THE ELECTROMAGNETIC INTERACTION IN THE HEISENBERG THEORY

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The interaction of nucleons at large distances is considered on the basis of a nonlinear Lagrangian of general form. By means of the Heisenberg commutation function it is shown that there are forces with the Coulomb dependence on the distance and with a fine-structure constant equal to 1/138 (scalar theory). The causes of the absence of electromagnetic forces in the vector, tensor, and axial-vector theories are analyzed. Deviations from the Coulomb law and their effects on renormalization are discussed.

THE quantization of the wave equation

$$\gamma_\mu \partial\psi / \partial x_\mu - l^2 \psi(\bar{\psi}\psi) = 0 \tag{1}$$

by means of the commutation function

$$S(x) = (2\pi)^{-4} \int e^{iqx} \frac{\varkappa^3}{q^2 (q^2 + \varkappa^2)} \left[\frac{\varkappa \gamma_\nu q_\nu}{q^2} - i \right] d^4 q \tag{2}$$

has made it possible to show[1] that a theory based on these two propositions contains forces with the Coulomb dependence on distance and with the fine-structure constant[2,3]

$$\alpha \approx {}^1/_{267}. \tag{3}$$

This is a great achievement of the theory, since for the first time electromagnetic forces have been represented as a consequence of other, more primary laws. A distinguishing feature of the work is that the calculated fine-structure constant has a value close to that found experimentally.

We shall take as the basis of the theory a Lagrangian of general form

$$L = \bar{\psi}\gamma_\mu \frac{\partial \psi}{\partial x_\mu} + \frac{1}{2} l^2 \sum_n C_n : \bar{\psi} O_n \psi \cdot \bar{\psi} O_n \psi :. \tag{4}$$

Here the colons indicate the normal product of the operators, and O_n are the 16 Dirac matrices

$$O_n = 1, \quad \gamma_\mu, \quad \gamma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu], \quad i\gamma_5\gamma_\mu, \quad \gamma_5, \tag{5}$$

which satisfy the relations

$$O_n^+ = O_n, \qquad \mathrm{Sp}\, O_m O_n = 4\delta_{mn}. \tag{6}$$

Thus the objections connected with the arbitrary choice of the Lagrangian in the papers of Heisenberg and others[1-3] are removed, and the introduction of the normal product removes from the theory the so-called vacuum divergences[4] and the indefinite quantity $S_F(0)$.

The Lagrangian (4) retains its form when the two operators ψ in the normal product are interchanged, if the coefficients C are subjected to the transformation

$$C_n \to C_n' = -\tfrac{1}{4} \sum_m C_m A_{mn}, \tag{7}$$

where the A_{mn} are defined by the identity

$$O_m \Gamma_n O_m = A_{mn} \Gamma_n \tag{8}$$

[Γ_n is any one of the matrices (5)]. The coefficients A_{mn} are shown in Table I. The requirement of invariance with respect to the transformation (7) is satisfied if the C_n appear only in the combinations

TABLE I

A_{mn}		Γ_n: 1	γ_s	$\gamma_{\mu\nu}$	$i\gamma_5\gamma_\mu$	γ_5
O_m	1	1	1	1	1	1
	γ_s	4	−2	0	2	−4
	$\gamma_{\mu\nu}$	6	0	−2	0	6
	$i\gamma_5\gamma_\mu$	4	2	0	−2	−4
	γ_5	1	−1	1	−1	1

$$B_m = \sum_n C_n (A_{nm} - 4\delta_{nm}). \tag{9}$$

1. PHOTONS AND THEIR PROPERTIES

The wave function of a boson

$$\varphi_{\alpha\beta}(x, y) = \langle 0 / T\psi_\alpha(x) \bar{\psi}_\beta(y) | \Phi \rangle. \tag{1.1}$$

obeys the equation

$$\varphi_{\alpha\beta}(x, x) = \int H_{\alpha\varepsilon\eta\beta}(x, u)\, \varphi_{\varepsilon\eta}(u, u)\, d^4 u, \tag{1.2}$$

which can be derived from the field equation

$$\gamma_\mu \partial\psi / \partial x_\mu + l^2 \sum_n C_n : O_n\psi \cdot \bar{\psi} O_n \psi: = 0 \qquad (1.3)$$

by means of the Tamm-Dancoff method. One thus finds that[5]

$$H_{\alpha\varepsilon\eta\beta}(x, u) = -\tfrac{1}{2} i l^2 \sum_N \tfrac{1}{4} B_N \{G(x, u)\Gamma^N S(u, x) + S(x, u)\Gamma^N G(u, x)\}_{\alpha\beta} (\Gamma^N)_{\eta\varepsilon}$$
$$= \tfrac{1}{4} \sum_{M, N} H_{MN}(x, u)(\Gamma^M)_{\alpha\beta}(\Gamma^N)_{\eta\varepsilon}. \qquad (1.4)$$

Here

$$H_{MN}(x, u) = -\tfrac{1}{2} i l^2 \tfrac{1}{4} B_N \operatorname{Sp}\{\Gamma^M G(x, u)\Gamma^N S(u, x) + \Gamma^M S(x, u)\Gamma^N G(u, x)\}$$
$$= (2\pi)^{-4} \int e^{ip(x-u)} H_{MN}(p)\, d^4p; \qquad (1.5a)$$

$$H_{MN}(p) = -\tfrac{1}{2} i l^2 \tfrac{1}{4} B_N (2\pi)^{-4} \operatorname{Sp} \int d^4q \{\Gamma^M G(p+q)\Gamma^N S(q) + \Gamma^M S(q)\Gamma^N G(-p+q)\}. \qquad (1.5b)$$

In the momentum representation for the function

$$\varphi_M(p) = \operatorname{Sp}\Gamma^M \varphi(p)$$
$$= \operatorname{Sp}\Gamma^M \int e^{-ipx} \langle 0 | T\psi(x)\bar{\psi}(x) | \Phi\rangle\, d^4x \qquad (1.6)$$

(1.2) has the form of a linear homogeneous equation

$$\varphi_M(p) = \sum_N H_{MN}(p)\varphi_N(p). \qquad (1.7)$$

The spinor structure of G and S is such that

$$\frac{il^2}{(2\pi)^4} \int \Gamma^M G(p+q)\Gamma^N S(q)\, d^4q = \Gamma^M(\gamma p)\Gamma^N B(p^2) + \Gamma^M \gamma_\nu \Gamma^N \gamma_\nu C(p^2) + \Gamma^M(\gamma p)\Gamma^N(\gamma p) D(p^2). \qquad (1.8)$$

Therefore

$$H_{MN}(p) = -\tfrac{1}{4} B_N \operatorname{Sp}\{\Gamma^M(\gamma p)\Gamma^N(\gamma p) D(p^2) + \Gamma^M\gamma_\nu\Gamma^N\gamma_\nu C(p^2) + \tfrac{1}{2}[\Gamma^M(\gamma p)\Gamma^N - \Gamma^N(\gamma p)\Gamma^M] B(p^2)\}. \qquad (1.9)$$

Equation (1.7) with the coefficients H_{MN} given by Eq. (1.9) can be used to determine the mass spectrum of the bosons with nonvanishing rest mass.[5] The condition for the existence of such particles is that the determinant of the system (1.7) be equal to zero.

Equation (1.7) is also valid for photons, for which $p^2 = 0$. Here, however, the functions B and C are singular:

$$B \approx -\frac{i}{2}\left(\frac{\varkappa l}{4\pi}\right)^2 \ln|p^2|, \qquad C \approx \frac{1}{4}\left(\frac{\varkappa l}{4\pi}\right)^2 \ln|p^2| \qquad (1.10)$$

(the singularity of D is of no significance). Because of this a regular solution of Eq. (1.7) exists only in cases in which the coefficients of the singular functions go to zero:

$$\sum_N B_N \operatorname{Sp}\Gamma^M \gamma_\nu \Gamma^N \gamma_\nu \varphi_N = 0, \qquad (1.11a)$$

$$\sum_N B_N \operatorname{Sp}\Gamma^M(\gamma p)\Gamma^N \varphi_N = 0, \qquad (1.11b)$$

$$\sum_N B_N \operatorname{Sp}\Gamma^N(\gamma p)\Gamma^M \varphi_N = 0. \qquad (1.11c)$$

These three cases extend and improve Eq. (19) of Heisenberg's paper.[1] According to Eqs. (8) and (6) the first of these equations is equivalent to the equation

$$B_M A_{VM} \varphi_M = 0, \qquad (1.12)$$

from which it follows that there are two types of solutions:

1) $A_{VM} = 0$, $B_M \neq 0$ is the general solution, which exists for all the types of nonlinear term. This solution has the form

$$\varphi_M = a(p_\mu e_\nu - p_\nu e_\mu), \qquad (1.13)$$

and it follows from Eq. (1.11b) that

$$(p \cdot e) = 0. \qquad (1.14)$$

Thus we get an antisymmetric tensor of the second rank with two independent transverse polarizations. The conclusion about transversality loses its validity if

$$B_T = C_S - 6C_T + C_P = 0, \qquad (1.15)$$

which occurs, for example, for the nonlinear terms $(\bar{\psi}\gamma_5\gamma_\mu\psi)^2$ and $(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2$ studied in reference 6.

2) $B_M = 0$, $A_{VM} \neq 0$; these are particular solutions, whose existence depends on the choice of the type of nonlinear term. By choice of the coefficients C_n one can construct any additional solution. In the cases of the pure types ($C_n = \pm\delta_{nn_0}$) they can all be enumerated:

$$S, P: \quad \text{none}$$
$$V, A: \quad \gamma_{\mu\nu};$$
$$T: \quad \gamma_\mu, \ \gamma_5\gamma_\mu. \qquad (1.16)$$

The polarization of the tensor solution in a mixture of the types V, A can also be longitudinal, since the equations (1.11b), (1.11c) are satisfied identically.

It is important to emphasize that all the additional solutions with their nonphysical properties are automatically removed from the theory. In fact, if we average the nonlinear term of the Lagrangian (4) over a single-nucleon state, we get an effective interaction Lagrangian of the form

$$l^2 \sum_n C_n\left(\frac{1}{4} A_{nN} - \delta_{nN}\right) :\bar{\psi}\Gamma_N\psi: \varphi_N = \frac{1}{4} l^2 \sum_N B_N :\bar{\psi}\Gamma_N\psi: \varphi_N, \tag{1.17}$$

which describes the coupling of the boson with a spinor particle having the charge $\frac{1}{4}l^2 B_N$. If the Lagrangian admits an additional solution of the N-th kind, then $B_N \equiv 0$, and the corresponding term in Eq. (1.17) drops out. Thus nonphysical photons cannot convey an interaction. Furthermore, in a mixture of the types V, A even the physical photons give no coupling, since in this case the main and additional solutions coincide.

It will be shown below that the vertex operator contains the factor

$$B_V = C_S - 6C_V + 2C_A - C_P, \tag{1.18}$$

which vanishes in the tensor type of theory, because of the presence of an additional solution. Therefore we can draw the general conclusion: the presence of additional solutions in any type of Lagrangian leads to the absence of electromagnetic interaction.

As is well known, Lagrangians that are mixtures of the vector and axial-vector types form a class invariant with respect to the transformation[7]

$$\psi' = e^{i\alpha\gamma_5}\psi. \tag{1.19}$$

The absence of electromagnetic forces in this class of Lagrangians speaks against the inclusion of the Touschek transformation in the invariance group of the Lagrangian.

2. THE INTERACTION IN A LOWER APPROXIMATION

It has been shown above that for particles with zero rest mass the theory gives just the kinematic properties that are observed for actual photons. The proof of their full identity is completed by the calculation of the propagation function

$$\begin{aligned} D_{MN}(x, y) &= \langle 0 | T\varphi_M(x)\varphi_N(y) | 0\rangle \\ &= \langle 0 | T\psi_\alpha(x)\Gamma^M_{\beta\alpha}\bar{\psi}_\beta(x)\psi_\gamma(y)\Gamma^N_{\delta\gamma}\bar{\psi}_\delta(y) 0\rangle \\ &= -\Gamma^M_{\beta\alpha}S^F_{\alpha\delta}(x, y)\Gamma^N_{\delta\gamma}S^F_{\gamma\beta}(y, x) \\ &= -\operatorname{Sp}\Gamma^M S_F(x, y)\Gamma^N S_F(y, x) \\ &= (2\pi)^{-4}\int D_{MN}(p)\exp[ip(x-y)]\,d^4p, \end{aligned} \tag{2.1}$$

$$D_{MN}(p) = -(2\pi)^{-4}\operatorname{Sp}\int d^4q\,\Gamma^M S(p+q)\Gamma^N S(q) \tag{2.2}$$

(the index F on the Fourier transforms S_F is omitted throughout in what follows).

The spinor structure of S(q) is such that

$$\begin{aligned} &\int d^4q\,\Gamma^M S(p+q)\Gamma^N S(q) \\ &= \Gamma^M(\gamma p)\Gamma^N(\gamma p)K(p^2) + \Gamma^M\gamma_\nu\Gamma^N\gamma_\nu L(p^2) \\ &+ \Gamma^M\Gamma^N M(p^2) + [\Gamma^M(\gamma p)\Gamma^N - \Gamma^M\Gamma^N(\gamma p)]\,N(p^2). \end{aligned} \tag{2.3}$$

Therefore

$$\begin{aligned} D_{MN}(p) = &-4(2\pi)^{-4}\delta_{MN}[\varepsilon_N p^2 K(p^2) + A_{VN}L(p^2) + M(p^2)] \\ &-4(2\pi)^{-4}\delta_{MN'}(1-\varepsilon_N)p_4 N(p^2), \end{aligned} \tag{2.4}$$

where ϵ_N and $\Gamma_{N'}$ are defined by the equations

$$(\gamma p)\Gamma_N(\gamma p) = \varepsilon_N p^2\Gamma_N, \tag{2.5}$$

$$\Gamma_{N'} = \gamma_4\Gamma_N. \tag{2.6}$$

The invariant functions K, L, M, N, calculated by means of Eq. (2) in the limit of small p^2/κ^2 (i.e., at distances much larger than the Compton wavelength of the nucleon), take the values

$$\frac{p^2K}{f(p^2)} \to 1, \quad \frac{L(p^2)}{f(p^2)} \to -\frac{1}{2}, \quad \frac{M(p^2)}{f(p^2)} \to 0, \quad \frac{P_4N(p^2)}{f(p^2)} \to 0; \tag{2.7}$$

$$f(p^2) = -\frac{i\pi^2\varkappa^4}{p^2}. \tag{2.8}$$

Thus in the infrared region of momenta

$$D_{MN}(p) = \delta_{MN}\frac{4i\pi^2\varkappa^4}{(2\pi)^4p^2}\left(\varepsilon_N - \frac{1}{2}A_{VN}\right), \tag{2.9}$$

and we return to the Feynman propagation function. Since along with this the Lorentz condition holds as a consequence of Eq. (1.14), all the laws of linear electrodynamics are satisfied at large distances. Sizable deviations from linearity occur for $p^2 \sim \kappa^2$, at distances related to the structure of the core of the nucleon. Even before this, however, at $p^2 \sim M_\pi^2$, the structure of the meson cloud around the nucleon begins to show up (cf. Sec. 4).

By means of the propagation function that has been found one can calculate the matrix element for scattering of two nucleons

$$\mathfrak{M} = -\bar{u}(p', q')W(p', q'; p, q)u(p, q). \tag{2.10}$$

Since in this case we can use (in lowest approximation) the effective interaction (1.17), we have

$$W(p'q', pq) = \sum_{M,N}\frac{1}{4}l^2B_M\frac{1}{4}l^2B_N D_{MN}(p'-p) \times \Gamma^M\Gamma^N\delta(p'-p+q'-q), \tag{2.11}$$

$$\mathfrak{M} = -i\pi^2\frac{(\varkappa l)^4}{4(2\pi)^4}\frac{1}{(p'-p)^2}\sum_N B_N^2\varepsilon_N\bar{u}(p')\Gamma^N u(p) \times \bar{u}(q')\Gamma^N u(q)\delta(p'-p+q'-q). \tag{2.12}$$

This formula is the same as the result of Heisenberg, Kortel, and Mitter,[2] which they obtained in a more complicated way. Comparing Eq. (2.12) with the matrix element of quantum electrodynamics,

$$\mathfrak{M}_{\text{ed}} = ie^2 \frac{\bar{u}\gamma_\mu u \cdot \bar{u}\gamma_\mu u}{(p'-p)^2} \delta(p'-p+q'-q), \qquad (2.13)$$

we conclude that there are tensor forces with the Coulomb dependence on the distance and with the fine-structure constant

$$\alpha = e^2/4\pi = \pi(\varkappa l/4\pi)^4 B_T^2.$$

In the scalar theory this quantity is equal to 1/2.7, i.e., 50 times the observed value. In the vector and axial vector theories there is no interaction.

Both the numerical discrepancy and the incorrect spin dependence are due to the absence of the vertex operator. The difference between this operator and unity must also be taken into account in the lowest approximation.

3. THE VERTEX OPERATOR

The vertex operator describing the emission (absorption) of a photon of type M can be written in the form

$$V_M = \langle\Phi'|\varphi_M(x)|\Phi\rangle = \langle\Phi'|\psi_\alpha(x)\Gamma^M_{\beta\alpha}\bar{\psi}_\beta(x)|\Phi\rangle, \qquad (3.1)$$

where Φ, Φ' are the states of the nucleon before and after the scattering.

Owing to the completeness of the set of eigenfunctions,

$$\langle 0|\psi_\mu(y)\psi_\alpha(x)\bar{\psi}_\beta(x)|\Phi\rangle = \sum_{\Phi'}\langle 0|\psi_\mu(y)|\Phi'\rangle\langle\Phi'|\psi_\alpha(x)\bar{\psi}_\beta(x)|\Phi\rangle, \qquad (3.2)$$

we have the equation

$$u_\mu e^{-iE't}\langle\Phi'|\psi_\alpha(x)\bar{\psi}_\beta(x)|\Phi\rangle = \int dy e^{-i\mathbf{p}'\mathbf{y}}\langle 0|\psi_\mu(y)\psi_\alpha(x)\bar{\psi}_\beta(x)|\Phi\rangle \qquad (3.3)$$

(u_μ is the spinor for the physical nucleon). The right member of Eq. (3.3) is easily calculated in the first approximation of the Tamm-Dancoff method:

$$\langle 0|\psi_\mu(y)\psi_\alpha(x)\bar{\psi}_\beta(x)|\Phi\rangle = \frac{il^2}{3}\sum_N \frac{B_N}{4}\int G_{\mu\rho}(y,z)\Gamma^N_{\rho\sigma}S_{\sigma\beta}(z,x) \times S_{\alpha\lambda}(x,z)\Gamma^N_{\lambda\nu}\langle 0|\psi_\nu(z)|\Phi\rangle d^4z + \ldots$$

$$= \frac{il^2}{3}(2\pi)^{-8}\sum_N\frac{B_N}{4}\int G_{\mu\rho}(q)\Gamma^N_{\rho\sigma}S_{\sigma\beta}(r)S_{\alpha\lambda}(p-q+r) \times \Gamma^N_{\lambda\nu}u_\nu e^{iqy+i(p-q)x}d^4q d^4r + \ldots, \qquad (3.4)$$

The terms indicated by the row of dots are omitted because they make no contribution to the infrared asymptotic behavior of the vertex operator. By using Eqs. (3.3) and (3.4) we get (for $t_x = t_y$)

$$V_M = \frac{il^2}{3}(2\pi)^{-5}\sum_N\frac{B_N}{4}e^{i(p-p')x}\int dq_0 G(p+q)\Gamma_N \times\int d^4r S(q+r)\Gamma^M S(r)\Gamma^N. \qquad (3.5)$$

Using Eq. (2.3) and averaging over the spins of the nucleons, we get ($q^2/\kappa^2 \to 0$)

$$\int S(q+r)\Gamma^M S(r)d^4r = -\frac{i\pi^2\varkappa^3}{q^2}(\hat{q}\Gamma^M - \Gamma^M\hat{r}). \quad (3.6)$$

For the tensor photon, with $\Gamma_M = \gamma_{\mu\nu}$, we have

$$\hat{q}\gamma_{\mu\nu} - \gamma_{\mu\nu}\hat{q} = -2i(\gamma_\mu q_\nu - \gamma_\nu q_\mu),$$

$$\Gamma^N(\hat{q}\gamma_{\mu\nu} - \gamma_{\mu\nu}\hat{q})\Gamma^N = A_{NV}(\hat{q}\gamma_{\mu\nu} - \gamma_{\mu\nu}\hat{q}), \qquad (3.7)$$

$$\sum_N B_N A_{NV} = -4B_V.$$

The result is then

$$V_T = -\frac{(\varkappa l)^2}{3}(2\pi)^{-5}B_V e^{i(p-p')x}\pi^2\varkappa \times\int dq_0\frac{\hat{p}+\hat{q}}{(p+q)^2}\cdot\frac{\hat{q}\gamma_{\mu\nu}-\gamma_{\mu\nu}\hat{q}}{q^2}. \qquad (3.8)$$

Calculating the momentum integral for $p^2 \to 0$,[3] we find

$$V_T = C\frac{(p'-p)_\mu}{|\mathbf{p}'-\mathbf{p}|}\gamma_\nu \exp[i(p-p')x], \qquad (3.9)$$

$$C = \frac{1}{12}\left(\frac{\varkappa l}{4\pi}\right)^2 B_V. \qquad (3.10)$$

The introduction of the vertex operator in the matrix element brings the latter to the form

$$\mathfrak{M} = -\sum_{M,N}\frac{l^2}{4}B_M\frac{l^2}{4}B_N D_{MN}(p'-p) \times \mathrm{Sp}\,\Gamma^M\gamma_{\mu\nu}\,\mathrm{Sp}\,\Gamma^N\gamma_{\rho\sigma}C^2\bar{u}'\frac{(\Delta p)_\mu}{|\Delta p|}\gamma_\nu u \times\bar{u}'\frac{(\Delta q)_\rho}{|\Delta q|}\gamma_\sigma u\delta(p'-p+q'-q') = \delta(p'-p+q'-q) \times l^4 B_N^2 D_N(p'-p)C^2\bar{u}'\gamma_\nu u\cdot\bar{u}'\gamma_\nu u \qquad (3.11)$$

($\Gamma^N = \gamma_{4\nu}$), i.e., we get vector forces with the fine-structure constant

$$\alpha = \pi(\varkappa l/4\pi)^4 B_T^2\cdot 16C^2 = (\pi/9)(\varkappa l/4\pi)^8 B_T^2 B_V^2, \quad (3.12)$$

which in the scalar theory is equal to 1/202. The remaining numerical disagreement with experiment is due to the omission of higher-order corrections to D_{MN} (cf. Sec. 4).

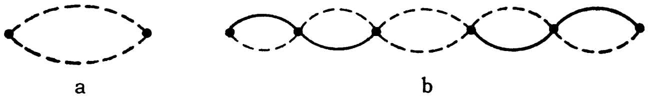
It must be particularly emphasized that as the result of the emission of tensor photons vector forces are produced. This is explained by the invariance of the vertex operator under charge conjugation and reflections of the space coordinates (cf. reference 3).

4. CORRECTIONS TO THE PHOTON PROPAGATION FUNCTION IN THE CHAIN APPROXIMATION

The expression for the photon propagation function

$$D_{\alpha\beta\mu\nu}(x, y) = \int [\delta_{\alpha\varepsilon}\delta_{\eta\beta}\delta(x, u) + L_{\alpha\varepsilon\eta\beta}(x, u)] \times D^0_{\varepsilon\eta\tau\varkappa}(u, v)[\delta_{\mu\tau}\delta_{\varkappa\nu}\delta(v, y) + L_{\mu\tau\varkappa\nu}(v, y)]\, d^4u\, d^4v \tag{4.1}$$

(D^0 is the propagation function calculated in Sec. 2) contains the operator L, which describes the effect of the higher approximations at one of the junctions of the diagram (Fig. 1, a) that represents the propagation function D^0 in lowest approximation.



If we confine ourselves to the "chain" approximation, the function D_{MN} can be represented by the diagram of Fig. 1, b; to this diagram there corresponds the following integral equation for L:

$$L_{\alpha\varepsilon\eta\beta}(x, u) = H_{\alpha\varepsilon\eta\beta}(x, u) + \int H_{\alpha\rho\sigma\beta}(x, v) L_{\rho\varepsilon\eta\sigma}(v, u)\, d^4v. \tag{4.2}$$

The operator H (x, u) that appears here corresponds to one link of the chain; it has already been encountered in Eq. (1.2).

The two equations (4.1) and (4.2) are decidedly simplified by the substitution

$$L_{\alpha\varepsilon\eta\beta}(x, u) = -\delta_{\alpha\varepsilon}\delta_{\eta\beta}\delta(x, u) + Q_{\alpha\varepsilon\eta\beta}(x, u) \tag{4.3}$$

and passage to the momentum representation,

$$Q_{\alpha\varepsilon\eta\beta}(x, u) = (2\pi)^{-4}\int Q_{\alpha\varepsilon\eta\beta}(p)\, e^{ip(x-u)}\, d^4p. \tag{4.4}$$

They are thus reduced to linear algebraic equations:

$$D_{\alpha\beta\mu\nu}(p) = Q_{\alpha\varepsilon\eta\beta}(p)\, D^0_{\varepsilon\eta\tau\varkappa}(p)\, Q_{\mu\tau\varkappa\nu}(p), \tag{4.5}$$

$$Q_{\alpha\varepsilon\eta\beta}(p) = \delta_{\alpha\varepsilon}\delta_{\eta\beta} + H_{\alpha\rho\sigma\beta}(p)\, Q_{\rho\varepsilon\eta\sigma}(p). \tag{4.6}$$

Expressing Q (p) in terms of the Dirac matrices

$$Q_{\alpha\varepsilon\eta\beta}(p) = \frac{1}{4}\sum_{M,N} Q_{MN}(p)(\Gamma^M)_{\alpha\beta}(\Gamma^N)_{\eta\varepsilon} \tag{4.7}$$

and using Eqs. (1.4) and (6), we bring Eq. (4.6) to the form

$$Q_{MN}(p) = \delta_{MN} + \sum_K H_{MK}(p)\, Q_{KN}(p), \tag{4.8}$$

which is entirely analogous to Eq. (1.7). The only difference is the presence of the inhomogeneous term δ_{MN}. In view of this relation between the equations (1.7) and (4.8) it can be asserted that the eigenvalues of the equation for $\varphi_N(p)$ are the poles of Q (p); that is, that the boson masses are the poles of Q (p).

Equation (4.8) can be solved easily, since $H_{MK} \neq 0$ for M = K or M = K′ [cf. Eqs. (1.9) and (2.6)]. In our case of $p^2/\kappa^2 \to 0$ the solution is particularly simple, since

$$p^2D(p^2) \to \frac{1}{2}(\varkappa l/4\pi)^2, \quad p_4B(p^2) \to 0, \quad C(p^2) \to 0, \tag{4.9}$$

$$H_{MK}(p) \to -\frac{1}{2}(\varkappa l/4\pi)^2\delta_{MK}B_M\varepsilon_M. \tag{4.10}$$

The result is

$$Q_{MN} = \frac{\delta_{MN}}{1 + {}^1/_2(\varkappa l/4\pi)^2 B_M\varepsilon_M} = Q_M\delta_{MN}. \tag{4.11}$$

Using the expansions

$$D_{\alpha\varepsilon\mu\nu} = \sum_{A,B} D_{AB}(\Gamma^A)_{\alpha\beta}(\Gamma^B)_{\mu\nu}, \tag{4.12}$$

$$D^0_{\varepsilon\eta\tau\varkappa} = \sum_{A,B} D^0_{AB}(\Gamma^A)_{\varepsilon\eta}(\Gamma^B)_{\tau\varkappa} \tag{4.13}$$

and Eq. (4.7), we get

$$D_{AB}(p) = \sum_{M,N} Q_{AM}(p)\, D^0_{MN}(p)\, Q_{BN}(p). \tag{4.14}$$

Owing to Eq. (4.11) we have in the infrared region

$$D_{AB}(p) = Q_AQ_B D^0_{AB}(p) = Q^2_A D^0_A(p)\delta_{AB}. \tag{4.15}$$

Thus inclusion of the higher-order corrections by the summation of "chain" diagrams leads to the appearance of a correction factor

$$Q^2_A = [1 + \frac{1}{2}(\varkappa l/4\pi)^2 B_A\varepsilon_A]^{-2} \tag{4.16}$$

in the expression for the propagation function. This factor depends on the sign of the nonlinear term; it contains B_A to the first power, whereas B_A occurs squared in all the other expressions. Because of this there is a possibility of determining the sign of the nonlinear term. The value of the correction in the scalar theory is 0.17; that is, it is far from the nearest zero of the denominator. It increases very rapidly, however, with approach to the threshold for production of a vector meson. Below the threshold the main part is played by the virtual production of mesons, which smears out the structure of the nucleon and leads to other nonlocal effects.

In view of the fact that the threshold for the production of mesons by electrons is considerably higher than that for nucleons, deviations from the Coulomb law are easier to detect in the scattering of nucleons. Thus nonlinear (and nonlocal) effects appear at lower energies in mesodynamics than in electrodynamics. This in turn is closely connected with the large difference between the coupling constants, which receives a natural explanation in a unified nonlinear theory.[1] An intermediate position between these two cases is occupied by the scattering of electrons by nucleons.

The appearance of a correction factor in the propagation function can be understood in the sense of a charge renormalization, although here all quantities are finite. The presence of additional poles in the renormalized propagation function is due to an actual physical effect: the influence of the meson field on the electromagnetic field. This dispenses with the difficulty that several writers[8] have pointed out in the interpretation of the additional poles. The difficulty with the vanishing of the renormalized charge is also removed. It could occur only at energies sufficient for penetration into the nucleon core, where there is of course no point in talking about the Coulomb law. In this region mesic forces play the decisive role. The continuous transition between electromagnetic and mesic forces underlines the necessity of a unified theory of both types of interaction.

5. THE FINE-STRUCTURE CONSTANT

Inserting the renormalizing factor (4.16) in Eq. (3.12), we get the following expression for the fine-structure constant of the electromagnetic interaction:

$$\alpha = \frac{\pi}{9}\left(\frac{\varkappa l}{4\pi}\right)^8 B_T^2 B_V^2 \left[1 - \frac{1}{2}\left(\frac{\varkappa l}{4\pi}\right)^2 B_T\right]^{-2} \tag{5.1}$$

($\epsilon_T = -1$, since $\Gamma^N = \gamma_{4\nu}$; cf. Sec. 3).

Using the values of the constants calculated in reference 5, we get for the pure types of nonlinear term the results shown in Table II (the small numerical disagreement with the result of Ascoli and Heisenberg[3] is due to an improvement in the value of the constant κl). Comparison with the experimental value 1/137.03 is strong evidence for the Lagrangian

$$L = \bar{\psi}\gamma_\mu \partial\psi/\partial x_\mu + \frac{1}{2} l^2 (\bar{\psi}\psi)(\bar{\psi}\psi). \tag{5.2}$$

Precisely this Lagrangian was proposed in reference 2, but a mistake that got into the calculations led to a change of the sign of the nonlinear term.[3]

In the pseudoscalar theory the forces are larger by an order of magnitude. The absence of forces in the vector, axial-vector, and tensor theories has been analyzed above (Sec. 1); it is due to the presence of longitudinal photons in these theories.

The accuracy of the expression (5.1) can be improved by the calculation of higher approximations to the vertex operator and by an estimate of the degree of correctness of the chain approximation for the photon propagation function.

TABLE II

Type of theory	Sign of nonlinear term	
	+	−
S	1/138.1	1/277.4
P	1/10.8	1/37.2
V, A, T	0	0

6. CONCLUSION

As the result of the present treatment we conclude that it is possible to reduce the laws of electromagnetic forces to other, more primitive, laws. In the limiting case of small momenta the electrodynamics constructed on the basis of Heisenberg's theory goes over into the ordinary quantum electrodynamics with the fine-structure constant 1/138, and with the correct polarization properties of the photon. The last two facts are closely interconnected; the admission of a longitudinal polarization makes the charge vanish.

Moreover, with increase of the energies of the interacting particles departures from the Coulomb law appear because of the virtual production of mesons and nucleon-antinucleon pairs. Beginning at a certain value of the energy a separate treatment of the electromagnetic and mesic forces becomes impossible, and the concept of electric charge loses its meaning; the internal structure of the particles begins to manifest itself. Such difficulties of the linear theory as the vanishing of the renormalized charge, and so on, occur only beyond this threshold value of the energy and are of no physical interest. With further increase of the energy electrodynamics goes over continuously into the mesodynamics of vector mesons.

The existence of an upper limit to the applicability of the separate theory also shows the necessity of a unified theory of the elementary particles.

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