$$E = e\varphi = \frac{e^2}{r_0\sqrt{2}} \sinh \frac{r_0\sqrt{2}}{r} = \frac{m_0c^2}{\sqrt{2}} \sinh\left(\frac{\sqrt{2}}{m_0c^2} E_1\right).$$

If now we assume that this relation between the energies in the linear and nonlinear theories holds not only for the energy of the field of a stationary point charge, but in general for every electromagnetic energy, and apply it to the energy of electromagnetic quanta, then

$$\omega = (\omega_0/\sqrt{2}) \sinh(\sqrt{2}\,\omega_1/\omega_0), \qquad (2)$$

where ω_1 is the frequency in the linear theory, and $\omega_0 = m_0c^2/\hbar$ is the critical frequency at which the quantum energy equals the rest energy of the particle with which the quantized field interacts.

In view of the obviously preliminary nature of the present calculation, there is no point in carrying out exact computations of Δm_q; for an approximate quantitative estimate, it suffices to use the simplified formula from the first papers of Weisskopf,[2] according to which

$$\Delta m_q = \frac{\pi e^2 h}{m_0c^2}\left(\int_0^{\omega_0} \frac{1}{\omega} dN + \omega_0^2 \int_{\omega_0}^{\infty} \frac{1}{\omega^3} dN\right), \qquad dN = \frac{8\pi\omega^2 d\omega}{(2\pi c)^3}.$$

Making here the change indicated in Eq. (2) (it is clear that the change is to be made only in the matrix elements and not in dN), we also change the intermediate limit of the integrations: instead of ω_0, we write $\eta\omega_0$, choosing the factor $\eta \sim 1$ in such a way that the integrands have the same value at the place where they are joined. The substitution leads to the expression

$$\Delta m_q = \frac{\alpha}{2\pi} m_0 2\sqrt{2}\left(\int_0^{\eta} \frac{\xi^2 d\xi}{\sinh\sqrt{2}\,\xi} + 2\int_{\eta}^{\infty} \frac{\xi^2 d\xi}{\sinh^3\sqrt{2}\,\xi}\right), \quad \alpha = \frac{e^2}{hc},$$

where $\eta \approx 0.81$. Numerical integration gives a value ≈ 0.374 for the quantity in brackets, so that we have $2 \times 2^{1/2} \times 0.374 \approx 1.06$; therefore we get finally

$$\Delta m_q \approx (\alpha/2\pi)\, m_0.$$

We note that the final value of Δm_q is gotten just from the strong singularity of φ at the origin. This result cannot be given by nonlinear theories with a finite potential at the origin (the Born-Infeld type of theory).

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PARAMAGNETIC ABSORPTION AND ROTATION OF PLANE OF POLARIZATION FOR CERTAIN SALTS IN THE MICROWAVE BAND

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SEVERAL recently-reported experimental investigations[1-4] are devoted to a study of paramagnetic rotation of the plane of polarization in the microwave band, for the case when the preferred direction in the medium (the gyration direction) is perpendicular to the direction of the propagation of the incident wave. For several substances, critical relations were obtained for the angle of rotation of the plane of polarization, β, as a function of the constant field H_0. Curves of this type can be obtained also by other means without directly measuring the angle β. In fact (see references 1 and 6), starting with general considerations, we can obtain the following expression for the angle of rotation of the plane of polarization per unit length of the paramagnet

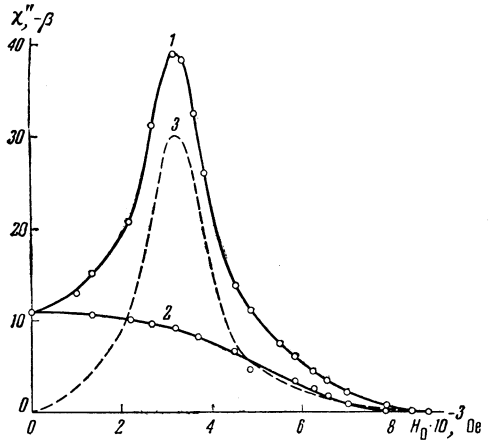
$$\beta = -(\pi\omega\sqrt{\varepsilon}/c)(\chi''_{\perp} - \chi''_{\parallel}) \sin 2\alpha, \qquad (1)$$

where α is the angle between the constant (H_0) and high frequency fields; $\chi''_{\perp}$ and $\chi''_{\parallel}$ are the imaginary parts of the magnetic susceptibility of the paramagnet for perpendicular and parallel fields. To explain the dependence of β on H_0 it is necessary to know the corresponding dependences of the imaginary parts of the magnetic susceptibility, $\chi''_{\perp}$ and $\chi''_{\parallel}$, on the field, and these are readily obtained by experiment. Certain results of such experiments are listed below.

The apparatus used in the present investigation, with which we could obtain the dependence of χ'' on H_0 for all angles α, was analogous to the apparatus described by us earlier.[5] The only difference was, first, that in addition to being able to use a cylindrical cavity in the H_{011} mode we could also use a rectangular cavity in the H_{102} mode, and could thus reduce considerably the electromagnet gap, and second, that the generator portion of the apparatus was rigidly coupled to the measuring portion. This eliminated completely the possibility of contact error inherent in the rotating flange of the previous version of the apparatus. To vary the angle α, the entire apparatus was rotated as a unit.

The experiments were performed at room tem-

perature and at 9150 Mcs, using powdered paramagnetic salts placed in polystyrol containers in the antinode of the magnetic field of the cavity. By suitably locating the cavity in the constant magnetic field it was possible to make the high frequency and constant fields either parallel or perpendicular to each other. Keeping the cavity position fixed, we plotted point by point the absorption vs. magnitude of the constant magnetic field, first with perpendicular fields and then with parallel fields. This was followed by control experiments: the constant field was left unchanged and the apparatus was rotated many times so as to make the fields alternately parallel and perpendicular, and the values of absorption $\chi''_{\perp}$ and $\chi''_{\parallel}$ corresponding to these positions were measured. Such experiments were carried out at various values of the constant field and, particularly, at fields considerably greater than the resonant values, at which the values of $\chi''_{\perp}$ and $\chi''_{\parallel}$ are close to each other. The experiments were performed with powdered salts $MnSO_4 \cdot H_2O$, $MnCl_2 \cdot 4H_2O$, $GdSO_4 \cdot 8H_2O$, GdF_3, $GdCl_3 \cdot 6H_2O$, $GdBr_3 \cdot 6H_2O$, $GdI_3 \cdot 6H_2O$.



It was established by these experiments that, for all the foregoing salts, at fields ranging from 0 to 6000 oersteds, the absorption curves in perpendicular fields lie always above the corresponding curves in parallel fields. We observed no intersecting curves. The diagram shows the absorption curves in perpendicular (curve 1) and parallel (curve 2) fields (the ordinate units are arbitrary) for $MnCl_2 \cdot 4H_2O$. The absorption curves for all other substances listed above are analogous to those shown in the diagram. The only difference is in the widths of the resonance curves, in the position of the maximum, and in the value of the initial absorption $\chi''(0)$. Thus, for $MnSO_4 \cdot H_2O$ and $GdI_3 \cdot 6H_2O$, the values of initial absorption are quite small and the permanent fields hardly influence the absorption in parallel fields, while the paramagnetic resonance is sharply pronounced in perpendicular fields. Many other paramagnetic salts exhibit absorption in parallel and perpendicular fields analogous to that we obtained for $MnCl_2 \cdot 4H_2O$. This follows from the work of Hadders, Locher, and Gorter.[7]

The diagram shows also the dependence of the angle $(-\beta)$ of rotation of the plane of polarization (in arbitrary units) on the value of the permanent field (curve 3), determined as a difference of ordinates of curves 1 and 2, in accordance with Eq. (1). It is seen from the diagram that the rotation curve does not reverse sign over a large interval of fields. (For other substances listed above, the curves of rotation will obviously be analogous.) Imamutdinov, Neprimerov, and Shekun[1] obtained for $MnCl_2 \cdot 4H_2O$ a polarization-plane rotation curve of different shape, one that reverses sign at fields of approximately 4×10^3 oersteds. Obviously this difference in the shapes of the curves calls for further study and it appears to us that it would be interesting to determine the effect of the type of wave propagating in the substance on the rotating ability.

In view of the weak dependence of absorption in parallel fields on the permanent field, the shape of the rotation curve for the salt $MnSO_4 \cdot H_2O$ is close to the shape of the paramagnetic resonance curve, in good agreement with the data of Gozzini.[3]

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