account, the equation for the function f_0 (see references 1 and 7) becomes an integro-differential and nonlinear equation. Its solution can be obtained by an iteration method. Choosing as the zeroth approximation, $f_0^{(0)}$, the Maxwellian distribution function with an electron temperature defined in the usual way (see, for instance, reference 3), we find that in that approximation the coefficients $A_1^{(0)}$ and $A_2^{(0)}$ are given by the following expressions:

$$A_1^{(0)} = \frac{kT_e}{m} A_2^{(0)} = \frac{kT_e}{m} A(\sqrt{u}), \text{ where } u = \frac{mv^2}{2kT_e},$$

$$A(x) = \Phi(x) - \frac{2}{\sqrt{\pi}} x \cdot e^{-x^2} \tag{2}$$

[$\Phi(x)$ is the error integral]. Substituting now (1) and (2) into the equation for the function f_0, we find easily its solution and thus obtain the first iteration $f_0^{(1)}$. In the case of a strong constant electric field, for instance ($E \gg kT\sqrt{\delta}/el$):†

$$f_0^{(1)} = C \exp\left\{- \int_0^{mv^2/2kT_e} \frac{u^2 + pA(\sqrt{u})}{2u + pA(\sqrt{u})} du\right\}. \tag{3}$$

Here $T_e = eEl/\sqrt{6\delta}$ is the temperature of the electrons, l the mean free path of the electrons which is independent of the velocities, δ the average fraction of energy lost by an electron in one collision (in the case of elastic collisions $\delta = 2m/M$). Finally,

$$p = \frac{2\nu_{ee}(\sqrt{2kT_e/m})l}{\delta\sqrt{2kT_e/m}} = \frac{12\pi e^2 N_e}{lE^2} \ln\left(\frac{k^{3/2} T_e T^{1/2}}{e^3 N_e^{1/2}}\right).$$

The parameter p characterizes the influence of the interelectronic collisions on the distribution function. For small values of p the function $f_0^{(1)}$ is the same as the one given by Druyvestein,[8] and for large p the same as the Maxwellian one, as should be the case. From the graphs given in the figure it is clear that in the region of large u (i.e., in the "tail" of the distribution function) the deviations from the Maxwellian distribution are appreciable even for $p = 100$.

Calculations show that the next iterations lead only to an unimportant change in the distribution function: the difference between $f_0^{(1)}$ and $f_0^{(2)}$ is a maximum for $p \sim 10$, but in that case $0.9 \le f_0^{(1)}/f_0^{(2)} \le 1.0$ (while $0.5 \le f_0^{(0)}/f_0^{(1)} \lesssim \infty$). We note also that for large values of u the functions $f_0^{(1)}$ and $f_0^{(2)}$ practically coincide; in that case the function f_0 is given approximately by the following expression:

$$f_0 = C \exp\left\{-\frac{u^2}{4} + \frac{pu}{4} - \frac{p(p+4)}{8} \ln\left(1 + \frac{2u}{p}\right)\right\}. \tag{4}$$

The influence of the interelectronic collisions on the distribution function of electrons in semiconductors can be taken into account in a similar way. In a strong electric field, in particular, the same expression (3) is valid for $f_0^{(1)}$ (one needs only bear in mind that in semiconductors $\delta = 2mv_s^2/kT$, where v_s is the sound velocity[1]).

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*The influence of the collisions between the electrons on the directed (current) part of the distribution function in a strongly ionized plasma was considered by Landshoff[2] and the author.[3] It is inappreciable in the case of a weakly ionized plasma.

†A similar expression for the function $f_0^{(1)}$ is also obtained in a variable electric field and also when there is a constant magnetic field present (see reference 7).

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ANGULAR CORRELATION IN INTERNAL CONVERSION, INCLUDING EFFECTS OF SCREENING AND OF THE FINITE SIZE OF THE NUCLEUS

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As has been shown in papers by Rose et al.[1] and by Dolginov,[2] the angular correlation of a conversion electron with any subsequent radiation x can be expressed in the following way:

$$W_{e-x}(\vartheta) = \sum_{\nu=0}^{\nu_m} b_\nu A_\nu (\gamma - x) P_\nu (\cos \vartheta)$$

using the notation of Rose. Here b_ν are the angular correlation coefficients. If $b_\nu = 1$, we get the well-known expansion of the angular correlation function between the γ-ray quantum and the radiation x in terms of Legendre polynomials $P_\nu(\cos\vartheta)$. The formulas for the calculation of the b_ν in the case of the K shell are:

$$b_\nu^{(L)}(e) = 1 + \frac{\nu(\nu+1)}{2L(L+1) - \nu(\nu+1)} \frac{L}{2L+1} \frac{|L+1+T_e|^2}{L(L+1)+|T_e|^2},$$

$$T_e = \frac{\exp(i\delta_L)}{\exp(i\delta_{-L-1})} \frac{[L(R_1+R_2) - (2L+1)R_3 - R_4]_{\varkappa=L}}{[R_1+R_2+2R_4]_{\varkappa=-L-1}};$$

$$b_\nu^{(L)}(m) = 1 + \frac{\nu(\nu+1)}{2L(L+1) - \nu(\nu+1)} \frac{L(L+1)}{2L+1}$$

$$\times \frac{|1-T_m|^2}{L+1+L|T_m|^2},$$

$$T_m = \frac{\exp(i\delta_{L+1})}{\exp(i\delta_{-L})} \frac{(R_3'+R_4')_{\varkappa=L+1}}{(R_3'+R_4')_{\varkappa=-L}},$$

$$L = 1, 2, 3, 4, 5;\ \nu = 0, 2, 4.$$

Here $b_\nu^{(L)}(e)$ are the coefficients for electric transitions, $b_\nu^{(L)}(m)$ are those for magnetic transitions, L is the degree of the multipole, δ_L and δ_{-L} are phase shifts, and R_i and R_i' are radial integrals.

All of the coefficients b_ν $(\nu > 2)$ can be obtained from b_2 by the formula

$$b_\nu(\pi) - 1 = \frac{\nu(\nu+1)[L(L+1)-3]}{3[2L(L+1)-\nu(\nu+1)]}[b_2(\pi) - 1],$$

where $\pi = e$ or m.

The paper by Rose et al.[1] presents curves of $b_2(\pi)$ for 12 elements, $10 \leq Z \leq 96$, obtained on the assumption of the Coulomb field of a point nucleus. We have obtained curves of $b_2(\pi)$ for Z = 81, 84, 88, 92, using the phase shifts[3] and radial integrals calculated with allowance for effects of screening and of the finite size of the nucleus, as in the work of Sliv and Band[4] on internal conversion coefficients. Let us compare these curves over the range of variation of the energy k from 0.5 to 2.0 mc^2.

All of our curves are displaced toward smaller Z as compared with those of Rose (for example, our curves for Z = 92 are displaced relative to those of Rose toward the curves for Z = 88). For most of the cases the maximum shift is insignificant [for $b_2^{(1)}(e)$, $b_2^{(3)}(e)$, $b_2^{(4)}(e)$, $b_2^{(2)}(m)$, $b_2^{(3)}(m)$, $b_2^{(4)}(m)$ it is 1 percent, for $b_2^{(5)}(e)$, 0.2 percent, and for $b_2^{(5)}(m)$, 0.1 percent], and occurs at energy $k = 2.0\ mc^2$. The exceptions are $b_2^{(2)}(e)$ and $b_2^{(1)}(m)$. The maximum shift of $b_2^{(2)}(e)$ reaches 7 percent at k = 0.5; that of $b_2^{(1)}(m)$ at k = 2.0 for Z = 88 is 60 percent, and for Z = 92 it is 100 percent. (See Figs. 1 and 2; the curves of Rose are shown as dashed lines.)

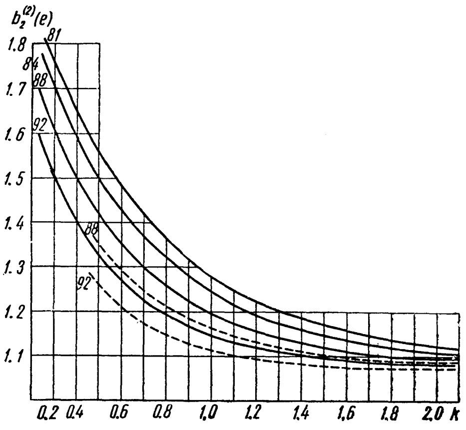

FIG. 1

These deviations agree with previously obtained results,[4] since the tables of internal conversion coefficients (ICC) and our curves for $b_2^{(L)}$ have been obtained with the same radial integrals. The largest deviations of the ICC from the Coulomb values were found in the case of magnetic dipole transitions (L = 1) for heavy elements.

At present, analogous calculations of the coefficients b_2 are being made for the L shell.

In conclusion the writer expresses her deep gratitude to A. Z. Dolginov and L. A. Sliv for valuable advice and suggestions that have been used in the preparation of this paper.

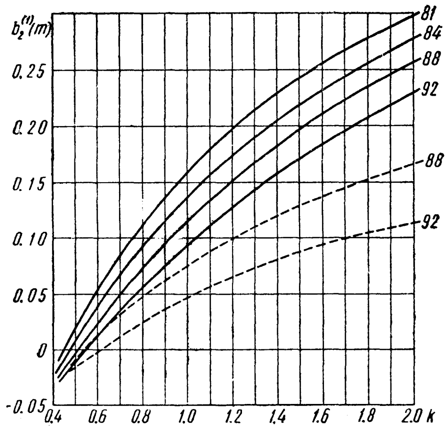

FIG. 2

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MEASUREMENTS OF THE TRANSVERSE PROTON RELAXATION TIME IN AQUEOUS SOLUTIONS OF PARAMAGNETIC SALTS BY THE SPIN-ECHO METHOD

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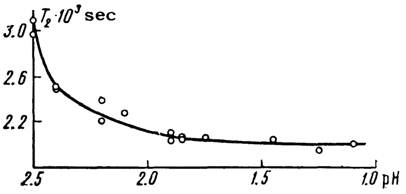
THE spin-echo effect[1,2] makes it possible to determine experimentally the absolute values of the longitudinal and transverse nuclear relaxation times T_1 and T_2. The advantage of the spin-echo method over other methods is particularly evident in measurements of nuclear relaxation times in low-viscosity liquids.[2]

To obtain the spin-echo effect, we constructed apparatus for the quantitative measurement of the transverse relaxation time T_2 of protons in water, in the presence of paramagnetic ions. The measurement was carried out at 12.2 Mcs in the field of a permanent magnet. A radio-frequency magnetic field with an amplitude of ~ 3.7 oersteds was applied to the sample in the form of two successive short rectangular pulses, with approximate durations of 16 and 32 microseconds, insuring 90° and 180° nutations respectively in the magnetic polarization of the water protons. The interval between the pulses could be varied from 0.3 to 2 milliseconds.

The solution to be studied, contained in a glass tube, was placed in a radio-frequency induction head of the "through-flow" type,[3] whose receiving coil picked up the induced nuclear echo signal, which was then amplified by a receiver with a 120 kcs bandwidth. The amplified output voltage from the receiver was applied to an oscillograph, and the echo signal on its screen was photographed. The non-uniformity of the permanent magnet field was estimated from the width of the echo signal. It proved to be equal to 1.2 oersted in the region where the sample (1 cm^3 in volume) was located.

In a field of the above non-uniformity, the effect of the self-diffusion of water molecules during the experimentally-determined time T_2 (about 10^{-3} sec) was negligible.

As a check on the operation of the apparatus, experiments were carried out on the hydrolysis of iron in nitric acid solution. In all the experiments the concentrations of Fe^{+++} and nitrate ions were constant and equal to 0.02 M and 0.6 M respectively. The results of the T_2 measurements for water protons in $Fe(NO_3)_3$ solutions are shown in the figure as a function of the acidity (pH). It is well known[4] that the proton relaxation effect in solutions of paramagnetic salts depends on the degree to which the ions are bound into a complex. The values of T_2 at higher pH are considerably larger than at high acidity. The gradual reduction in T_2 with increasing acidity indicates an increase in the effectiveness of the paramagnetic iron ions in shortening the transverse relaxation time of protons. This can be explained by the fact that when the pH is reduced the concentration of hydroxyl ions in the solution is decreased, leading to the dissociation of the iron hydro-complex. With increasing relative amounts of free iron ions in the solution, which are more effective in causing relaxation, the time T_2 must become shorter, as is well confirmed by the trend of the data shown in the figure.



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