THE INFLUENCE OF COLLISIONS BETWEEN ELECTRONS ON THEIR VELOCITY DISTRIBUTION IN GASES AND IN SEMICONDUCTORS IN AN ELECTRIC FIELD

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It is well known[1] that in a plasma located in an electric field the velocity distribution function of the electrons will be on the whole symmetrical, i.e., will depend only upon the absolute magnitude of the velocity. Its form is determined for very small degrees of ionization of the plasma by the collisions of the electrons with the heavy particles (atoms, molecules). For higher degrees of ionization an important and even a basic role is played by the collisions between the electrons themselves which must, of course, lead to the approach of the distribution function to the Maxwellian. It is the aim of the present paper* to study the influence of the interelectronic collisions on the symmetric part of the distribution function $f_0(v, t)$.

Following Landau,[4] and also taking the symmetry of the main part of the distribution function into account, we can write the integral for interelectronic collisions S_{ee} in the following form (compare references 5 and 6):

$$S_{ee}(f_0, f_0) = -\frac{1}{v^2}\frac{\partial}{\partial v}\left\{v^2\nu_{ee}\left[A_1(f_0)\frac{\partial f_0}{\partial v} + A_2(f_0)\,v f_0\right]\right\}. \quad (1)$$

Here $\nu_{ee}(v)$ is the frequency of collisions between electrons, and A_1 and A_2 coefficients defined by the relations:

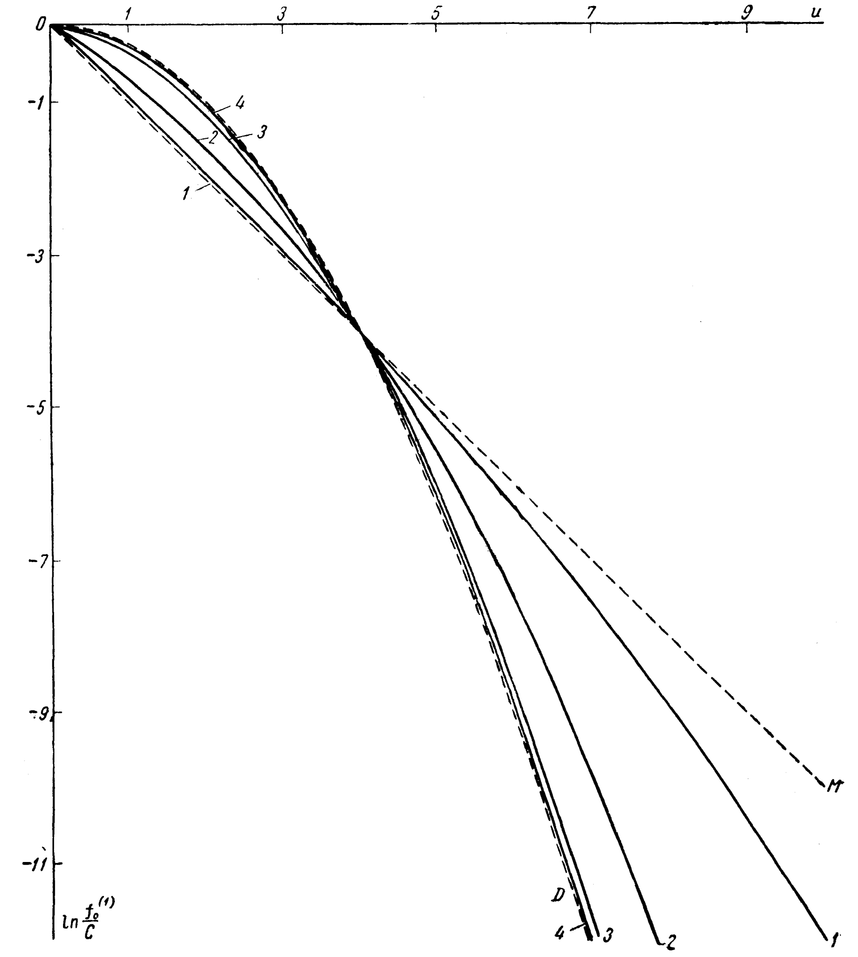
$$A_1 = \frac{4\pi}{3N_e}\left\{\int_0^v v_1^4 f_0(v_1)\,dv_1 + v^3\int_v^\infty v_1 f_0(v_1)\,dv_1\right\},$$

$$A_2 = \frac{4\pi}{N_e}\int_0^v v_1^2 f_0(v_1)\,dv_1,$$

$$\nu_{ee} = (4\pi e^4 N_e/m^2 v^3)\ln\left[k^{3/2}T_e T^{1/2}/e^3 N_e^{1/2}\right], \quad (1')$$

where e, m are the charge and mass of an electron, N_e and T_e the density and temperature of the electrons, and T the temperature of the heavy particles.

Taking collisions between the electrons into



The dependence of $\ln(f_0^{(1)}/C)$ on $u = mv^2/2kT_e$ for different values of the parameter p: curve 1) p = 100; curve 2) p = 10; curve 3) p = 1; curve 4) p = 0.1. The dotted curves are the Maxwellian distribution (M) and the Druyvestein distribution (D).

account, the equation for the function f_0 (see references 1 and 7) becomes an integro-differential and nonlinear equation. Its solution can be obtained by an iteration method. Choosing as the zeroth approximation, $f_0^{(0)}$, the Maxwellian distribution function with an electron temperature defined in the usual way (see, for instance, reference 3), we find that in that approximation the coefficients $A_1^{(0)}$ and $A_2^{(0)}$ are given by the following expressions:

$$A_1^{(0)} = \frac{kT_e}{m} A_2^{(0)} = \frac{kT_e}{m} A(\sqrt{u}), \text{ where } u = \frac{mv^2}{2kT_e},$$

$$A(x) = \Phi(x) - \frac{2}{\sqrt{\pi}} x \cdot e^{-x^2} \quad (2)$$

[$\Phi(x)$ is the error integral]. Substituting now (1) and (2) into the equation for the function f_0, we find easily its solution and thus obtain the first iteration $f_0^{(1)}$. In the case of a strong constant electric field, for instance ($E \gg kT\sqrt{\delta}/el$):†

$$f_0^{(1)} = C \exp\left\{-\int_0^{mv^2/2kT_e} \frac{u^2 + pA(\sqrt{u})}{2u + pA(\sqrt{u})} du\right\}. \quad (3)$$

Here $T_e = eEl/\sqrt{6\delta}$ is the temperature of the electrons, l the mean free path of the electrons which is independent of the velocities, δ the average fraction of energy lost by an electron in one collision (in the case of elastic collisions $\delta = 2m/M$). Finally,

$$p = \frac{2\nu_{ee}(\sqrt{2kT_e/m})l}{\delta\sqrt{2kT_e/m}} = \frac{12\pi e^2 N_e}{lE^2} \ln\left(\frac{k^{3/2}T_e T^{1/2}}{e^3 N_e^{1/2}}\right).$$

The parameter p characterizes the influence of the interelectronic collisions on the distribution function. For small values of p the function $f_0^{(1)}$ is the same as the one given by Druyvestein,[8] and for large p the same as the Maxwellian one, as should be the case. From the graphs given in the figure it is clear that in the region of large u (i.e., in the "tail" of the distribution function) the deviations from the Maxwellian distribution are appreciable even for p = 100.

Calculations show that the next iterations lead only to an unimportant change in the distribution function: the difference between $f_0^{(1)}$ and $f_0^{(2)}$ is a maximum for $p \sim 10$, but in that case $0.9 \le f_0^{(1)}/f_0^{(2)} \le 1.0$ (while $0.5 \le f_0^{(0)}/f_0^{(1)} \lesssim \infty$). We note also that for large values of u the functions $f_0^{(1)}$ and $f_0^{(2)}$ practically coincide; in that case the function f_0 is given approximately by the following expression:

$$f_0 = C \exp\left\{-\frac{u^2}{4} + \frac{pu}{4} - \frac{p(p+4)}{8} \ln\left(1 + \frac{2u}{p}\right)\right\}. \quad (4)$$

The influence of the interelectronic collisions on the distribution function of electrons in semiconductors can be taken into account in a similar way. In a strong electric field, in particular, the same expression (3) is valid for $f_0^{(1)}$ (one needs only bear in mind that in semiconductors $\delta = 2mv_s^2/kT$, where v_s is the sound velocity[1]).

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*The influence of the collisions between the electrons on the directed (current) part of the distribution function in a strongly ionized plasma was considered by Landshoff[2] and the author.[3] It is inappreciable in the case of a weakly ionized plasma.

†A similar expression for the function $f_0^{(1)}$ is also obtained in a variable electric field and also when there is a constant magnetic field present (see reference 7).

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ANGULAR CORRELATION IN INTERNAL CONVERSION, INCLUDING EFFECTS OF SCREENING AND OF THE FINITE SIZE OF THE NUCLEUS

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As has been shown in papers by Rose et al.[1] and by Dolginov,[2] the angular correlation of a conversion electron with any subsequent radiation x can