tion the question remains open whether we are dealing with the conversion of a virtual or a real photon into an electron-positron pair. One of the possible explanations is the decay $\mu^+ \to e^+ + \nu + \tilde{\nu} + \gamma$ with subsequent conversion of the photon into a pair.

The present event was observed in scanning about 50,000 muon decays. Thus the relative probability of a "three-electron" decay of a muon may be estimated as $p(3e)/p(e) \leq 2 \times 10^{-5}$. If the data of other authors, who have observed a large number of μ-e decays and have not discovered the "three-electron" decay, are considered, then the estimated probability of such a process must be reduced to a few times 10^{-6}. The reliability of this number is not great, since only one case of "three-electron" decay has been observed, and therefore it is impossible to absolutely exclude the possibility of an accidental superposition of tracks.

The probability $p(3e)/p(e)$ of the order of 10^{-6} can be obtained by assuming a second-order radiative process: emission of a virtual photon during the escape of the electron with its subsequent conversion into an electron-positron pair. The energy of such an electron-positron pair may be estimated from the angle formed by the tracks of the pair, which is about 8°, approximately the same for all three possible pairs of electron tracks, and is equal to 15 Mev.

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HYDRODYNAMICS OF SOLUTIONS OF STRANGE PARTICLES IN HELIUM II NEAR THE λ POINT

D. G. SANIKIDZE

Institute of Physics, Academy of Sciences, Georgian S.S.R.

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We have derived the equations for the hydrodynamics of solutions of strange particles in helium II in the immediate vicinity of the λ-transition point. In contradistinction to the usual set of equations, from the point of view considered here (compare reference 1) ρ_s is not a given function of p, T, and the concentration c, but is determined from these equations themselves which also describe the process by which ρ_s approaches its equilibrium value. As in the paper by Ginzburg and Pitaevskiĭ[1] the superfluid part of the liquid is described by a complex function $\psi(x, y, z, t) = \eta e^{i\varphi}$ defined in such a way that

$$\rho_s = m|\psi|^2, \qquad \mathbf{v}_s = (\hbar/m)\nabla\varphi$$

(m is the mass of a He^4 atom).

The derivation is analogous to the one used by Pitaevskiĭ[2] in deriving the equations for the hydrodynamics of pure helium II near the λ point. We shall, therefore, not give the calculations but write down the final result:

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\Delta\psi + \left[\left(\frac{\partial\varepsilon}{\partial\rho}\right)_{\rho_s, S, c} + \left(\frac{\partial\varepsilon}{\partial\rho_s}\right)_{\rho, S, c} - \frac{Z}{\rho}c\right]m\psi - i\Lambda\left[\frac{1}{2}\left(-\frac{i\hbar}{m}\nabla - \mathbf{v}_n\right)^2 + \left(\frac{\partial\varepsilon}{\partial\rho_s}\right)_{\rho, S, c}\right]m\psi; \quad (1)$$

$$\partial\rho/\partial t + \operatorname{div}(\rho - m|\psi|^2)\mathbf{v}_n + (i\hbar/2)(\psi\Delta\psi^* - \psi^*\Delta\psi) = 0; \quad (2)$$

$$\partial(\rho c)/\partial t + \operatorname{div}(\rho c\mathbf{v}_n + \mathbf{g}) = 0; \quad (3)$$

$$\frac{\partial}{\partial t}\left\{(\rho - m|\psi|^2)v_{ni} + \frac{i\hbar}{2}\left(\psi\frac{\partial\psi^*}{\partial x_i} - \psi^*\frac{\partial\psi}{\partial x_i}\right)\right\} = -\frac{\partial}{\partial x_k}\left\{(\rho - m|\psi|^2)v_{ni}v_{nk} + \frac{\hbar^2}{2m}\left(\frac{\partial\psi}{\partial x_i}\frac{\partial\psi^*}{\partial x_k} - \psi\frac{\partial^2\psi^*}{\partial x_i\partial x_k} + \text{c.c.}\right) + p\delta_{ik} - \eta\left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i} - \frac{2}{3}\delta_{ik}\operatorname{div}\mathbf{v}_n\right)\right\}; \quad (4)$$

$$\frac{\partial S}{\partial t} + \operatorname{div}\left[S\mathbf{v}_n + \frac{1}{T}\left(\mathbf{q} - \frac{\mathbf{g}Z}{\rho}\right)\right] = \frac{R}{T}. \quad (5)$$

The dissipative function of the liquid is

$$R = \frac{2\Lambda}{\hbar}\left|\left[\frac{1}{2}\left(-\frac{i\hbar}{m}\nabla - \mathbf{v}_n\right)^2 + \left(\frac{\partial\varepsilon}{\partial\rho_1}\right)_{\rho, S, c}\right]m\psi\right|^2 + \frac{1}{2}\eta\left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_{nl}}{\partial x_l}\right)^2 - \mathbf{q}\frac{\nabla T}{T} - \mathbf{g}T\nabla\frac{Z}{\rho T}.$$

The impurity current **g** and heat current **q** are expressed by the usual equations.[3]

In the case of small gradients of ρ_s Eqs. (1) to (5) go over into the following ones:

$$\mathbf{v}_s + \nabla\left\{\frac{v_s^2}{2} + \left(\frac{\partial\varepsilon}{\partial\rho}\right)_{\rho_s, S, c} + \left(\frac{\partial\varepsilon}{\partial\rho_s}\right)_{\rho, S, c} - \frac{Z}{\rho}c - \frac{\hbar\Lambda}{2m\rho_s}\operatorname{div}\rho_s(\mathbf{v}_s - \mathbf{v}_n)\right\} = 0, \quad (6)$$

$$\partial\rho/\partial t + \operatorname{div}(\rho_s\mathbf{v}_s + \rho_n\mathbf{v}_n) = 0, \quad (7)$$

$$\partial(\rho c)/\partial t + \operatorname{div}(\rho c\mathbf{v}_n + \mathbf{g}) = 0, \quad (8)$$

$$\frac{\partial}{\partial t}(\rho_n v_{ni} + \rho_s v_{si}) + \frac{\partial}{\partial x_k}\Big\{\rho_n v_{ni} v_{nk} + \rho_s v_{si} v_{sk} + p\delta_{ik} - \eta\left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i} - \frac{2}{3}\delta_{ik}\,\mathrm{div}\,\mathbf{v}_n\right)\Big\} = 0, \quad (9)$$

$$\frac{\partial S}{\partial t} + \mathrm{div}\Big[S\mathbf{v}_n + \frac{1}{T}\Big(\mathbf{q} - \frac{\mathbf{g}Z}{\rho}\Big)\Big] = \frac{R}{T}, \quad (10)$$

$$\frac{\partial \rho_s}{\partial t} + \mathrm{div}\,\rho_s\mathbf{v}_s = -\frac{\Lambda m}{2\hbar}\left\{\frac{(\mathbf{v}_n - \mathbf{v}_s)^2}{2} + \left(\frac{\partial \varepsilon}{\partial \rho_s}\right)_{\rho, S, c}\right\}\rho_s. \quad (11)$$

The dissipative function of the liquid is

$$R = \frac{\hbar\Lambda}{2m\rho_s}[\mathrm{div}\,\rho_s(\mathbf{v}_s - \mathbf{v}_n)]^2 + \frac{2\Lambda m}{\hbar}\left[\frac{1}{2}(\mathbf{v}_s - \mathbf{v}_n)^2 + \left(\frac{\partial \varepsilon}{\partial \rho_s}\right)_{\rho, S, c}\right]^2\rho_s + \frac{1}{2}\eta\left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_{kl}}{\partial x_l}\right)^2 - \mathbf{q}\frac{\nabla T}{T} - \mathbf{g}T\nabla\frac{Z}{\rho T}.$$

Formally Eqs. (6) to (10) are the same as the usual set of equations for the hydrodynamics of solutions of extraneous particles in helium II except, however, that ρ_S is not given in them but is an independent quantity for which the approach to its equilibrium value is described by the additional Eq. (11). In the given equations the quantity

$$(\partial\varepsilon/\partial\rho_s)_{\rho_s, S, c} + (\partial\varepsilon/\partial\rho)_{\rho, S, c} - Zc/\rho,$$

where $Z = (\partial\epsilon/\partial c)_{\rho,S,\rho_S}$, plays the role of the chemical potential of He^4 in the solution.

The parameter Λ entering into the equations could be estimated from a comparison of the absorption coefficient for first sound evaluated from Eqs. (6) to (11) with the measured value of the absorption coefficient in He^3-He^4 solutions near the λ point. There are not, however, at the present time any such experimental data. An estimate made by Pitaevskiĭ[2] for pure helium II gives $\Lambda \approx 15$.

For definite applications of the theory it is necessary to know the function $\epsilon(\rho, S, \rho_S, c)$, which can be determined from experimental data on the dependence of the superfluid component of He^3-He^4 solutions near the λ point on p, T, c, and on its density.

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[1] V. L. Ginzburg and L. P. Pitaevskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1240 (1958), Soviet Phys. JETP **7**, 858 (1958).

[2] L. P. Pitaevskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 408 (1958), Soviet Phys. JETP **8**, 282 (1959).

[3] I. M. Khalatnikov, Usp. Fiz. Nauk **60**, 69 (1956), Fortschr. Phys. **5**, 287 (1957).

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*THE PROPAGATION OF OSCILLATIONS ALONG VORTEX LINES IN ROTATING HELIUM II**

É. L. ANDRONIKASHVILI and D. S. TSAKADZE

Tbilisi State University

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According to the theory developed by Feynman[1] on the basis of Onsager's hypothesis, there should appear in rotating helium II vortex lines parallel to the axis of rotation and running through the whole of the liquid. Experiments performed by us to confirm this hypothesis[2] showed that rotating helium II possesses, when twirled around, a quite appreciable elasticity. The presence of such an elasticity is also confirmed by the experiments of Hall.[3]

In the interpretation of these experiments we assumed that transverse elastic waves were propagated along the vortex lines. This point of view was confirmed by Hall (private communication), who observed a periodic change in the frequency of the oscillation of a light disc suspended in rotating helium II under such conditions that the liquid level above it was changing continuously. At the same time the length of the vortices, which on the one side were fastened to the surface of the disc and on the other side to the free surface of the liquid, was also changing, as assumed by Hall. The periodic changes in the frequency of the oscillations were within a range of one per cent.

In contradistinction to Hall, we measured the magnitude of the logarithmic decrement of the damping δ of the oscillations of an elastically suspended disc, which were performed at the same time as the rotation together with the helium II. The damping decrement was measured by a method described earlier.[4] The time dependence of the distance between the disc and the liquid surface was studied by a periscope system of mirrors and a cathetometer.