ON DIRECT PRODUCTION OF ELECTRON-POSITRON PAIRS BY HIGH-ENERGY ELECTRONS

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The absolute number of spurious tridents for 10^{10}, 10^{11}, and 10^{12} ev primary electrons is computed by the Monte Carlo method for two types of the bremsstrahlung spectrum: for the one given by the Bethe-Heitler formula and for that described by the Migdal formulae, which take into account the Landau-Pomeranchuk and the Ter-Mikaelyan effects. It is shown that it is feasible to measure the energy of fast electrons by determining the energy dependence of the mean transverse distance between the vertices of electron-positron pairs produced by bremsstrahlung γ quanta. The value of the cross section for the direct production of electron-positron pairs calculated by Bhabba is confirmed experimentally.

1. INTRODUCTION

MANY investigations[1-13] have been devoted to the determination of the cross section for the direct pair production by high-energy electrons in the energy range 0.5 — 100 Bev. Different authors obtained different cross sections for direct pair production. The cross sections measured in references 1, 3, 6, 7, and 8, are several times larger than the value calculated by Bhabha,[14] while, in the other experiments, no such discrepancy is observed.

The basic difficulty in an experimental determination of the cross section for direct production of electron-positron pairs by high-energy electrons lies in the necessity of excluding the so-called spurious tridents.*

In the majority of the experiments mentioned above, a correction for the number of spurious tridents, calculated by Kaplon and Koshiba,[1] has been used. However, this correction for the number of spurious tridents represents only a rough approximation, since a number of factors, which substantially influence the number of the spurious tridents, have been neglected. In the calculations of Kaplon and Koshiba, the change in the energy of the electron due to the emission of bremsstrahlung quanta had been neglected. This energy decrease, however, has a direct bearing on the electron scattering, which determines essentially the number of spurious tridents. In addition, the decrease in the electron energy softens the energy spectrum of the emitted photons and leads to an increase of the angles at which they are emitted. The conversion mean free path was assumed to be the same for γ quanta of all energies, and to be equal to its minimum value, i.e., $9/7$ radiation lengths, although, for photons of comparatively small energies (≤ 1 Bev), its marked increase should be taken into account. The angular distribution of photons was assumed to be Gaussian, which also is an insufficiently accurate approximation.

Finally, the results of the calculation are given as a ratio of the number of spurious tridents to the total number of pairs produced in the conversion of γ quanta of the electron bremsstrahlung. The use of a correction of such a form for the number of spurious tridents can lead to ambiguity, since the pairs produced by electron bremsstrahlung at sufficiently large distances from the track of the electron have a rather low detection efficiency in contrast to the pairs near the track, and spurious tridents are detected with an efficiency of almost 100%. On the other hand, the presence of secondary pairs of the electromagnetic cascade is always possible near the electron track, and a suitable procedure of distinguishing these pairs from those produced by the bremsstrahlung radiation of the primary electron does not exist. It should still be noted that the number of pairs of high-energy electrons produced by the

*We shall use the following terminology: trident — an event of a direct production of an electron-positron pair by an electron; spurious trident — an electron-positron pair produced in the conversion of a γ quantum of the bremsstrahlung radiation of an electron at a very small distance from its track. The tracks of the electron and of the pair produced by the γ quantum cannot be distinguished in experiments from the event of a direct production of an electron-positron pair by an electron; an observed trident is either a trident or a spurious trident.

bremsstrahlung is subject to marked fluctuations even for a strictly fixed value of the electron energy.

The difficulty in applying the correction was partially avoided in the experiment of Fay,[2] who calculated the average number of bremsstrahlung pairs for the energy range of the measurements of the cross section for the production of tridents by electrons.

In addition to Kaplon and Koshiba,[1] the correction for the spurious tridents has also been calculated by Block and King,[3] but their work is also not free from the above-mentioned errors, and, in addition, they assumed that the bremsstrahlung quanta are emitted only at the beginning of the electron trajectory, which is not the actual case.

In connection with the above, a calculation of the number of spurious tridents has been carried out in reference 4 by means of the Monte Carlo method. The calculation has been carried out, as in reference 1, for nuclear emulsions. After reference 4 had already gone to press, the authors had the occasion to get acquainted with the work of Weill, Gailloud, and Rosselet,[5] who calculated the number of spurious tridents by numerical integration.

In the present article, a new, improved calculation by the Monte Carlo method is presented. A detailed comparison (where this is possible) with the calculations of Weill et al.[5] is carried out. The absolute number of spurious tridents obtained is used in a calculation, based on our experimental data, of the cross section for direct pair production by electrons.

2. CALCULATION OF THE ABSOLUTE NUMBER OF SPURIOUS TRIDENTS BY THE MONTE CARLO METHOD

The calculation was carried out for electrons of three primary energies: 10^{10}, 10^{11}, and 10^{12} ev. A total of 105 case histories of the passage of an electron through one radiation length, assumed to equal to 2.9 cm in the emulsion, were traced for each of these energies.

It was assumed that the electron, with a given initial energy, is produced at the point $x = y = z = 0$ (Fig. 1) and moves along the x axis. Its passage through matter was followed up to $x = 2.9$ cm. This limitation was dictated by the fact that, for $x > 2.9$ cm, a cascade initiated by the primary high-energy electron develops intensely, which makes it difficult to follow the electron track in the emulsion. Apart from this, a marked degradation of the energy of the initial electron occurs at such distances.

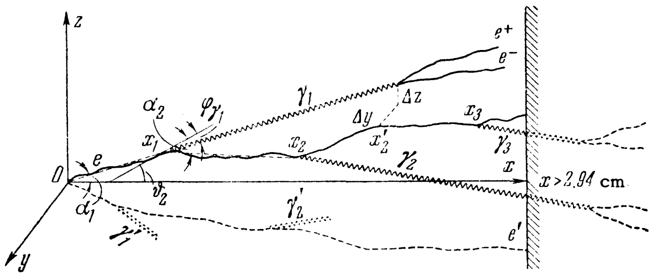

FIG. 1. The Monte Carlo case history.

The coordinates x_i of the points where γ quanta were emitted by an electron of energy E_i were found by using the value of the total cross section for the bremsstrahlung, according to Bethe and Heitler. This cross section was obtained by integrating the differential energy spectrum, for the case of complete screening, from the electron energy $2mc^2$ to E_i.[15a] The choice of the lower energy limit is due to the fact that the only γ quanta of interest are those with energies sufficient to produce electron-positron pairs. The bremsstrahlung cross section, as well as all the cross sections used in the calculations for the elementary processes were computed for the NIKFI-R nuclear emulsion. However, the results are equally correct for the Ilford G-5 emulsion, since the composition of both types is almost identical.

The energy of the emitted γ quantum was determined from the differential energy spectrum of Bethe and Heitler.[15a] Two projections of the angle $\overline{\varphi}$, at which the bremsstrahlung γ quantum was emitted, were found by using the expression for the angular distribution in the ultrarelativistic case.[16] For each emitted energy quantum $E_{\gamma i}$, the decrease in the electron energy beyond the point x_i was taken into account: the energy of the electron beyond the point x_i was taken as $E_{i+1} = E_i - E_{\gamma i}$. In general, it was assumed that, in the interval $x_i \le x \le x_{i+1}$, the electron energy was $E_{i+1} = E_0 - \sum_1^i E_{\gamma k}$, E_0 being the initial electron energy. All elementary processes along the path $x_i \le x \le x_{i+1}$ were found for an electron energy equal to E_{i+1}.

The x coordinates of the conversion points x'_m were found by means of the equations of reference 15b for the pair production cross section (the formulae were integrated numerically) for γ quanta emitted at points $x_1, x_2 \ldots, x_i$. Electron pairs with $x' > 2.9$ cm were not considered any further. The electron coordinates y_i, z_i, y'_m, z'_m due to the multiple scattering were then found. In the multiple scattering process, two angles were taken into account: the angle of the chord $\overline{\alpha}$, and the angle of the tangent $\overline{\vartheta}$. The distribution of the projection of the angle $\overline{\alpha}$, was obtained from the Gaussian distribution for y and z ($\alpha_y = y/x$, $\alpha_z = y/z$). However, for fixed y and z, the distribution of the projection of the angles of the tangent is no longer Gaussian. In such a case, it is necessary to use the Fermi distribution[15C] which, for the projection on y and for a given y, is given by the formula

$$P(x, y, \vartheta_y)\, dy d\vartheta_y = \frac{2\sqrt{3}}{\pi\theta_s^2 x^2} \exp\left[-\frac{4}{\theta_s^2 x}\left(\vartheta_y^2 - \frac{3y}{x}\vartheta_y + \frac{3y^2}{x^2}\right)\right].$$

However, it can be shown that the distribution for the projection of the angle $\vartheta'_y = \vartheta_y - {}^3\!/_2\,\alpha_y$ is Gaussian. In view of this fact, we first found the angles ϑ', and then the projections of the true angle of the tangent were found from the relation $\vartheta_1 = \vartheta'_i + {}^3\!/_2\,\alpha_{i-1}$. (See Fig. 1; ϑ_1 was always assumed to be equal to zero).

Each of the coordinates of the electron y_n and z_n at any point x_n is expressed as:

$$y_n = \sum_1^n x_i\left(\sum_1^n \vartheta_{iy} + \alpha_{nu}\right);$$

$$z_n = \sum_1^n x_i\left(\sum_1^n \vartheta_{iz} + \alpha_{nz}\right).$$

where x_i denotes the distance between x_i and x_{i-1}. Coordinates of the conversion points of the γ quanta and the vertices of the electron pairs are equal to

$$y'_k = \sum_1^l x_i\left(\sum_1^l \vartheta_{iy} + \alpha_{ly}\right) + s\left(\sum_1^{l+1} \vartheta_{iy} + \varphi_{ly}\right),$$

$$z'_k = \sum_1^l x_i\left(\sum_1^l \vartheta_{iz} + \alpha_{lz}\right) + s\left(\sum_1^{l+1} \vartheta_{iz} + \varphi_{lz}\right),$$

where s is the path traversed by the γ quantum before conversion, and l is the index denoting the point at which it was emitted. In this notation, the projections of the distance between the electron and the vertex of the pair are equal to

$$\Delta y = y_n - y'_k, \qquad \Delta z = z_n - z'_k.$$

To be able to compare the results of the present calculation with those of Weill et al.,[5] a value of 32 Mev-deg $\times\ (100\mu)^{-1/2}$ was used for the multiple-scattering constant, which also takes into account inelastic scattering. In addition, to explain the influence of the multiple scattering constant on the number of spurious tridents, the calculations were carried out also with the value 26.2 Mev-deg $\times\ (100\mu)^{-1/2}$.

As in reference 4, the calculations were carried out for two forms of the bremsstrahlung spectrum of the electron. However, instead of the bremsstrahlung spectrum of Ter-Mikaelyan, which was used in reference 4, we used the spectrum given by the formulae of Migdal[17] (the numerical calculations of reference 18 were actually used), since the latter takes into account both the Landau-Pomeranchuk[19] and Ter-Mikaelyan[20] effects.

To ascertain the contribution of the process (in which the vertices of the pairs produced by bremsstrahlung γ quanta of one of the electrons of a high-energy pair are found near the track of the other electron, so that the corresponding Δy and Δz satisfy the criteria of spurious trident production) to the number of the spurious tridents, the case history of an additional electron, shown dotted in Fig. 1 was followed. It is assumed that the additional electron is emitted only at an angle $\overline{\omega}$ with respect to the direction of the main electron, given by the formula of Stearns[21]

$$\overline{\omega} = (2mc^2/E)\ln(E/mc^2),$$

where E denotes the total energy of both electrons. For the additional electron, the radiation energy loss was taken into account. The projections of the distances between the additional electron and the vertices of the pairs produced by the main electron were calculated.

The ionization loss, Compton scattering and nuclear photoeffect were neglected in the calculation, because of their small role in the energy range under consideration. The small energy losses of the electron in the production of tridents and the increase of the length of the electron due to multiple scattering were also neglected.

3. RESULTS OF THE CALCULATION

In reference 1, the pairs produced in the conversion of the bremsstrahlung γ quanta, with vertices at a distance $\rho = (\Delta y^2 + \Delta z^2)^{1/2} \leq 0.2\,\mu$ from the primary electron, were considered as spurious tridents. In reference 4, three different criteria were considered: $\rho \leq 0.2$, 0.3, and $0.4\,\mu$. In reference 5, the following criterion of a spurious trident is introduced: $\Delta y \leq 0.2\,\mu$, $\Delta z \leq 0.44\,\mu$. The difference between the projections on y and z is due to the shrinking of the emulsion along z.

To work out a single criterion for the production of a spurious trident, and for the convenience of comparison with the work of Weill et al.,[5] the results of the present calculations are carried out for the criterion $\Delta y \leq 0.2\,\mu$, $\Delta z \leq 0.44\,\mu$. Results will be given below also for the criterion $\Delta y \leq 0.3\,\mu$, $\Delta z \leq 0.66\,\mu$, which can be useful for the determination of spurious tridents in especially unfavorable conditions of observation and, in

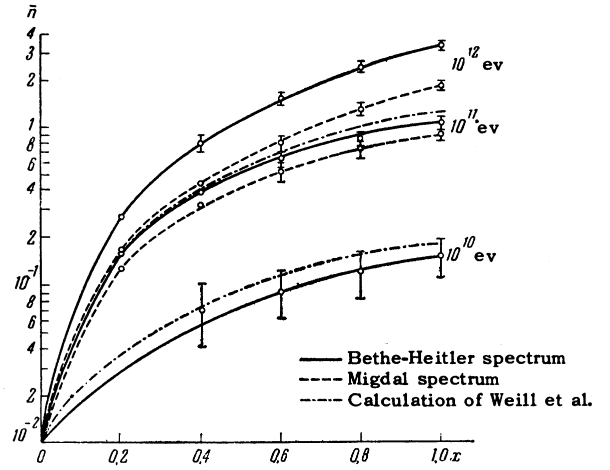

FIG. 2. Dependence of the average number of spurious tridents $\bar{n}$ on the transverse distance $\bar{x}$ (in radiation units) for various electron energies.

addition, show the dependence of the number of spurious tridents on the criterion used.

The dependence of the mean number of spurious tridents $\bar{n}$ on the distance from the point $x = y = z = 0$ where the primary electron was produced is shown in Fig. 2. The errors shown represent the standard deviations from the average calculated from the data of the Monte Carlo method.

The comparison of the present data with the results of Weill et al.[5] should essentially be carried out only for the case of the Bethe-Heitler bremsstrahlung spectrum and for an electron energy of 10^{10} and 10^{11} ev, since neither the 10^{12} ev energy nor the possible Migdal spectrum were considered in reference 5. For an energy of 10^{10} ev, the results of both calculations coincide within the limit of errors; for 10^{11} ev, our curve calculated by the Monte Carlo method lies somewhat lower. This can be explained by the fact that, in our calculation, the radiation energy loss was taken into account, which leads to an increase in the average angle of multiple scattering, and one should therefore expect a corresponding decrease in the number of spurious tridents. The results of both calculations can, therefore, be regarded as being fully in agreement. It should be noted that, although at 10^{11} ev the number of spurious tridents for the Bethe-Heitler and Migdal spectra are in agreement within the limits of errors, a marked discrepancy is obtained for 10^{12} ev electrons (see Fig. 3). Therefore, at energies $\approx 10^{12}$ ev, the question whether the bremsstrahlung spectrum according to Migdal[17] is correct or not is of primary importance in explaining the number of spurious tridents.

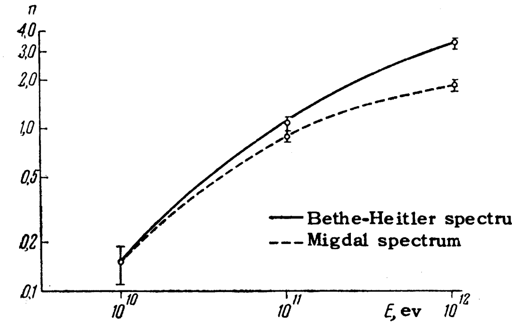

FIG. 3. Dependence of the number of spurious tridents $\bar{n}$ on electron energy.

Using the weaker criterion $\Delta y \le 0.3\,\mu$, $\Delta z \le 0.66\,\mu$, the number of spurious tridents per radiation length increased on the average by $\sim 20\%$ for 10^{10} and 10^{11} ev, and by 3% for 10^{12} ev as compared with the data in Fig. 2. For the normal criterion, $\Delta y \le 0.2\,\mu$, $\Delta z \le 0.44\,\mu$, using a multiple scattering constant equal to 26.2 Mev-deg $\times$ $(100\,\mu)^{-1/2}$ instead of the 32 Mev-deg $\times$ $(100\,\mu)^{-1/2}$ used in the present calculation, the number of spurious tridents increases, on the average, by 23% for 10^{10} and 10^{11} ev and by 5% for 10^{12} ev.

This increase in the number of spurious tridents upon changing the criterion and the multiple scattering constant can be explained by the transverse distribution of the pairs shown in Fig. 4, from which it follows that the number of pairs does not vary greatly if r is changed, for all pairs, by a factor of 1.2 — 1.5 from the value $r \approx 0.2\,\mu$. The picture, however, is greatly different for electron energies of $\approx 5 \times 10^{11}$ ev, where the maximum of the transverse distribution of pairs shifts to the region $\rho \approx 0.2 - 0.3\,\mu$. In the reduction of experimental data on electrons of such energies, a detailed analysis of the selection criteria for spuri-

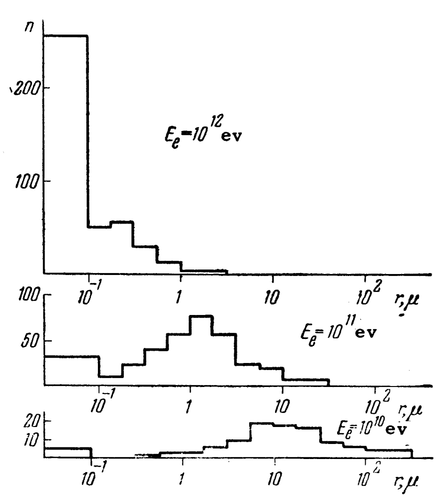

FIG. 4. Differential transverse distribution of pairs; n — number of pairs, r — distance of the pair vertices from the axis.

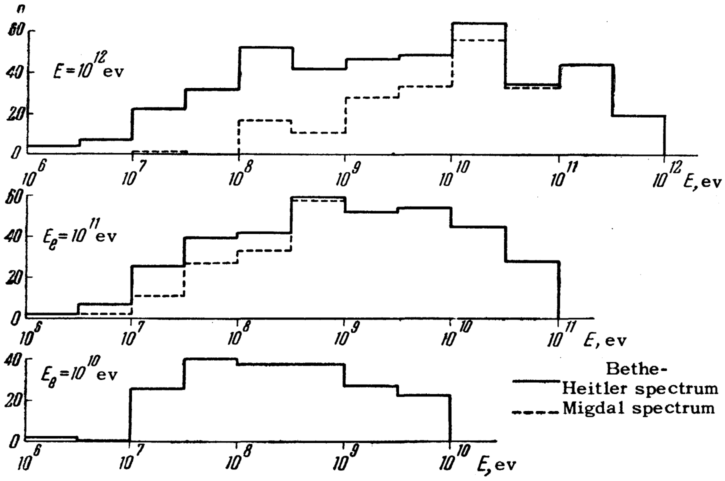

FIG. 6. Differential energy spectrum of electron-positron pair; solid line – according to Bethe-Heitler; dotted line – according to Migdal; n – number of pairs.

ous tridents should therefore be carried out.

A possible source of the increased number of spurious tridents, in the case where electron pairs are radiated, is the conversion of γ quanta bremsstrahlung of a single electron of the pair just next to the track of another electron, i.e., the production of a spurious trident involving the second electron. Such a cause of increase in the number of spurious tridents can be especially important for such values of the pair energy at which its components diverge to a small extent at distances comparable to one radiation length.

The information obtained by means of 105 additional electrons showed only one additional production of a spurious trident. From this fact one can conclude that the mutual production of spurious tridents (taking possible fluctuations into account) increases the number of spurious tridents on each electron of the pair in not more than a few percent of the cases.

The integral energy spectrum of primary electrons after their passage through one radiation length in the emulsion is shown in Fig. 5.

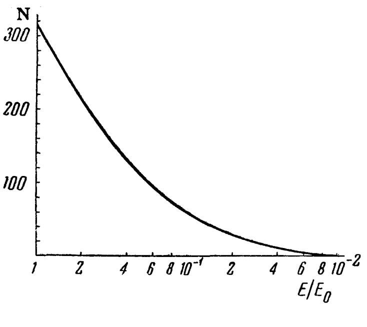

FIG. 5. Integral energy spectrum of primary electrons after traversing one radiation length; N – number of electrons, $\Sigma N = 315$.

It should be noted that, in the limits of standard deviations, a decrease of the energy by a factor of E was obtained for electrons traversing one radiation length, which is in agreement with the results of the cascade shower theory in approximation A.

The differential energy spectrum of electron pairs produced in the conversion of bremsstrahlung radiation γ quanta of primary electrons is shown in Fig. 6. The energy spectra of spurious tridents can be useful in accumulating a large amount of data for good statistical accuracy and may help explain the distribution of the transverse momenta with which the spurious tridents are produced.

The differential transverse distribution of pairs, the dependence of the average transverse distance of the pairs from the axis, and the average number of pairs produced by electrons per radiation length are shown in Figs. 4 and 7. In view of the fact that a possible new method of electron energy measurement in the range $10^{10} - 10^{12}$ ev follows from Fig. 7, we shall discuss briefly the problem of energy measurement.

In the study of the isolated electron-photon and photon components of explosion showers in nuclear emulsions, one encounters the important problem

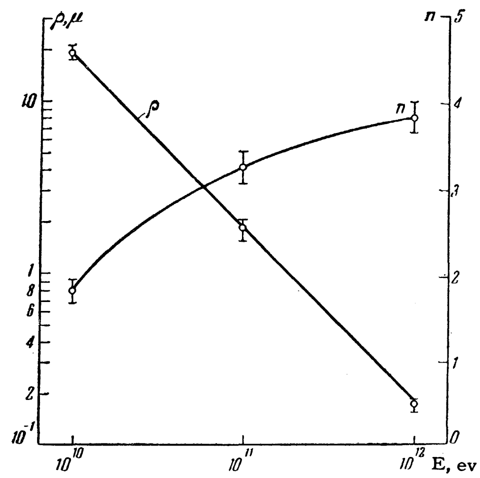

FIG. 7. Dependence of the average transverse distance of pairs from the axis, ρ, and of the average number of pairs n produced per radiation length on the electron energy.

of measuring the energy of an electron-positron pair above 10^{10} ev. The most accurate method is the measurement of the primary electron energy from the characteristics of the electromagnetic cascade initiated by it. In that case one usually measures the energy spectrum of the cascade electrons at a certain distance from its origin. Then, from the known values of $N(E_0, E > E_k, x)$ (number of electrons at a distance x with energy $> E_k$), the energy of the electron or of the pair producing the cascade is determined from theoretical cascade curves. This method becomes the more accurate the bigger the x that can be used to study the distribution of electrons of the cascade. However, the emulsion layers have limited dimensions and, therefore, it is impossible to use the optimal x for the energy measurement. In addition, events in which only the initial part of the cascade is recorded in the emulsion ($x \leq 1.5-2$ radiation lengths) are very frequent. In these cases, because of large fluctuations in the number of secondary electrons, the energy cannot be determined at all from the cascade development. Finally, the energy measurements by the above method are very difficult for the photon component of explosion showers, where it is very often impossible to distinguish between separate cascades.

Another method of energy measurement is through a measurement of the relative multiple scattering. By such a method, however, it is possible to carry out measurements only up to $\approx 10^{11}$ ev, and it is necessary to have close, almost parallel, tracks of high-energy particles.

A unique method is that of energy measurements from the magnitude of the decreased ionization at the initial portion of the track of very energetic pairs ($E_0 \approx 10^{12}$ ev) (the Chudakov-Perkins effect). However, the accuracy of this method is not high, because, in final analysis it is the angle of emission of the pair components, which has a rather large distribution, which is really measured. In addition, marked fluctuations of ionization are also always present.

The possibility of the measurement of the energy of primary electrons on the basis of the transverse distribution of vertices of secondary pairs is very attractive. It should be expected that the characteristics of the transverse development should depend strongly on the initial electron energy. Unfortunately, calculations of such a type are almost completely lacking in the literature, apart from a few qualitative considerations, which is clearly due to the difficulties connected with an analytical solution of such problems.

The dependence of the average transverse distance of the vertices of the pairs (produced in the conversion of bremsstrahlung γ quanta of the primary electron) from the original axis, which coincides with the direction of motion of the electron in the very beginning of its trajectory, has been calculated. This axis (the x axis in Fig. 1) clearly coincides with the axis of the developing cascade. For the calculation of the average r, only the pairs whose vertices were at $x \leq 2.9$ cm were used. The dependence of the average number of pairs on the electron energy was also obtained. The result, however, is shown only for orientation purposes, since the average number of pairs produced in the conversion of bremsstrahlung γ quanta depends only weakly on the energy of the radiating electron. The number of these pairs has a Poisson distribution for the electron energies used in the present calculation. It is evident that this result will not be correct for the total number of pairs, since the additional cascade pairs markedly increase the fluctuations in the number of pairs. The errors shown in Fig. 7 are standard deviations from the average, and were computed from the data of the Monte Carlo calculations.

The straight line obtained for the energy dependence of the mean transverse distance of pair vertices from the axis, makes it possible to carry out an estimate of the energy of the primary electron, provided we have data on pairs produced by γ quanta of the primary electron or on a pair at a distance $x = 2.9$ cm from the point of origin. For the x axis, one should naturally take the center of mass of all possible trajectories. The proposed method makes it possible to measure the energy of the primary electron pair (events most often found in the experiment) with standard deviation of 64% for the energy of 10^{10} ev, 44% for 10^{11} ev, and 39% for 10^{12} ev. One should bear in mind that the accuracy of the energy estimate depends on the number of pairs produced in the conversion of γ quanta bremsstrahlung radiation of the primary electron. The shown errors are calculated for the average number of pairs for each energy.

The accuracy of the method decreases for measurements of the energy of a single electron, owing to the decrease of the average number of pairs by a factor of two. In that case, however, the energy cannot be measured, in general, from the decrease of ionization in the beginning of the pair trajectories, nor can the method of the relative multiple scattering measurement be used as a rule (the second energy trace is missing). Therefore, one should think that the proposed method will be useful in that case, too.

The electron pairs of the second generation may represent an additional source of error. These pairs can be produced in the conversion of bremsstrahlung γ quanta of all these electron pairs which, in turn, have been produced in the conversion of bremsstrahlung γ quanta of the primary electron. Such cascade pairs may lead to an underestimate of the energy of the primary electron.

From the results of the present calculation, we tried to estimate the approximate number of cascade pairs. For a primary electron of 10^{12} ev, one should expect about three cascade pairs for 8 pairs produced by bremsstrahlung γ quanta (for $x < 2.9$ cm). The average distance of cascade pairs from the axis is larger, by about a factor of ten, than the average distances shown in Fig. 7. The mean longitudinal distance x at which cascade pairs are produced amounts to ≈ 2.6 cm, while the average x for the bremsstrahlung pairs is ≈ 2 cm.

The probability of a cascade pair production up to $x = 2$ cm is small, and can, in practice, be neglected. From the pairs observed in experiments up to $x = 2$ cm, one can calculate their average transverse distance $\bar{r}_2$. It should then be taken into account that $\bar{r}_2$ will be increased to $x = 2.9$ cm. If we assume that r increases as $x^{3/2}$ (in fact, after the emission of a γ quantum, its distance from the axis increases only linearly), then $\bar{r}$ at a given point x should be $r_x = \bar{r}_2 x^{3/2}/2\sqrt{2}$. Furthermore, we can introduce the condition that all pairs for $r > 4r_x$ be excluded, since they are due to cascade development. This rule is analogous to the exclusion of the deviations larger than 4 times the average value in the multiple-scattering measurement.

Since the cascade pairs have, on the average, a smaller energy than the pairs produced by bremsstrahlung γ quanta of the primary electron, we can introduce an additional condition $E \ll \bar{E}_2$, where E is the energy of the pair studied and $\bar{E}_2$ is the average energy of the pairs. $\bar{E}_2$ can also be estimated from Fig. 5.

We note, finally, that beyond $x = 2$ cm the number of cascade pairs increases as the square of x, in contrast to bremsstrahlung pairs, which are roughly linearly distributed with respect to x. For an energy of 10^{11} ev, one should expect one such pair per 6.5 pairs of bremsstrahlung radiation. The same exclusion procedure can be applied here. For 10^{10} ev, one can neglect the contribution of cascade pairs.

Thus, one can hope that the procedure of the exclusion of cascade pairs will make it possible to avoid an underestimate of the electron energy. A small fraction of the electron pairs near the axis, with a comparatively large energy, will be difficult to exclude by the above procedure. However, because of the small transverse distances from the axis, this fraction of cascade pairs will not introduce a large error.

4. EXPERIMENTAL RESULTS

We recorded a high-energy nuclear interaction* of the type $2 + 31\,\alpha$ in a stack of stripped nuclear emulsions irradiated in the stratosphere. The energy (measured against the value of the angle containing half of all the shower tracks) was found to be 9×10^{13} ev. High-energy electron-positron pairs ($E_0 \gtrsim 10$ Bev) produced in the photon component of the star were used for the experimental measurements of the mean free path λ_{exp} of trident production by electrons. The electron-positron pairs traversed ≈ 1.5 cm in single emulsion layers, and the shower traversed about 8.5 cm in the whole stack.

Tracks of shower particles of the narrow cone and the photon component of the nuclear interaction were followed up to their exit from the stack. In scanning, the coordinates of the vertices of the electron-positron pairs were found and recorded. For further measurements, only the electron-positron pairs sharply distinguishable from the main mass of tracks were chosen, a procedure necessary for a reliable identification of visible tridents.

The measurements of the relative multiple scattering were used mainly to estimate the energy of the electron-positron pairs. The measurements were carried out using a Koristka MS-2 microscope. In calculating the electron energy from the average values of the second differences of relative multiple scattering, the results of reference 22 were used. It is shown there that the usual procedure of energy determination, by measurement of the relative scattering of the components of high-energy electron-positron pairs, leads to a serious underestimate.

For two electron-positron pairs, the average second differences of the relative multiple scattering were not larger than the noise level ($0.15 \pm 0.02\,\mu$) even for maximum cell length ($4500\,\mu$). For such pairs, the ionization near their vertices was measured by the grain counting method. The Perkins-Chudakov effect was not detected for these pairs, probably because the energy was less than 1000 Bev. In addition, the method described in Sec. 3 was used for the measurements of the energy in these two cases. The energies of the

*The interaction has been found by A. Ya. Burtsev.

electron-positron pairs estimated in such a way were found to be (1 to 3) $\times 10^{11}$ ev, which did not contradict the estimates obtained by scattering and ionization measurements.

Since a large number of high-energy electrons is present in the photon component of a nuclear interaction, an increase in the number of spurious tridents over the calculated value is possible, owing to the process discussed in Sec. 3. This increase can occur in the conversion of γ quanta near the track of the investigated electron. For an estimate of the probability of such an event, 24 cm of shower-particle tracks of the narrow cone in the region of intensive development of the photon component were studied, and no electron pairs satisfying the spurious trident criterion were observed on this length.

As a result of an experimental study of the photon-component of the nuclear shower, 25 events of production of apparent tridents were observed. The total length of the investigated track, for electrons with an average energy of 20 Bev, equals 42.1 radiation lengths. After excluding the spurious tridents along each electron track, according to the results of the Monte Carlo calculations (Fig. 2), it was found that 9.3 ± 5.5 tridents were produced on the given total length of 42.1 radiation lengths, which corresponded to a mean free path $\lambda_{\text{exp}} = 4.5^{+6.5}_{-1.6}$ radiation lengths. The standard deviation in the determination of the number of tridents is $\Delta N_t = \sqrt{(\Delta N_{VT})^2 + (\Delta p)^2}$, where ΔN_{VT} is the standard deviation in the number of apparent tridents. The tridents were assumed to be distributed according to the Poisson law, which was confirmed by the data of the Monte Carlo calculation. Δp is the error in the calculated Monte Carlo curves for the number of spurious tridents, and it is usually small as compared with ΔN_{VT}.

Since the mean free path for trident production at 20 Bev equals, according to the Bhabha theory, $\lambda_{\text{theor}} = 7$ radiation lengths, one can consider that the measured λ_{exp} is in agreement with that value. These results contradict to a certain degree the results of Weill et al.,[5] according to which λ_{exp}, outside the limit of errors, is substantially smaller than the corresponding theoretical value. However, in reference 5, the error of ΔN_T was clearly underestimated, since, from the data presented there, it follows that $\Delta N_T < \Delta N_{VT}$.

The obtained data on the production of tridents by electrons of an average energy of 20 Bev are not in contradiction with the predictions of quantum electrodynamics.

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