EFFECT OF INTERACTION ON THE PHASE MOTION OF ELECTRONS IN A SYNCHROTRON

L. V. IOGANSEN

Moscow State Pedagogical Institute

Submitted to JETP editor March 4, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 299-300 (July, 1959)

MANY laboratories have recently built electronic accumulating systems for energies of hundreds of millions of electron volts, with the number of bunched electrons reaching $N \sim 10^{14}$. In the calculations, the forces of electromagnetic interaction of the electrons in the bunches are disregarded. In the present note we calculate approximately the effect of these forces on the phase motion of the electrons and obtain an estimate of the equilibrium angular dimensions of the bunch, determined by the interaction forces. We use in our calculations Eq. (3) below for the tangential forces of interaction in the bunch; this equation is correct if the angular dimensions of the bunch are $\vartheta_0 \ll 1$ (reference 1). We do not take into account the screening of the electron interaction by the walls of the chamber and by the magnet. This estimate is therefore valid only for sufficiently small bunches. For bunches whose dimensions are on the order of the transverse diameter of the chamber, the specific nature of the construction of the accelerator must be taken into account. The problem of the influence of the interaction forces on the motion of electrons in a synchrotron was first raised and examined by I. E. Tamm for a specific case (Report of the Physics Institute, U.S.S.R. Academy of Sciences, 1948).

For a circular orbit, limiting ourselves to the ultrarelativistic case $\gamma = E/mc^2 \gg 1$, we obtain, by using the usual derivation, a linearized phase equation in the form

$$\ddot{\psi} + \eta\dot{\psi} + \Omega^2\psi + \frac{c}{(1-n)\,aE_s}\,[W_{s\,\mathrm{coh}} + c\bar{f}_\tau(\psi)] = 0, \quad (1)$$

where ψ is the phase of the electron, measured from the phasing point ψ_s, a is the radius of the stable orbit, and n is the decrement index of the magnetic field. The phasing point is determined from the condition

$$\frac{c}{2\pi a}\,eV_0\sin\psi_s = \dot{E}_s + W_{\gamma s} + W_{s\,\mathrm{coh}},$$

where V_0 is the amplitude of the resonator voltage, E_s the energy of the stable electron, $W_{\gamma s} = (2e^2c/3a^2)\,\gamma^4$ is the energy lost by a single electron to radiation,

$$W_{s\,\mathrm{coh}} = -c\int_{-\infty}^{+\infty}\bar{f}_\tau(\psi)\,\varphi(\psi)\,d\psi \sim Ne^2c/a^2\vartheta_0^{4/3} \quad (2)$$

is the mean energy lost by a single electron to coherent radiation, and

$$\eta = \left\{\left(\frac{3-4n}{1-n}\right)\frac{W_{\gamma s}}{E_s} + \frac{\dot{E}_s}{E_s} + \frac{1}{(1-n)}\,\frac{W_{s\,\mathrm{coh}}}{E_s}\right\},$$

$$\Omega^2 = \frac{c^2eV}{2\pi(1-n)\,a^2E_s}, \qquad V = V_0\cos\psi_s,$$

$$\bar{f}_\tau(\psi) = -\frac{d}{d\psi}\Phi(\psi) = -\frac{d}{d\psi}\left\{\frac{2}{3^{1/3}}\,\frac{Ne^2}{a^2}\int_0^\infty\varphi(\psi-\xi)\,\frac{d\xi}{\xi^{1/3}}\right\}. \quad (3)$$

The distribution of particles by phases, $\varphi(\psi)$, depends on the character of the phase motion and on the distribution of particles by parameters of this motion. As the phase motion proceeds, $\varphi(\psi)$ will in general change. As a result, $\bar{f}_\tau(\psi)$ will also change, making it difficult to obtain an exact solution of Eq. (1). We shall give below a qualitative estimate of the solutions of (1), assuming $\varphi(\psi)$ to be specified and constant. In this case Eq. (1) describes dissipative motion in a well with a potential

$$U(\psi) = \frac{\Omega^2}{2}\psi^2 + \frac{c}{(1-n)\,aE_s}\,[W_{s\,\mathrm{coh}}\psi - c\Phi(\psi)]. \quad (4)$$

We obtain from (2), by the mean-value theorem, $W_{s\,\mathrm{coh}} = -c\bar{f}_\tau(\psi_1)$, where ψ_1 is not an extremal point. Consequently the sum $W_{s\,\mathrm{coh}} + c\bar{f}_\tau(\psi)$ vanishes in the region $|\psi| \lesssim \vartheta_0$ at least at two points, and is negative between these two points. The second term in (4) is therefore essentially a straight line with a positive slope, but must have a well in the region $|\psi| \lesssim \vartheta_0$. The edge of the well lies near $\psi = -\vartheta_0$, and the ends of the bunch always project outside this well. The shape of the potential (4) depends on the ratio of the first and second terms near the point $\psi = -\vartheta_0$. This ratio is best characterized by the dimensionless parameter

$$p = \frac{\Omega^2\vartheta_0}{cW_{s\,\mathrm{coh}}/(1-n)\,aE_s} \sim \left(\frac{aV}{2\pi Ne}\right)\vartheta_0^{7/3}. \quad (5)$$

If $p \gg 1$, the first term in (4) predominates and the potential differs little from an ordinary parabola. To the contrary, when $p \ll 1$, the second term predominates. In this case the potential has two minima, one in the front, in the region $\psi \leq \vartheta_0$ and one in the rear, in the region $\psi < -\vartheta_0$; these are separated by a hump near the point $\psi = -\vartheta_0$. The bunch is located essentially in the forward well, but its ends always project beyond its limits. In the intermediate case $p \sim 1$ we get a point of inflection instead of the rear well and the hump.

It is easy to visualize qualitatively the variation of the dimensions of the bunch in the accumulating system. First, as long as the bunch is large, $p \gg 1$ and the particles perform the ordinary damped oscillations in an almost-parabolic potential. The bunch becomes compressed, the potential (4) is deformed, and the oscillations become distorted. If it becomes possible to compress the bunch in some manner so that $p \ll 1$, the potential will already have two minima. Thanks to the fact that the ends of the bunch project beyond the forward potential wall, the bunch begins to overflow backwards and increases in size. Consequently, there should exist an equilibrium bunch, with angular dimensions of an order of magnitude determined by the condition $p \sim 1$ or

$$\vartheta_0 \sim (2\pi Ne/aV)^{1/3}, \qquad \gamma \gg 1; \qquad \vartheta_0 \ll 1. \qquad (6)$$

In one of the installations now being designed, $N \sim 10^{14}$, $V_0 \sim 10^5$ v, and $a \sim 10^2$ cm (reference 2). It is assumed that the equilibrium angular dimensions of the bunch are determined by the swing of the phase oscillations due to quantum fluctuations of the radiation, and are small at these parameters. Inserting the numerical values in (6) we get $\vartheta_0 \sim 2$. This means that the interaction forces cannot be neglected. However, the estimate (6) itself can no longer be applied. To determine the dimensions of the bunch under these conditions and to answer the question whether the phase stability is disturbed, it is necessary to perform the calculation with allowance for the forces of interaction between the electrons without assuming the bunch to be small, and to take into account the interaction between the bunch and the walls of the chamber and the magnet.

In conclusion, I express my sincere gratitude to Prof. M. S. Rabinovich for valuable advice.

[1] L. V. Iogansen and M. S. Rabinovich, J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 118 (1959), Soviet Phys. JETP this issue, p. 83.

[2] G. K. O'Neill, Stanford University Report, 1958.

Translated by J. G. Adashko
47

DETERMINATION OF THE p-p SCATTERING MATRIX AT 90°

S. B. NURUSHEV

Joint Institute for Nuclear Research

Submitted to JETP editor March 4, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 301-302 (July, 1959)

To determine the p-p scattering matrix at 90° it is necessary to perform five experiments. If we measure at this angle of the value of the cross section I, the coefficient of spin correlation C_{nn}, and the Wolfenstein parameters[1] D, R, and A, then the amplitudes and phases of the components of the p-p scattering matrix can be determined from the relations

$$b^2 = |B|^2/4I = \tfrac{1}{2}(1 - C_{nn}),$$

$$c^2 = 2|C|^2/I = \tfrac{1}{4}(1 + C_{nn} + 2D),$$

$$h^2 = |H|^2/2I = \tfrac{1}{4}(1 + C_{nn} - 2D),$$

$$\sin\delta_C = -(R + A)/2bC, \quad \cos\delta_H = (A - R)/2bh,$$

where B, C, and H are given by

$$B = |B|e^{i\varphi_B}, \quad C = |C|e^{i(\delta_C + \varphi_B)}, \quad H = |H|e^{i(\delta_H + \varphi_B)}.$$

The symbols used are the same as in Wolfenstein's paper.[1]

Since the experimental data are incomplete, we can only estimate the region of possible values of the amplitudes. If we assume $D(90°) = -0.75 \pm 0.15$ for an energy of 140 Mev, which follows from an extrapolation of the data of Taylor,[2] then $0 \le b^2 \le 40\%$, $0 \le c^2 \le 20\%$, and $75 \le h^2 \le 95\%$. For 315 Mev, an estimate was made by Wolfenstein.[3] Combining the experimental data at energies of 382 Mev[4] and 415 Mev[5] and referring them to 400 Mev, we obtain $b^2 = (30 \pm 4)\%$, $c^2 = (56 \pm 5)\%$, and $h^2 = (14 \pm 5)\%$. Using the value of the correlation tensor $C_{kp} = 0.63 \pm 0.10$, measured at 90° and 382 Mev,[6] we can determine the phase difference $\delta_C - \delta_H$, which equals 90°. For 635 Mev, as follows from reference 7, $0 \le b^2 \le 24\%$, $76 \le c^2 \le 100\%$, and $0 \le h^2 \le 12\%$. From this we can determine the possible values of the correlation tensor C_{nn} and of the parameters R and A at 635 Mev, namely $52 \le C_{nn} \le 100\%$, $|R| \le 27\%$, and $|A| \le 21\%$.

It follows from this estimate that in the energy range under consideration the principal contribution to the cross section is made by the triplet interaction. Furthermore, the tensor-like triplet term h^2 predominates in the lower interval,