tial V, the solution u is inserted instead of the exact solution y. This procedure gives the well-known Born approximation:

$$\eta_l = -\frac{\pi m}{\hbar^2}\int_0^\infty V(r) J^2_{l+1/2}(kr)\, r dr. \qquad (7)$$

Applying this same procedure to Eqs. (1b) and (1c), we get the well-known Kapteyn integral[3]

$$\int_0^\infty J_\mu(\alpha t) J_\nu(\alpha t)\frac{dt}{t} = \begin{cases} \dfrac{2}{\pi}\dfrac{\sin[\pi(\nu-\mu)/2]}{\nu^2-\mu^2}, & \mu \neq \nu \\ \dfrac{1}{2\mu}, & \mu = \nu. \end{cases} \qquad (8)$$

Finally, applying this procedure to Eqs. (1a) and (1c), where the solution for v is given by Eq. (6), and using Eq. (8), we get the Pais approximation for the phase shifts:

$$\frac{2l+1-2\eta_l/\pi}{2l+1-4\eta_l/\pi}\eta_l = -\frac{\pi m}{\hbar^2}\int_0^\infty V(r) J^2_{l+1/2-2\eta_l/\pi}(kr)\, r dr. \qquad (9)$$

Pais obtained this formula by means of a variational principle. We see that one can obtain the approximate formulas of Born and Pais for the phase shifts by using a single kind of procedure.

The examples considered below show the accuracies of the Born and Pais approximations (it must be emphasized that the Pais formula (9) is incorrect for the zeroth-order phase shift). For the Gauss potential $V(r) = -V_0 \exp(-\alpha^2 r^2)$, Eqs. (7) and (9) give

$$\eta_{l\,\mathrm{Born}} = \eta_l^{(1)} = \frac{\pi M V_0}{4\hbar^2\alpha^2}\exp\left(-\frac{k^2}{2\alpha^2}\right) J_{l+1/2}\left(\frac{k^2}{2\alpha^2}\right),$$

$$\frac{2l+1-2\eta_l/\pi}{2l+1-4\eta_l/\pi}\eta_l = \frac{\pi M V_0}{4\hbar^2\alpha^2}\exp\left(-\frac{k^2}{2\alpha^2}\right) J_{l+1/2-2\eta_l/\pi}\left(\frac{k^2}{2\alpha^2}\right). \qquad (10)$$

Considering the scattering of a neutron by a proton (M is the mass of the proton) at 100 Mev, and choosing for the constants the values $V_0 = 45$ Mev and $\alpha^2 = 0.266 \times 10^{26}\ \mathrm{cm}^{-2}$, we get the following values of the phase shifts by the Pais method: $\eta_1 = 0.534$, $\eta_2 = 0.221$, whereas the Born method gives $\eta_1 = 0.487$ and $\eta_2 = 0.197$. Since the second Born approximation gives better results than the first, we shall compare the values obtained above for the phase shifts with the results of the second Born approximation,[4] $\eta_1 = 0.552$ and $\eta_2 = 0.213$. We see that the Pais method gives considerably better results than the first Born approximation.

For large values of l the phase shifts η_l calculated by the Born and Pais methods approach each other, as can be seen from Eqs. (7) and (9).

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THE RADIATIVE CORRECTION TO THE MASS OF THE ELECTRON IN NONLINEAR ELECTRODYNAMICS

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It has been shown in a previous paper[1] that if in analogy with Einstein's theory of the gravitational field one describes the electromagnetic (vector) field as the curvature of an auxiliary "space" with the metric $ds = \gamma_i dx^i$, then one can obtain a nonlinear Lagrangian of the electromagnetic field, which in the case of a static spherically symmetrical field leads to the potential

$$\varphi = (e/r_0\sqrt{2})\sinh(r_0\sqrt{2}/r), \qquad r_0 = e^2/m_0c^2. \qquad (1)$$

This gives for the classical (unquantized) field mass of a stationary electron $m_{cl} \approx m_0/5$, where m_0 is the experimental rest mass of the electron (the value $m_{cl} \approx m_0/3$ is erroneously given in reference 1).

To calculate the radiative (quantum) correction Δm_q to the mass of the electron, caused by the interaction of the stationary electron with the photon and electron-positron backgrounds, we must first of all find the wave solution of the field equations corresponding to the nonlinear Lagrangian in question. Since this is practically unfeasible because of the great mathematical difficulties, it is not without interest to try to give at least a preliminary and approximate estimate of the size of Δm_q. The idea of the calculation is as follows.

In the nonlinear theory under consideration, one gets in accordance with Eq. (1) for the energy of a stationary charge e situated in the field of another charge e, not the value $E_1 = e\varphi_1 = e^2/r$, but instead

$$E = e\varphi = \frac{e^2}{r_0\sqrt{2}} \sinh \frac{r_0\sqrt{2}}{r} = \frac{m_0c^2}{\sqrt{2}} \sinh\left(\frac{\sqrt{2}}{m_0c^2} E_1\right).$$

If now we assume that this relation between the energies in the linear and nonlinear theories holds not only for the energy of the field of a stationary point charge, but in general for every electromagnetic energy, and apply it to the energy of electromagnetic quanta, then

$$\omega = (\omega_0/\sqrt{2}) \sinh(\sqrt{2}\,\omega_1/\omega_0), \tag{2}$$

where ω_1 is the frequency in the linear theory, and $\omega_0 = m_0c^2/\hbar$ is the critical frequency at which the quantum energy equals the rest energy of the particle with which the quantized field interacts.

In view of the obviously preliminary nature of the present calculation, there is no point in carrying out exact computations of Δm_q; for an approximate quantitative estimate, it suffices to use the simplified formula from the first papers of Weisskopf,[2] according to which

$$\Delta m_q = \frac{\pi e^2 h}{m_0c^2}\left(\int_0^{\omega_0} \frac{1}{\omega} dN + \omega_0^2 \int_{\omega_0}^{\infty} \frac{1}{\omega^3} dN\right), \qquad dN = \frac{8\pi\omega^2 d\omega}{(2\pi c)^3}.$$

Making here the change indicated in Eq. (2) (it is clear that the change is to be made only in the matrix elements and not in dN), we also change the intermediate limit of the integrations: instead of ω_0, we write $\eta\omega_0$, choosing the factor $\eta \sim 1$ in such a way that the integrands have the same value at the place where they are joined. The substitution leads to the expression

$$\Delta m_q = \frac{\alpha}{2\pi} m_0 2\sqrt{2}\left(\int_0^{\eta} \frac{\xi^2 d\xi}{\sinh\sqrt{2}\,\xi} + 2\int_{\eta}^{\infty} \frac{\xi^2 d\xi}{\sinh^3\sqrt{2}\,\xi}\right), \quad \alpha = \frac{e^2}{hc},$$

where $\eta \approx 0.81$. Numerical integration gives a value ≈ 0.374 for the quantity in brackets, so that we have $2 \times 2^{1/2} \times 0.374 \approx 1.06$; therefore we get finally

$$\Delta m_q \approx (\alpha/2\pi)\, m_0.$$

We note that the final value of Δm_q is gotten just from the strong singularity of φ at the origin. This result cannot be given by nonlinear theories with a finite potential at the origin (the Born-Infeld type of theory).

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PARAMAGNETIC ABSORPTION AND ROTATION OF PLANE OF POLARIZATION FOR CERTAIN SALTS IN THE MICROWAVE BAND

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SEVERAL recently-reported experimental investigations[1-4] are devoted to a study of paramagnetic rotation of the plane of polarization in the microwave band, for the case when the preferred direction in the medium (the gyration direction) is perpendicular to the direction of the propagation of the incident wave. For several substances, critical relations were obtained for the angle of rotation of the plane of polarization, β, as a function of the constant field H_0. Curves of this type can be obtained also by other means without directly measuring the angle β. In fact (see references 1 and 6), starting with general considerations, we can obtain the following expression for the angle of rotation of the plane of polarization per unit length of the paramagnet

$$\beta = -(\pi\omega\sqrt{\varepsilon}/c)\,(\chi''_\perp - \chi''_\parallel) \sin 2\alpha, \tag{1}$$

where α is the angle between the constant ($\mathbf{H}_0$) and high frequency fields; $\chi''_\perp$ and $\chi''_\parallel$ are the imaginary parts of the magnetic susceptibility of the paramagnet for perpendicular and parallel fields. To explain the dependence of β on H_0 it is necessary to know the corresponding dependences of the imaginary parts of the magnetic susceptibility, $\chi''_\perp$ and $\chi''_\parallel$, on the field, and these are readily obtained by experiment. Certain results of such experiments are listed below.

The apparatus used in the present investigation, with which we could obtain the dependence of χ'' on H_0 for all angles α, was analogous to the apparatus described by us earlier.[5] The only difference was, first, that in addition to being able to use a cylindrical cavity in the H_{011} mode we could also use a rectangular cavity in the H_{102} mode, and could thus reduce considerably the electromagnet gap, and second, that the generator portion of the apparatus was rigidly coupled to the measuring portion. This eliminated completely the possibility of contact error inherent in the rotating flange of the previous version of the apparatus. To vary the angle α, the entire apparatus was rotated as a unit.

The experiments were performed at room tem-