ON THE PROBLEM OF INVESTIGATING THE INTERACTION BETWEEN π MESONS AND HYPERONS

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It is shown that use of the unitary property of the S matrix makes it possible to obtain some information about the scattering of π mesons by Λ and Σ hyperons from an analysis of the data on the interaction of K mesons with nucleons. The possibility of studying the π-Λ and π-Σ interactions by examining peripheral collisions of hyperons with nucleons is discussed.

THE study of the interactions of π mesons with hyperons is of special interest in connection with the determination of the symmetry properties of the interactions of π mesons with various baryons.

1. Let us consider the reactions

$$\widetilde{K} + N \rightarrow \widetilde{K} + N, \tag{1a}$$

$$\widetilde{K} + N \rightarrow \Sigma(\Lambda) + \pi, \tag{1b}$$

$$\Sigma(\Lambda) + \pi \rightarrow \Sigma(\Lambda) + \pi \tag{1c}$$

in a range of K-meson energies in which one can neglect channels in which two pions are produced. Since the elements of the S matrix for the reactions (1) are connected with each other by the condition of unitarity, the question arises as to what information about the scattering amplitudes $\Sigma(\Lambda) + \pi \rightarrow \Sigma(\Lambda) + \pi$ can be obtained by studying the cross sections and polarizations in processes (1a) and (1b). The first part of the present paper contains an attempt to answer this question.

In what follows we assume that the spin of the K meson is zero and that the hyperon spin is $\frac{1}{2}$. We further assume that the interactions are invariant under space inversion, time reversal, and rotations in isotopic space.

The reactions (1) are described by elements of the T matrix ($iT = S - 1$) diagonal in the isotopic-spin quantum number,

$$T^0 = \begin{pmatrix} a_K^0 & a_{K\Sigma}^0 \\ a_{\Sigma K}^0 & a_\Sigma^0 \end{pmatrix}, \quad T^1 = \begin{pmatrix} a_K^1 & a_{K\Sigma}^1 & a_{K\Lambda} \\ a_{\Sigma K}^1 & a_\Sigma^1 & a_{\Sigma\Lambda} \\ a_{\Lambda K} & a_{\Lambda\Sigma} & a_\Lambda \end{pmatrix}, \tag{2}$$

where $a_K^0(a_K^1)$ is the amplitude for scattering $\widetilde{K} + N \rightarrow \widetilde{K} + N$ in the state with the indicated value of the isotopic spin, 0 (1); $a_{K\Sigma}^0(a_{K\Sigma}^1)$ is the amplitude for the reaction $\widetilde{K} + N \rightarrow \Sigma + \pi$ in the state with the isotopic spin 0 (1), and so on.

The spin structure of the scattering amplitude a_α can be represented in the form

$$a_\alpha = A_\alpha + iB_\alpha (\sigma [\mathbf{n} \times \mathbf{n}']), \tag{3}$$

where $\mathbf{n}$ ($\mathbf{n}'$) is the unit vector parallel to the momentum of the particles in the initial (final) state, in the center-of-mass system; A_α and B_α are two complex functions of the energy and of $\mathbf{n}\cdot\mathbf{n}'$.

The reaction amplitude $a_{\alpha\beta}$ has the form

$$a_{\alpha\beta} = A_{\alpha\beta} + iB_{\alpha\beta} (\sigma [\mathbf{n} \times \mathbf{n}']), \tag{4}$$

when the product of the intrinsic parities of all four particles in the initial (final) states is $\Pi = +1$, and the form

$$a_{\alpha\beta} = A_{\alpha\beta} (\sigma\mathbf{n}) + B_{\alpha\beta} (\sigma\mathbf{n}'), \tag{5}$$

when $\Pi = -1$. Here $A_{\alpha\beta}$ and $B_{\alpha\beta}$ are two complex functions of the energy and of $\mathbf{n}\cdot\mathbf{n}'$.

Let us turn to the analysis of the conditions for determining the T matrix from the experimental data. It can be seen from Eqs. (2), (3), and (4) that the number of real scalar functions involved in the matrixes T^0 and T^1 is $13 \times 4 = 52$. The invariance of the interaction under time reversal means that the S matrix is symmetric, and this reduces the number of functions determining the T matrix from 52 to 36. It can be shown further that when the conditions for the S matrix to be unitary are taken into account, the number of independent real functions is decreased by a factor of two and becomes 18.

The same result is obtained if we use the general formulas obtained in reference 1.

Let us now consider what information can be obtained by studying only processes (1a) and (1b). The number of real functions characterizing these processes is $5 \times 4 = 20$. They satisfy four relations of unitarity. Therefore only 16 of them are independent.

Reaction	Amplitude
(a) $K^- + p \to K^- + p$	$1/2\,(a_K^1 + a_K^0)$
(b) $K^- + p \to K^0 + n$	$1/2\,(a_K^1 - a_K^0)$
(c) $K_2^0 + p \to K_1^0 + p$	$1/2\,(a_K^1 - \tilde{a}_K^0)$
(d) $K_2^0 + p \to K_2^0 + p$	$1/2\,(a_K^1 + \tilde{a}_K^0)$
(e) $K^- + p \to \Lambda + \pi^0$	$a_{K\Lambda}$
(f) $K^- + p \to \Sigma^- + \pi^+$	$-(a_{K\Sigma}^0/\sqrt{6} + a_{K\Sigma}^1/2)$
(g) $K^- + p \to \Sigma^0 + \pi^0$	$a_{K\Sigma}^0/\sqrt{6}$
(h) $K^- + p \to \Sigma^+ + \pi^-$	$-(a_{K\Sigma}^0/\sqrt{6} - a_{K\Sigma}^1/2)$

The table shows 8 reactions of types (1a) and (1b) and their amplitudes. The symbol K_2^0 (K_1^0) denotes the long-lived (short-lived) K^0 meson; a_K^0 is the scattering amplitude of the K^0 mesons, which is determined in the analysis of the scattering of K^+ mesons by nucleons. In what follows we assume that the amplitude a_K^0 is already known.

In reality the reactions (c) and (d) in the table are the same process. By studying the time dependences of the scattering cross section and of the polarization after scattering (i.e., the dependences on the distance to the target), one can determine the amplitudes of reactions (c) and (d) separately.

By measuring the differential cross sections and the polarization of the nucleons in reactions (a) — (d) as shown in the table, we can completely fix the scattering amplitudes a_K^0 and a_K^1. The experimental data on the cross sections and polarizations of the hyperons in reactions (e) — (h), together with the four relations of unitarity, enable us to determine the reaction amplitudes $a_{K\Sigma}^0$, $a_{K\Sigma}^1$, and $a_{K\Lambda}$, apart from a common phase factor.

Since the expressions for the cross sections and polarizations, and also the unitarity relations for reactions (1a) and (1b) are invariant under the replacement

$$a_{K\Sigma}^0 \to e^{i\delta_0(E)} a_{K\Sigma}^0, \qquad a_{K\Sigma}^1 \to e^{i\delta_0(E)} a_{K\Sigma}^1,$$
$$a_{K\Lambda} \to e^{i\delta_1(E)} a_{K\Lambda}, \tag{6}$$

we cannot determine two phase factors $e^{i\delta_0}$ and $e^{i\delta_1}$, which are functions of the energy alone. This last follows from the relations that the amplitudes (4) and (5) satisfy by virtue of the unitary property of the S matrix.

Since the number of independent real functions involved in T^0 and T^1 is 18, and 16 of them are determined apart from two phase factors through the study of processes (1a) and (1b), for the complete reconstruction of the scattering amplitudes of pions by Λ and Σ hyperons in the states with isotopic spins 0 and 1, we need to determine in addition two more real functions of the energy and $\mathbf{n}\cdot\mathbf{n}'$, and two phase factors.

For each state with total angular momentum j and orbital angular momentum $l = j \pm \frac{1}{2}$, the T^0 matrix can be written in the form

$$-i\begin{pmatrix} \rho_K^0 \exp(2i\delta_K^0) - 1 & i\rho_{K\Sigma}^0 \exp(i\delta_{K\Sigma}^0) \\ i\rho_{K\Sigma}^0 \exp(i\delta_{K\Sigma}^0) & \rho_\Sigma^0 \exp(2i\delta_\Sigma^0) - 1 \end{pmatrix}, \tag{7}$$

where the ρ_α are certain positive functions of the energy, and the δ_α are the phases of the corresponding processes.

From the conditions for unitarity of the S matrix it follows that

$$\delta_{K\Sigma}^0 = \delta_K^0 + \delta_\Sigma^0, \qquad \rho_\Sigma^0 = \rho_K^0 = \{1 - (\rho_{K\Sigma}^0)^2\}^{1/2}. \tag{8}$$

The quantities ρ_K^0, δ_K^0, $\delta_{K\Sigma}^0$ can be determined apart from a common phase factor by studying processes (1a) and (1b). The quantities ρ_Σ^0 and δ_Σ^0 are then determined to the same accuracy from the relations (8).

Thus for π-Σ scattering the difference of the phases in the various states with zero isotopic spin is completely determined by the study of the reactions with K particles.*

For the states with isotopic spin (1) we have instead of the matrix (7)

$$-i\begin{pmatrix} \rho_K e^{2i\delta_K} - 1 & \rho_{K\Sigma} e^{i\delta_{K\Sigma}} & \rho_{K\Lambda} e^{i\delta_{K\Lambda}} \\ \rho_{K\Sigma} e^{i\delta_{K\Sigma}} & \rho_\Sigma e^{2i\delta_\Sigma} - 1 & \rho_{\Sigma\Lambda} e^{i\delta_{\Sigma\Lambda}} \\ \rho_{K\Lambda} e^{i\delta_{K\Lambda}} & \rho_{\Sigma\Lambda} e^{i\delta_{\Sigma\Lambda}} & \rho_\Lambda e^{2i\delta_\Lambda} - 1 \end{pmatrix}. \tag{9}$$

Here and in what follows, we shall write instead of $\rho_\alpha^1(\delta_\alpha^1)$ simply $\rho_\alpha(\delta_\alpha)$.

From the unitarity conditions we get

$$\rho_{K\Sigma}^2 + \rho_\Sigma^2 + \rho_{\Sigma\Lambda}^2 = 1, \qquad \rho_{K\Lambda}^2 + \rho_{\Sigma\Lambda}^2 + \rho_\Lambda^2 = 1, \tag{10}$$

$$\cos(2\delta_\Sigma + 2\delta_K - 2\delta_{K\Sigma}) = \frac{(\rho_K\rho_{K\Sigma})^2 + (\rho_{K\Sigma}\rho_\Sigma)^2 - (\rho_{K\Lambda}\rho_{\Sigma\Lambda})^2}{2\rho_\Sigma\rho_K(\rho_{K\Sigma})^2}, \tag{11}$$

$$\cos(\delta_{\Sigma\Lambda} + 2\delta_K - \delta_{K\Lambda} - \delta_{K\Sigma}) = \frac{(\rho_{K\Lambda}\rho_{\Sigma\Lambda})^2 + (\rho_K\rho_{K\Sigma})^2 - (\rho_{K\Sigma}\rho_\Sigma)^2}{2\rho_{K\Lambda}\rho_{\Sigma\Lambda}\rho_K\rho_{K\Sigma}}, \tag{12}$$

$$\cos(2\delta_\Lambda + 2\delta_K - 2\delta_{K\Lambda}) = \frac{(\rho_K\rho_{K\Lambda})^2 + (\rho_{K\Lambda}\rho_\Lambda)^2 - (\rho_{K\Sigma}\rho_{\Sigma\Lambda})^2}{2\rho_\Lambda\rho_K(\rho_{K\Lambda})^2}. \tag{13}$$

*It may turn out that in carrying out an unambiguous analysis it will be helpful to take into account Coulomb effects and the energy dependence of the S matrix at low energies.

We note that the Minami ambiguity exists for the reactions in question.

Some possibilities for determining the parity of the K meson relative to the hyperons through the analysis of the reactions (1) have recently been discussed by Amati and Vitale.[2]

It is easy to convince oneself that even when ρ_K, $\rho_{K\Sigma}$, $\rho_{K\Lambda}$, δ_K, $\delta_{K\Sigma}$, and $\delta_{K\Lambda}$ are known, the unitarity relations (10) — (13) are insufficient for the reconstruction of the matrix T^1. For this we need to know one more parameter in each state (for example, ρ_Σ).

We note that the relations (10) — (13) lead to some interesting inequalities. Noting that $\rho_\alpha > 0$ and $|\cos\theta| < 1$, we get from Eqs. (10) and (11)

$$0 < \rho_\Sigma^2 = 1 - \rho_{K\Sigma}^2 - \rho_{\Sigma\Lambda}^2 < 1 - \rho_{K\Sigma}^2, \tag{14}$$

$$|(\rho_K\rho_{K\Sigma})^2 + (\rho_{K\Sigma}\rho_\Sigma)^2 - \rho_{K\Lambda}^2(1-\rho_{K\Sigma}^2-\rho_\Sigma^2)| < 2\rho_\Sigma\rho_K(\rho_{K\Sigma})^2. \tag{15}$$

Let us introduce the new notations

$$\rho_{K\Sigma}^2 + \rho_{K\Lambda}^2 = a, \quad \rho_K\rho_{K\Sigma}^2 = b, \quad (\rho_K\rho_{K\Sigma})^2 - \rho_{K\Lambda}^2(1-\rho_{K\Sigma}^2) = c.$$

Then Eq. (15) can be rewritten in the form

$$|a\rho_\Sigma^2 + c| < 2b\rho_\Sigma. \tag{15$'$}$$

From Eq. (15′) together with Eq. (14) we get

$$\max\left\{0;\ \frac{b}{a} - \frac{1}{a}\sqrt{b^2-ac}\right\} < \rho_\Sigma < \min\left\{\sqrt{1-\rho_{K\Sigma}^2};\ \frac{b}{a} + \frac{1}{a}\sqrt{b^2-ac}\right\}. \tag{16}$$

The inequality (15′) holds only for $b^2 - ac \geq 0$. Consequently, the observable quantities ρ_K, $\rho_{K\Lambda}$, and $\rho_{K\Sigma}$ must satisfy this inequality. Similarly we have

$$\max\left\{0;\ \frac{b_1}{a_1} - \frac{1}{a_1}\sqrt{b_1^2-a_1c_1}\right\} < \rho_{\Sigma\Lambda} < \min\left\{\sqrt{1-\rho_{K\Sigma}^2};\ \sqrt{1-\rho_{K\Lambda}^2};\ \frac{b_1}{a_1} + \frac{1}{a_1}\sqrt{b_1^2-a_1c_1}\right\}, \tag{17}$$

$$\max\left\{0;\ \frac{b_2}{a_2} - \frac{1}{a_2}\sqrt{b_2^2-a_2c_2}\right\} < \rho_\Lambda < \min\left\{\sqrt{1-\rho_{K\Lambda}^2};\ \frac{b_2}{a_2} + \frac{1}{a_2}\sqrt{b_2^2-a_2c_2}\right\}, \tag{18}$$

where

$$a_1 \equiv \rho_{K\Lambda}^2 + \rho_{K\Sigma}^2 = a, \qquad b_1 \equiv \rho_{K\Lambda}\rho_K\rho_{K\Sigma},$$
$$a_2 \equiv \rho_{K\Lambda}^2 + \rho_{K\Sigma}^2 = a, \qquad b_2 \equiv \rho_K(\rho_{K\Lambda})^2,$$
$$c_1 \equiv (\rho_K\rho_{K\Sigma})^2 - \rho_{K\Sigma}^2(1-\rho_{K\Sigma}^2),$$
$$c_2 \equiv (\rho_\Lambda\rho_{K\Lambda})^2 - \rho_{K\Sigma}^2(1-\rho_{K\Lambda}^2).$$

2. Recently Chew and Low, and Okun' and Pomeranchuk,[3] have suggested that peripheral collisions be studied as a method for determining interactions between unstable particles. We shall assume that this method can be used for the determination of the scattering amplitude for $\Sigma(\Lambda) + \pi \rightarrow \Sigma(\Lambda) + \pi$ through studies of the processes $\Sigma + N \rightarrow \Sigma(\Lambda) + N + \pi$ and $\Lambda + N \rightarrow \Sigma + N + \pi$. The key point of the method is that the amplitude for the reaction $\Sigma + N \rightarrow \Sigma(\Lambda) + N + \pi$, regarded as a function of $(p_N' - p_N)^2$, where p_N (p_N') is the four-vector momentum of the nucleon in the initial (final) state, has a pole in the nonphysical region, $(p_N' - p_N)^2 = \mu^2$ (μ is the mass of the π meson). It is shown that the virtual process

$$N \rightarrow N + \pi, \qquad \Sigma(\Lambda) + \pi \rightarrow \Sigma(\Lambda) + \pi \tag{19}$$

corresponds to the pole term, whose residue is proportional to the amplitude for $\pi - \Lambda(\Sigma)$ scattering. Assuming that in the physical region near the pole the reaction $\Sigma + N \rightarrow \Sigma(\Lambda) + N + \pi$ is determined by the process (19), one can extrapolate its amplitude into the nonphysical region and separate out the residue of the pole term.

To estimate the effect of other terms in the physical region near the pole, we shall formulate certain rules that must be fulfilled if the contribution of the pole term in actually predominant in this region.

a) In the region near the pole the amplitude of the reaction $\Sigma^+ + p \rightarrow \Sigma^+ + p + \pi^0$ is equal to that of the reaction $\Sigma^- + p \rightarrow \Sigma^- + p + \pi^0$. This rule follows from the invariance of the virtual process

$$\pi^0 + \Sigma^\pm \rightarrow \pi^0 + \Sigma^\pm \tag{20}$$

under rotations in the isotopic space. Similarly it can be shown that the amplitudes for the following pairs of processes are equal:

$$\Sigma^+ + p \rightarrow \Sigma^0 + p + \pi^+ \quad \text{and} \quad \Sigma^- + p \rightarrow \Sigma^0 + p + \pi^-,$$
$$\Lambda + p \rightarrow \Sigma^+ + n + \pi^0 \quad \text{and} \quad \Lambda + p \rightarrow \Sigma^0 + n + \pi^+,$$
$$\pi^+ + p \rightarrow \pi^+ + \pi^0 + p \quad \text{and} \quad \pi^- + p \rightarrow \pi^- + \pi^0 + p \quad \text{etc.}$$

b) Near the pole the amplitudes for the reactions

$$\Sigma^\pm(\Lambda) + p \rightarrow \Sigma^\pm(\Lambda) + p + \pi^0$$
$$\text{and} \quad \tilde{\Sigma}^\pm(\tilde{\Lambda}) + p \rightarrow \tilde{\Sigma}^\pm(\tilde{\Lambda}) + p + \pi^0$$

are equal to each other. This rule follows from the invariance of the virtual process (20) under charge conjugation. In our case it is not of any great practical importance, but it can be of interest in other cases. For example, it can be shown that the amplitudes for the reactions $K^\pm + N \rightarrow K^\pm + N + \pi^0$ are equal. This equality is useful for the determination of the interaction of K mesons with π mesons, and has been noted by Okun' and Pomeranchuk.

c) If the nucleons are unpolarized in the initial state, then they remain unpolarized in the final state also.

Let us consider a reaction of the type

$$\Sigma^+ + p \rightarrow \Lambda + p + \pi^+ \tag{21}$$

in the region near the pole $(p_\Sigma - p_\Lambda)^2 = \mu^2$. We assume that the dominant process in this region is

$$\Sigma^+ \to \Lambda + \pi^+, \qquad \pi^+ + p \to \pi^+ + p, \tag{22}$$

the amplitude for which is proportional to

$$\frac{\bar{u}(p_\Lambda)\Gamma u(p_\Sigma)}{(p_\Lambda - p_\Sigma)^2 - \mu^2} a_{\pi p}, \tag{23}$$

where $\Gamma = 1$ if the relative parity Π of the Σ and Λ particles is -1, and $\Gamma = \gamma_5$ if $\Pi = +1$; $a_{\pi p}$ is the amplitude for the scattering $\pi^+ + p \to \pi^+ + p$. The amplitude for the process (22) does not contain any dependence on the spin operators of the hyperons for $\Pi = -1$ (more exactly, it contains a term proportional to σ_Y, but with a small coefficient); on the other hand, if $\Pi = +1$, the amplitude is proportional to $\sigma_Y \cdot \mathbf{k}$, where $\mathbf{k}$ is the unit vector parallel to the difference

$$\mathbf{P}_\Sigma / (E_\Sigma + M_\Sigma) - \mathbf{P}_\Lambda / (E_\Lambda + M_\Lambda).$$

If in the initial state the Σ^+ is polarized (polarization vector $\mathbf{P}$), then it can be shown from Eq. (23) that in the final state the polarization vector $\mathbf{P}'$ of the Λ particle is given by

$$\mathbf{P}' = \mathbf{P} \text{ for } \Pi = -1,$$
$$\mathbf{P}' = 2(\mathbf{Pk})\mathbf{k} - \mathbf{P} \text{ for } \Pi = +1. \tag{24}$$

Thus if in the region where the pole term predominates one could measure the polarization of the Λ particles produced in the reaction (21) with polarized Σ, then it would be possible not only to evaluate the effect of the non-pole terms, but also to get information about the relative parity of the Λ and Σ hyperons.

In a number of cases the study of the polarization of the products from peripheral collisions can be a source of information about the parities of unstable particles.*

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*The possibility of determining the parities of particles by the study of peripheral collisions without considering the polarization has been discussed recently by Taylor.[4]