S.S.R.) **28**, 2217, 2455 (1958), Soviet Phys. "Tech. Physics" **3**, 2038, 2252 (1959).

[12] Sawada, Brueckner, Fukuda, and Brout, Phys. Rev. **108**, 507 (1958); R. Brout, Phys. Rev. **108**, 515 (1958).

[13] V. P. Silin, Физика металлов и металловедение (Physics of Metals and Metallography) **3**, 193 (1956).

[14] V. P. Silin, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 495 (1957), Soviet Phys. JETP **6**, 387 (1958).

[15] P. Nozieres and D. Pines, Phys. Rev. **111**, 442 (1958).

[16] L. D. Landau, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 59 (1957), Soviet Phys. JETP **5**, 101 (1957); I. M. Khalatnikov and A. A. Abrikosov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 110 (1957), Soviet Phys. JETP **6**, 84 (1958).

[17] V. P. Silin, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 781 (1958), Soviet Phys. JETP **7**, 538 (1958).

[18] V. P. Silin, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 1243 (1958), Soviet Phys. JETP **8**, 870 (1959).

Translated by H. Lashinsky
40