POLARIZATION EFFECTS IN THE ELASTIC SCATTERING OF ELECTRONS FROM DEUTERONS

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The differential scattering cross section and the change of electron polarization are calculated for the elastic scattering of polarized electrons on polarized deuterons.

INTRODUCTION

IF we look at the electron-nucleon interaction, to first order in the electromagnetic field but taking into account all mesonic-radiation corrections, the nucleon structure is given by two material form factors $a(q^2)$ and $b(q^2)$ (see for example the work of Akhiezer, Rozentsveĭg, and Shmushkevich[1]). Here $q^2 = (p_1 - p_2)^2$, where p_1 and p_2 are the four-momenta of the electron before and after scattering, $a(q^2)$ characterizes the charge distribution, and $b(q^2)$ the distribution of the anomalous nuclear magnetic moment.

In the static limit $(q^2 \rightarrow 0)$ we have $a_p = 1$ and $b_p = \mu_p$ for the proton, and $a_n = 0$ and $b_n = \mu_n$ for the neutron, where μ_p and μ_n are the anomalous magnetic moments of the proton and the neutron.

Experiments on the elastic scattering of electrons on deuterons[2,3] permit the possible determination of the neutron form factor. Both the nucleon form factors and the form factor of the deuteron as a whole enter into the formula for the electron-deuteron differential cross section.[4] Therefore, experiments with polarized particles, some of which are considered in the present work, are useful in determining these quantities.

DIFFERENTIAL ELASTIC SCATTERING CROSS SECTION

The matrix element for the electron-nucleon interaction has the following form:[1]

$$V_{if} = (\overline{U}_2 \Gamma_\mu U_1)\, g_\mu,$$
$$\Gamma_\mu = a(q^2)\,\gamma_\mu + \frac{ib(q^2)}{2M}\gamma_\mu \hat{q}, \qquad g_\mu = \frac{e^2}{q^2}(\overline{u}_2 \gamma_\mu u_1). \tag{1}$$

Capital letters indicate quantities related to the nucleon, while lower-case letters refer to the electron; M is the nucleon mass. In the nonrelativistic approximation, which will be applied later to the deuteron, we have

$$V_{if} = v_2^* [R_4 g_4 + \mathbf{R}\mathbf{g}]\, v_1, \qquad R_4 = a,$$
$$\mathbf{R} = -(i/2M)\{a(\mathbf{P}_1 + \mathbf{P}_2) - i(a+b)[\mathbf{q}\times\boldsymbol{\sigma}]\}; \tag{2}$$

v is a two-component spinor, and $\mathbf{P}_1$ and $\mathbf{P}_2$ are the initial and final nucleon momenta.

As applied to the bound nucleon, we can write instead of (2)

$$V_{if} = \int \psi_f^* V \psi_i \, d\tau, \tag{3}$$

$$V = g_4 a e^{i\mathbf{q}\mathbf{r}} - (\mathbf{g}/2M)\{a[\nabla e^{i\mathbf{q}\mathbf{r}} + e^{i\mathbf{q}\mathbf{r}}\nabla]$$
$$- i(a+b)([\nabla\times\boldsymbol{\sigma}]\, e^{i\mathbf{q}\mathbf{r}} - e^{i\mathbf{q}\mathbf{r}}[\nabla\times\boldsymbol{\sigma}])\}. \tag{4}$$

Corresponding to this, the matrix element for electron-deuteron scattering is written

$$V_{if} = \int \Psi_f^* [V_p + V_n]\, \Psi_i \, d\tau_p d\tau_n. \tag{5}$$

Here Ψ_i and Ψ_f are the deuteron wave functions before and after the collision, and V_p and V_n are determined by Eq. (4), where the form factors a and b correspond respectively to the proton and neutron. Writing the matrix element in the form (5) actually corresponds to using the impulse approximation for the deuteron. For elastic electron-deuteron scattering we get with the help of (5)

$$V'_{if} = \langle \chi_f | S | \chi_i \rangle. \tag{6}$$

Here χ_i and χ_f are deuteron spin functions,

$$S = A + \mathbf{B}(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)/2,$$
$$A = I_1\left(g_4 - \frac{i}{2M}\mathbf{g}\mathbf{q}\right) - \frac{1}{2M}\mathbf{I}_2\mathbf{g}, \qquad \mathbf{B} = I_3[\mathbf{q}\times\mathbf{g}];$$
$$I_1 = \int (a_p e^{i\mathbf{q}\mathbf{r}/2} + a_n e^{-i\mathbf{q}\mathbf{r}/2})\, |\varphi_d|^2 \, d\mathbf{r},$$
$$\mathbf{I}_2 = 2\int \varphi_d^* (a_p e^{i\mathbf{q}\mathbf{r}/2} - a_n e^{-i\mathbf{q}\mathbf{r}/2})\, \nabla\varphi_d \, d\mathbf{r},$$
$$I_3 = \frac{1}{2M}\int [(a_p + b_p)\, e^{i\mathbf{q}\mathbf{r}/2} + (a_n + b_n)\, e^{-i\mathbf{q}\mathbf{r}/2}]\, |\varphi_d|^2 \, d\mathbf{r} \tag{7}$$

$\mathbf{r}$ is the relative coordinate of the nucleons, σ_1 and σ_2 are proton and neutron spin operators, and φ_d is the deuteron coordinate wave function. The D-wave admixture in the deuteron ground state is neglected.

We look at the general case for the scattering of polarized electrons on polarized deuterons. The initial state is described by the density matrix ρ_0, represented as the direct product of the electron and neutron density matrices

$$\rho_0 = \rho_e \times \rho_d,$$
$$\rho_e = {}^1/_2\,(1 + i\gamma_5\hat{\zeta}_1)\,\eta^{(+)}(p_1),$$
$$\rho_d = \frac{1}{4}\left[1 + \frac{1}{3}\sigma_1\sigma_2 + \alpha(\sigma_1 + \sigma_2) + \beta_{im}(\sigma_{1i}\sigma_{2m} + \sigma_{2i}\sigma_{1m})\right].$$

Here

$$\eta^{(+)}(p) = \frac{i\hat{p} - m}{2\varepsilon}\gamma_4$$

is the positive energy state projection operator. The four-component vector $\zeta_\mu = (\boldsymbol{\zeta}, \zeta_4)$ describes the electron polarization. In the rest system $\zeta_\mu = (\boldsymbol{\zeta}^0, 0)$, and in an arbitrary system

$$\boldsymbol{\zeta} = \zeta_t^0 + (\varepsilon/m)\,\zeta_l^0, \qquad \zeta_4 = i\,(\mathbf{p}\zeta^0)/m,$$

where ζ_t^0 and ζ_l^0 are the transverse and longitudinal components of the vector $\boldsymbol{\zeta}^0$, $\mathbf{p}$ is the momentum; ϵ the energy, m the electron mass; α and β_{im} describe the deuteron polarization, where $\mathrm{Sp}\,\beta_{im} = \beta_{ii} = 0$. The term "polarization" in the present case includes that process usually called "alignment."

The following expression is obtained for the differential cross section*

$$\frac{d\sigma}{d\Omega} = \frac{1}{4}\left(\frac{e^2}{4\pi}\right)^2 \frac{\mathrm{Sp}\,\{\eta^{(+)}(p_2)\,S\rho_0S^+\}}{\varepsilon_1^2\sin^4(\vartheta/2)\,(1 + \xi\sin^2(\vartheta/2))}. \tag{8}$$

Here ϑ is the scattering angle, $\xi = \epsilon_1/M$. Carrying out the summation on the electron and nucleon spin indices, we get

$$\mathrm{Sp}\,\{\eta^{(+)}(p_2)\,S\rho_0S^+\} = |I_1|^2\cos^2\frac{\vartheta}{2} + |I_3|^2\Big\{\frac{2}{3}\mathbf{q}^2\Big(1 + \sin^2\frac{\vartheta}{2}\Big)$$
$$+ \beta_{im}\Big[(p_{1i}p_{2m} + p_{2i}p_{1m})\cos^2\frac{\vartheta}{2}$$
$$- p_{1m}p_{1i}\Big(1 + \sin^2\frac{\vartheta}{2} - \xi\sin^2\frac{\vartheta}{2}\Big)$$
$$- p_{2i}p_{2m}\Big(1 + \sin^2\frac{\vartheta}{2} + \xi\sin^2\frac{\vartheta}{2}\Big)\Big]\Big\}$$
$$+ \frac{m}{2\varepsilon_1\varepsilon_2}\{i\,|I_3|^2(\alpha\mathbf{q})\,[\zeta_4\mathbf{q}^2 - i\,(\varepsilon_1 - \varepsilon_2)\,(\zeta\mathbf{q})]$$
$$- 2I_1I_3\,[(\alpha\zeta)\,\mathbf{q}^2 - (\alpha\mathbf{q})\,(\zeta\mathbf{q})]\}. \tag{9}$$

In the computations it is everywhere assumed that $\epsilon_1 \gg m$ and $\vartheta \gg m/\epsilon_1$. The final expression for the differential cross section is conveniently written with the following choice of axes:

$$\mathbf{k} = \mathbf{p}_1/|\mathbf{p}_1|, \qquad \mathbf{n} = [\mathbf{p}_1\times\mathbf{p}_2]/|[\mathbf{p}_1\times\mathbf{p}_2]|, \qquad \mathbf{l} = [\mathbf{k}\times\mathbf{n}].$$

We get

$$d\sigma/d\Omega = (d\sigma/d\Omega)_0\,(1 - N/N_0), \tag{10}$$

where

$$N_0 = (a_p + a_n)^2 - \frac{2}{3}\mathbf{q}^2\frac{(a_p + a_n + b_p + b_n)^2}{4M^2}\Big(1 - \frac{2}{\cos^2(\vartheta/2)}\Big),$$

$$N = \frac{(a_p + a_n + b_p + b_n)^2}{4M^2}\Big\{(\zeta^0\mathbf{k})\tan\frac{\vartheta}{2}\Big[\frac{1}{2}\mathbf{q}^2\Big(2 - \xi\sin^2\frac{\vartheta}{2}\Big)$$
$$\times\Big[2(\alpha\mathbf{l}) + (\alpha\mathbf{k})(2 + \xi)\tan\frac{\vartheta}{2}\Big]$$
$$+ \frac{2M\varepsilon_2\sin\vartheta\,(a_p + a_n)}{a_p + a_n + b_p + b_n}\Big[2(\alpha\mathbf{k}) - (\alpha\mathbf{l})\tan\frac{\vartheta}{2}(2 + \xi)\Big]\Big]$$
$$+ \mathbf{q}^2\beta_{im}\Big[k_ik_m\Big(\cos^2\frac{\vartheta}{2} + \tan^2\frac{\vartheta}{2} + \xi\sin^4\frac{\vartheta}{2}\Big)$$
$$+ l_il_m\Big(1 + \sin^2\frac{\vartheta}{2} - \xi\sin^4\frac{\vartheta}{2}\Big)$$
$$+ l_ik_m\sin^2\frac{\vartheta}{2}\tan\frac{\vartheta}{2}(2 + \xi\cos\vartheta)\Big]\Big\}. \tag{11}$$

The scattering cross section for unpolarized particles is

$$\Big(\frac{d\sigma}{d\Omega}\Big)_0 = \frac{1}{4}\Big(\frac{e^2}{4\pi}\Big)^2\frac{f_d^2N_0\cos^2(\vartheta/2)}{\varepsilon_1^2\sin^4(\vartheta/2)\,(1 + \xi\sin^2(\vartheta/2))}. \tag{12}$$

The deuteron form factor is

$$f_d = \int|\varphi_d|^2e^{i\mathbf{qr}/2}\,d\mathbf{r} = \int_0^\infty u^2j_0(qr/2)\,dr,$$

where j_0 is the spherical Bessel function. In particular, if

$$\varphi_d = \sqrt{\gamma/2\pi}\,e^{-\gamma r}/r, \quad \text{then } f_d = (4\gamma/q)\tan^{-1}(q/4\gamma).$$

Formula (12) coincides with the expression given by Jankus.[4]

The following circumstance is essential: the ratio of the scattering cross sections for polarized and unpolarized particles does not depend on the deuteron form factor. It is evident from formula (11) that, in the approximation used, the differential cross section does not depend on the transverse component of the polarization of the electrons. It is interesting to note the following: if the cross section does not depend on the proton polarization in the scattering of unpolarized electrons by protons, then in the present case the influence of the deuteron polarization shows up even for the scattering of unpolarized electrons, and the problem has azimuthal symmetry.

THE CHANGE IN ELECTRON POLARIZATION

The basic expression for the final polarization of the electrons $\boldsymbol{\zeta}_2$ in the scattering of electrons

*The factor $e^2/\mathbf{q}^2$ has been removed from S.

with polarization ζ_1 from polarized deuterons has the following form:

$$\zeta_2 = \frac{i(\varepsilon_2/m)\,\mathrm{Sp}\,\{\gamma\gamma_4\gamma_5\gamma_i^{(+)}(p_2)\,Sp_0S^+\eta^{(+)}(p_2)\}}{\mathrm{Sp}\,\{\eta^{(+)}(p_2)\,Sp_0S^+\}}. \quad (13)$$

The method of computation is exactly the same as for deriving the differential cross section. We set down the result:

$$\zeta_2^0 = \mathbf{k}\,[C_{11}(\zeta_1^0\mathbf{k}) + C_{12}(\zeta_1^0\mathbf{n}) + C_{13}(\zeta_1^0\mathbf{l}) + D_{11}(\alpha\mathbf{k}) + D_{13}(\alpha\mathbf{l})] + \mathbf{n}\,[C_{22}(\zeta_1^0\mathbf{n}) + C_{23}(\zeta_1^0\mathbf{l})] + \mathbf{l}\,[C_{31}(\zeta_1^0\mathbf{k}) + C_{32}(\zeta_1^0\mathbf{n}) + C_{33}(\zeta_1^0\mathbf{l}) + D_{31}(\alpha\mathbf{k}) + D_{33}(\alpha\mathbf{l})]. \quad (14)$$

The coefficients in formula (14) have the following meaning:

$$C_{11} = \frac{\cos\vartheta}{N_0 - N}\Big\{(a_p + a_n)^2 + \frac{\mathbf{q}^2}{4M^2}(a_p + a_n + b_p + b_n)^2 \times\Big[\frac{2}{3}\Big(1 + 2\tan^2\frac{\vartheta}{2}\Big) - \beta_{im}\Big[k_ik_m\Big(\cos^2\frac{\vartheta}{2} + \tan^2\frac{\vartheta}{2} + \xi\sin^4\frac{\vartheta}{2}\Big) + l_il_m\Big(1 + \sin^2\frac{\vartheta}{2} - \xi\sin^4\frac{\vartheta}{2}\Big) + l_ik_m\tan\frac{\vartheta}{2}\sin^2\frac{\vartheta}{2}(2 + \xi\cos\vartheta)\Big]\Big]\Big\},$$

$$C_{22} = \frac{1}{N_0 - N}\Big\{(a_p + a_n)^2 + \frac{\mathbf{q}^2}{4M^2}(a_p + a_n + b_p + b_n)^2\Big[\frac{2}{3} - \beta_{im}\Big[k_ik_m\Big(1 + \tan^2\frac{\vartheta}{2} + \sin^2\frac{\vartheta}{2} - \xi\sin^4\frac{\vartheta}{2}\Big) - l_ik_m\tan\frac{\vartheta}{2}\sin^2\frac{\vartheta}{2}(2 + \xi\cos\vartheta) + l_il_m\Big(\cos^2\frac{\vartheta}{2} + 2\tan^2\frac{\vartheta}{2} + \xi\sin^4\frac{\vartheta}{2}\Big)\Big]\Big]\Big\},$$

$$C_{23} = \frac{\mathbf{q}^2(a_p + a_n + b_p + b_n)^2\tan(\vartheta/2)\beta_{im}}{4M^2(N_0 - N)} \times\Big\{n_il_m\tan\frac{\vartheta}{2}\Big(2 + \xi\cos^2\frac{\vartheta}{2}\Big) - k_in_m\Big(2 - \xi\sin^2\frac{\vartheta}{2}\Big)\Big\},$$

$$D_{11} = -\frac{\cos\vartheta\tan(\vartheta/2)(a_p + a_n + b_p + b_n)}{M(N_0 - N)}\Big\{\varepsilon_2\sin\vartheta(a_p + a_n) + \frac{(a_p + a_n + b_p + b_n)\mathbf{q}^2}{8M}\tan\frac{\vartheta}{2}\Big(2 + \xi\cos^2\frac{\vartheta}{2}\Big)\Big\},$$

$$D_{13} = \frac{\cos\vartheta\tan(\vartheta/2)(a_p + a_n + b_p + b_n)}{M(N_0 - N)} \times\Big\{\varepsilon_2\sin^2\frac{\vartheta}{2}(2 + \xi)(a_p + a_n) - \frac{(a_p + a_n + b_p + b_n)\mathbf{q}^2}{8M}\Big(2 - \xi\sin^2\frac{\vartheta}{2}\Big)\Big\},$$

$$C_{13} = C_{22}\sin\vartheta, \quad C_{12} = -C_{23}\sin\vartheta, \quad C_{31} = -C_{11}\tan\vartheta,$$
$$C_{33} = C_{22}\cos\vartheta, \quad C_{32} = -C_{23}\cos\vartheta, \quad D_{31} = -D_{11}\tan\vartheta,$$
$$D_{33} = -D_{13}\tan\vartheta.$$

It is obvious from these expressions that the final electron polarization does not depend on the deuteron form factor. It is more convenient to represent the final polarization in the coordinate system connected with the scattered electron:

$$\zeta_{2l} = \zeta_2\mathbf{p}_2/p_2, \quad \zeta_{2t}^{\perp} = \zeta_2\mathbf{n}; \quad \zeta_{2t}^{\parallel} = (\zeta_2[\mathbf{p}_2\times\mathbf{n}])/p_2,$$

ζ_{2l} is the longitudinal component, $\zeta_{2t}^{\perp}$ the transverse component perpendicular to the plane of the reaction, and $\zeta_{2t}^{\parallel}$ the transverse component lying in the plane of the reaction. For the incident electron $\zeta_{1l} = \zeta_1\mathbf{k}$, $\zeta_{1t}^{\perp} = \zeta_1\mathbf{n}$, and $\zeta_{1t}^{\parallel} = \zeta_1\mathbf{l}$. In terms of these the result is written

$$\zeta_{2l}^0 = [C_{11}\zeta_{1l}^0 + D_{11}(\alpha\mathbf{k}) + D_{13}(\alpha\mathbf{l})]/\cos\vartheta,$$
$$\zeta_{2t}^{0\parallel} = [C_{13}\zeta_{1t}^{0\parallel} + C_{12}\zeta_{1t}^{0\perp}]/\sin\vartheta,$$
$$\zeta_{2t}^{0\perp} = [C_{13}\zeta_{1t}^{0\perp} - C_{12}\zeta_{1t}^{0\parallel}]/\sin\vartheta.$$

It is evident that the longitudinal and transverse components of the polarization change independently of each other during the scattering.

If the deuteron is not polarized at first, we get

$$\zeta_{2l}^0 = \zeta_{1l}^0,$$
$$\zeta_{2t}^{0\parallel} = \zeta_{1t}^{0\parallel}\Big[1 - \frac{\mathbf{q}^2(a_p + a_n + b_p + b_n)^2\tan^2(\vartheta/2)}{3M^2N_0}\Big],$$
$$\zeta_{2t}^{0\perp} = \zeta_{1t}^{0\perp}\Big[1 - \frac{\mathbf{q}^2(a_p + a_n + b_p + b_n)^2\tan^2(\vartheta/2)}{3M^2N_0}\Big].$$

In this case the longitudinal component of the electron polarization does not change during the scattering, but both transverse components change in the same way. This statement is true also for the scattering of electrons on unpolarized protons, as follows from the formulas obtained before.[5]

If the electrons are initially unpolarized, we have

$$\zeta_{2l}^0 = [D_{11}(\alpha\mathbf{k}) + D_{13}(\alpha\mathbf{l})]/\cos\vartheta, \quad \zeta_{2t}^{0\perp} = \zeta_{2t}^{0\parallel} = 0.$$

The electrons appear longitudinally polarized. Other polarization effects may be interesting, for example, those arising in inelastic electron-deuteron scattering, which effects are now being calculated.

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