THERMOELECTRIC PHENOMENA IN STRONG MAGNETIC FIELDS IN METALS POSSESSING VARIOUS FERMI SURFACES

Yu. A. BYCHKOV, L. É. GUREVICH, and G. M. NEDLIN

Institute for Physical Problems, Academy of Sciences, U.S.S.R.; Leningrad Physico-Technical Institute, Academy of Sciences, U.S.S.R.

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The asymptotic behavior of the thermoelectric force, Peltier coefficients, and Thomson coefficients for metals with closed Fermi surfaces and open surfaces of the "corrugated cylinder" and "space net" types is investigated on the basis of the quasi-classical theory of kinetic phenomena in metals in strong magnetic fields, as developed by Lifshitz, Azbel', and Kaganov[1] and Lifshitz and Peschanskiĭ.[2]

LIFSHITZ, Azbel', Kaganov, and Peschanskiĭ[1,2] have given the theory of the asymptotic behavior of the kinetic coefficients in very strong magnetic fields, when the period of rotation of the electrons is much greater than the relaxation time. The asymptotic dependence of the coefficients on the magnetic field and on its direction relative to the crystal axes is then largely determined by the nature of the Fermi surface, and is very different for open and closed surfaces. The coefficients are weakly dependent on the actual form of the collision integral and on the dispersion law. The range of validity of this quasi-classical theory is indicated by Lifshitz et al. The asymptotic behavior of the electrical conductivity tensor was calculated in detail[1,2] and some aspects of the behavior of the heat conductivity tensor and of the Thomson coefficient were briefly considered[3] on the basis of the theory. In the present work we discuss in detail a number of thermoelectric phenomena.

As is well known, the electron distribution function, f, for a metal in an electric field and a temperature gradient differs from $f_0 = \{\exp[(\epsilon-\mu)/kT]+1\}^{-1}$ by an amount f_1 i.e., $f = f_0 + f_1$. f is derived from the solution of the corresponding kinetic equation. As a result of the additional term f_1 the current density vector, **j**, and the heat current vector, **q**, differ from zero, and are related to f_1 by the simple equations

$$\mathbf{j} = \frac{2e}{(2\pi\hbar)^3}\int \mathbf{v} f_1\, dp, \qquad \mathbf{q} = \frac{2}{(2\pi\hbar)^3}\int (\varepsilon - \zeta)\, \mathbf{v} f_1\, dp. \tag{1}$$

In general **j** and **q** can be written in the form[4]

$$j_i = \frac{a_{ik}}{T} E_k + b_{ik}\frac{\partial}{\partial x_k}\left(\frac{1}{T}\right),$$

$$q_i = \frac{c_{ik}}{T} E_k + d_{ik}\frac{\partial}{\partial x_k}\left(\frac{1}{T}\right). \tag{2}$$

In a magnetic field the kinetic coefficients are functions of the vector **H**. The coefficients in (2) are not independent, but are related by the well-known symmetry relations

$$a_{ik}(\mathbf{H}) = a_{ki}(-\mathbf{H}), \qquad d_{ik}(\mathbf{H}) = d_{ki}(-\mathbf{H}),$$

$$b_{ik}(\mathbf{H}) = c_{ki}(-\mathbf{H}). \tag{3}$$

If **E** and **q** are expressed in terms of **j** and ∇T, we obtain

$$E_i = \sigma_{ik}^{-1} j_k + \alpha_{ik}\frac{\partial T}{\partial x_k}, \qquad q_i = \beta_{ik} j^k - \varkappa_{ik}\frac{\partial T}{\partial x^k} \tag{4}$$

where

$$\sigma_{ik} = T^{-1} a_{ik}, \qquad \alpha_{ik} = T^{-1} a_{il}^{-1} b_{lk},$$

$$\beta_{ik} = c_{il} a_{lk}^{-1}, \qquad \varkappa_{ik} = T^{-2}(d_{ik} - c_{il} a_{lm}^{-1} b_{mk}), \tag{5}$$

which satisfy the symmetry relation connected with (3):

$$\sigma_{ik}(\mathbf{H}) = \sigma_{ki}(-\mathbf{H}), \qquad \varkappa_{ik}(\mathbf{H}) = \varkappa_{ki}(-\mathbf{H}),$$

$$T\alpha_{ik}(\mathbf{H}) = \beta_{ki}(-\mathbf{H}). \tag{6}$$

In these expressions σ_{ik} is the electrical conductivity tensor, κ_{ik} is the heat conductivity tensor, α_{ik} can be called the thermal emf tensor, and β_{ik} are the Peltier coefficients.[4] It can easily be shown that the Thomson effect and related phenomena are described by the quantity

$$\mu_{ik} = -\alpha_{ik} + \partial\beta_{ki}/\partial T.$$

In what follows we shall be interested in the asymptotic behavior of the thermal emf in a strong magnetic field. Evidently, knowing the dependence of α_{ik} on **H**, and using the symmetry relations, the asymptotic behavior of β_{ik} and μ_{ik} can easily be derived. In fact, we will first look for the asymptotic Peltier coefficients in each case, for the following reason. It can be seen from (5) that whereas α_{ik} is connected with a_{ik} and b_{ik} [i.e., it is found from the solution of the kinetic equation with an electric field (a_{ik}) and a temperature gradient (b_{ik}) present], the value of β_{ik} can be found if the solution of the kinetic equation in the presence of an electric field only is known so that α_{ik} can be determined by using relation (6).

1. CLOSED FERMI SURFACE

To find the dependence of the tensor β_{ik} on magnetic field one must know the behavior of a_{ik} and c_{ik}, and we shall base our calculation on the work of Lifshitz and Peschanskiĭ.[2] They examined in detail the asymptotic electrical conductivity tensor, $\sigma_{ik} = a_{ik}/T$, but it is clear that apart from some special cases (as can occur in the case of equal numbers of electrons and holes for closed surfaces), the form of the field dependence of a_{ik} and c_{ik} will be analogous. These cases will be examined separately.

a) Unequal numbers of electrons and holes. We use below the symbol $\gamma_0 = 1/\omega t_0$ where t_0 is some characteristic time of the order of the relaxation time, and ω is some frequency of revolution of an electron in its phase trajectory. We must look for the asymptote of quantities in a very high magnetic field, such that $\gamma_0 \ll 1$. According to Lifshitz and Peschanskiĭ, we have for this case

$$a_{ik} = \begin{pmatrix} \gamma_0^2 a_{xx}^{(0)} & \gamma_0 a_{xy}^{(0)} & \gamma_0 a_{xz}^{(0)} \\ -\gamma_0 a_{xy}^{(0)} & \gamma_0^2 a_{yy}^{(0)} & \gamma_0 a_{yz}^{(0)} \\ -\gamma_0 a_{xz}^{(0)} & -\gamma_0 a_{yz}^{(0)} & a_{zz}^{(0)} \end{pmatrix} \tag{7}$$

and the expansion of $a_{ik}^{(0)}$ in powers of γ_0 starts with the zero order term in general. The symmetry relations (3) are taken into account in (7). The most important fact turns out to be that the components a_{xy} and a_{yx} are independent of the collision integral and

$$a_{xy} = -a_{yx} = 2ecT\,(V_1 - V_2)/H\,(2\pi\hbar)^3 = ecT\,(n_1 - n_2)/H,$$

where V_1 and V_2 are the volumes in phase space occupied by electrons and holes, and n_1 and n_2 the corresponding numbers of electrons and holes. Using (1) we then obtain

$$c_{xy} = -c_{yx} = \frac{\pi^2k^2T^3}{3H}\frac{d}{d\varepsilon}(n_1 - n_2).$$

Evidently

$$c_{ik} = \begin{pmatrix} \gamma_0^2 c_{xx}^{(0)} & \gamma_0 c_{xy}^{(0)} & \gamma_0 c_{xz}^{(0)} \\ -\gamma_0 c_{xy}^{(0)} & \gamma_0^2 c_{yy}^{(0)} & \gamma_0 c_{yz}^{(0)} \\ -\gamma_0 c_{xz}^{(0)} & -\gamma_0 c_{yz}^{(0)} & c_{zz}^{(0)} \end{pmatrix},$$

$$a_{ik}^{-1} = \begin{pmatrix} d_{xx} & \gamma_0^{-1} d_{xy} & d_{xz} \\ -\gamma_0^{-1} d_{xy} & d_{yy} & d_{yz} \\ d_{xz} & d_{yz} & d_{zz} \end{pmatrix},$$

then

$$\beta_{ik} = \begin{pmatrix} \nu_{xx} & \gamma_0\nu_{xy} & \gamma_0\nu_{xz} \\ \gamma_0\nu_{yx} & \nu_{yy} & \gamma_0\nu_{yz} \\ \nu_{zx} & \nu_{zy} & \nu_{zz} \end{pmatrix}, \tag{8}$$

where

$$\nu_{xx} = \nu_{yy} = \frac{\pi^2k^2T^2}{3e}\frac{d}{d\varepsilon}\ln(n_1 - n_2). \tag{9}$$

Remembering that $\alpha_{ik} = [\beta_{ki}(-\mathbf{H})]/T$ and taking the symmetry relations into account, the coefficients of the thermal emf, α_{ik} are equal to

$$\alpha_{ik} = \frac{1}{T}\begin{pmatrix} \nu_{xx} & -\gamma_0\nu_{yx} & \nu_{zx} \\ -\gamma_0\nu_{xy} & \nu_{yy} & \nu_{zy} \\ -\gamma_0\nu_{xz} & -\gamma_0\nu_{yz} & \nu_{zz} \end{pmatrix}. \tag{10}$$

b) Equal numbers of electrons and holes. This corresponds to a whole range of metals. The expansion of the components a_{xy} in terms of γ_0 now starts with the quadratic term. i.e.,

$$a_{ik} = \begin{pmatrix} \gamma_0^2 a_{xx}^{(0)} & \gamma_0^2 a_{xy}^{(0)} & \gamma_0 a_{xz}^{(0)} \\ \gamma_0^2 a_{xy}^{(0)} & \gamma_0^2 a_{yy}^{(0)} & \gamma_0 a_{yz}^{(0)} \\ -\gamma_0 a_{xz}^{(0)} & -\gamma_0 a_{yz}^{(0)} & a_{zz}^{(0)} \end{pmatrix}. \tag{11}$$

If we make use of the fact that $d(n_1 - n_2)/d\epsilon \neq 0$, then $c_{xy} \sim \gamma_0$, as in the first case, i.e., the dependence of c_{ik} on **H** is unchanged. We then obtain

$$\beta_{ik} = \begin{pmatrix} \gamma_0^{-1}\nu_{xx} & \gamma_0^{-1}\nu_{xy} & \nu_{xz} \\ \gamma_0^{-1}\nu_{yx} & \gamma_0^{-1}\nu_{yy} & \nu_{yz} \\ \gamma_0^{-1}\nu_{zx} & \gamma_0^{-1}\nu_{zy} & \nu_{zz} \end{pmatrix}, \tag{12}$$

and

$$\alpha_{ik} = \frac{1}{T}\begin{pmatrix} -\gamma_0^{-1}\nu_{xx} & -\gamma_0^{-1}\nu_{yx} & -\gamma_0^{-1}\nu_{zx} \\ -\gamma_0^{-1}\nu_{xy} & -\gamma_0^{-1}\nu_{yy} & -\gamma_0^{-1}\nu_{zy} \\ \nu_{xz} & \nu_{yz} & \nu_{zz} \end{pmatrix}. \tag{13}$$

In the present case all the ν_{ik} depend on the angle between the vector **H** and the crystal axes, and their actual form is determined by the collision integral and the dispersion relation.

2. OPEN FERMI SURFACE

Lifshitz and Peschanskiĭ[2] have shown that in this case the kinetic coefficients are rapid functions of angle near certain particular directions of the magnetic field, when the character of the phase trajectory changes, i.e., goes from open to closed or vice versa. The field dependence of the coefficients varies rapidly near to these directions, and it is, in general, no longer possible to expand in powers of γ_0. These authors pointed out that there are three types of special field direction: a) directions for which a band of open trajectories exists, forming a one-dimensional set; b) directions giving open trajectories forming a two-dimensional region; c) an isolated direction of the vector **H** in the region of open trajectories, where the trajectories become closed.

We consider below the behavior of the thermoelectric coefficients near the special directions of all three types.

The "corrugated cylinder" is the simplest type of surface. For field directions which make a not too great angle with the cylinder axis, the trajectories are closed and the asymptotic expression for β_{ik} and, correspondingly, of α_{ik} will be just the same as for closed surfaces. The behavior of β_{ik}, however, changes rapidly when **H** approaches a direction perpendicular to the cylinder axis. The corresponding special field direction belongs to the first type. If we introduce $\eta = \gamma_0/\sin\theta$ (where θ is the angle between the cylinder axis and the plane perpendicular to the field vector), choose the direction of **H** as the z axis and take the x axis as lying in the plane passing through **H** and the cylinder axis, then[2]

$$a_{ik} = \begin{pmatrix} \gamma_0^2 b_{xx}(\eta) & \gamma_0 b_{xy}(\eta) & \gamma_0 b_{xz}(\eta) \\ -\gamma_0 b_{xy}(-\eta) & b_{yy}(\eta) & b_{yz}(\eta) \\ -\gamma_0 b_{xz}(-\eta) & b_{yz}(-\eta) & b_{zz}(\eta) \end{pmatrix}, \quad (14)$$

where b_{ik} is a function of the type

$$b_{ik} = (b_{ik}^{(0)} + \eta b_{ik}^{(1)} + \eta^2 b_{ik}^{(2)})/(1 + \eta^2 \lambda_{ik}),$$

where $b_{ik}^{(n)}$ and λ_{ik} are relatively slow functions of the angle θ. Then

$$a_{ik}^{-1} = \begin{pmatrix} \gamma_0^{-2} d_{xx}(\eta) & \gamma_0^{-1} d_{xy}(\eta) & \gamma_0^{-1} d_{xz}(\eta) \\ -\gamma_0^{-1} d_{xy}(-\eta) & d_{yy}(\eta) & d_{yz}(\eta) \\ -\gamma_0^{-1} d_{xz}(-\eta) & d_{yz}(-\eta) & d_{zz}(\eta) \end{pmatrix}, \quad (15)$$

$$c_{ik} = \begin{pmatrix} \gamma_0^2 f_{xx}(\eta) & \gamma_0 f_{xy}(\eta) & \gamma_0 f_{xz}(\eta) \\ -\gamma_0 f_{xy}(-\eta) & f_{yy}(\eta) & f_{yz}(\eta) \\ -\gamma_0 f_{xz}(-\eta) & f_{yz}(-\eta) & f_{zz}(\eta) \end{pmatrix}. \quad (16)$$

From this it is easy to obtain

$$\beta_{ik} = \begin{pmatrix} \nu_{xx}(\eta) & \gamma_0 \nu_{xy}(\eta) & \gamma_0 \nu_{xz}(\eta) \\ \gamma_0^{-1} \nu_{yx}(\eta) & \nu_{yy}(\eta) & \nu_{yz}(\eta) \\ \gamma_0^{-1} \nu_{zx}(\eta) & \nu_{zy}(\eta) & \nu_{zz}(\eta) \end{pmatrix}, \quad (17)$$

and the thermal emf is consequently

$$\alpha_{ik} = \frac{1}{T} \begin{pmatrix} \nu_{xx}(-\eta) & -\gamma_0^{-1} \nu_{yx}(-\eta) & -\gamma_0^{-1} \nu_{zx}(-\eta) \\ -\gamma_0 \nu_{xy}(-\eta) & \nu_{yy}(-\eta) & \nu_{zy}(-\eta) \\ -\gamma_0 \nu_{xz}(-\eta) & \nu_{yz}(-\eta) & \nu_{zz}(-\eta) \end{pmatrix}. \quad (18)$$

For $\eta \to \infty$ ν_{ik} tends to a finite limit. By comparing their behavior for two limiting values of θ (0 and $\pi/2$) we see that all the ν_{ik}, except ν_{yx}, ν_{yz}, and ν_{zx}, are relatively slowly varying functions of θ, i.e., they retain the same dependence on H for all angles. At the same time, ν_{yx}, ν_{yz}, and ν_{zx} vary rapidly in the neighborhood of $\theta = 0$. By considering this behavior for $\theta = 0$ and $\theta = \pi/2$ one can obtain

$$\nu_{zx} = \frac{\eta^2 \gamma(\eta) + \eta\sigma(\eta)}{1 + \eta^2 \nu(\eta)},$$

$$\nu_{yx} = \frac{\eta^2 \beta(\eta)}{1 + \eta^2 \lambda(\eta)}, \qquad \nu_{yz} = \frac{\eta\delta(\eta) + \eta^2 \rho(\eta)}{1 + \eta^2 \varphi(\eta)}, \quad (19)$$

where β, γ, δ etc. are slowly varying functions of angle, and their actual form depends on the dispersion law and on the collision integral.

For directions of **H** which make a not too large angle with the cylinder axis, we have

$$\beta_{xx} = \beta_{yy} = -\frac{\pi^2 k^2 T^2}{3e} \frac{d}{d\varepsilon} \ln R, \quad (20)$$

where R is the Hall constant.

Another surface which lends itself easily to investigation is the "space net" type of surface. For this all three types of special field direction exist, and we will not consider the properties of the first type which is exactly equivalent to the case of $\theta = 0$ for a corrugated cylinder. We will discuss the other two cases, which refer to the magnetic field direction close to the direction of the crystallographic axes and of the boundary of the two-dimensional region of field directions which give open trajectories. In these cases the required choice of angle ϑ can be obtained from the fact that a_{ik} must be of the form

$$a_{ik} = \begin{pmatrix} \gamma_0^2 a_{xx}^{(0)} & \gamma_0 a_{xy}^{(0)} & \gamma_0 a_{xz}^{(0)} \\ -\gamma_0 a_{xy}^{(0)} & \gamma_0^2 a_{yy}^{(0)} + \vartheta c_1 & \gamma_0 a_{yz}^{(0)} + \vartheta c_2 \\ -\gamma_0 a_{xz}^{(0)} & -\gamma_0 a_{yz}^{(0)} + \vartheta c_2 & a_{zz}^{(0)} \end{pmatrix}, \quad (21)$$

where a_{ik} is mainly determined by closed trajectories, apart from which there is a band of open trajectories with width proportional to the angle ϑ. Then

$$a_{ik}^{-1}=\begin{pmatrix} d_{xx}+\vartheta\gamma_0^{-2}d_{xx}^{(1)} & \gamma_0^{-1}d_{xy} & d_{xz}+\vartheta\gamma_0^{-1}d_{xz}^{(1)} \\ -\gamma_0^{-1}d_{xy} & d_{yy} & d_{yz} \\ d_{zx}+\vartheta\gamma_0^{-1}d_{zx}^{(1)} & d_{zy} & d_{zz} \end{pmatrix} \quad (22)$$

from which it is easy to deduce that

$$\beta_{ik}=\begin{pmatrix} \nu_{xx} & \gamma_0\nu_{xy} & \gamma_0\nu_{xz} \\ \gamma_0\nu_{yx}+\vartheta\gamma_0^{-1}\nu_{yx}^{(1)} & \nu_{yy} & \gamma_0\nu_{yz}+\vartheta\nu_{yz}^{(1)} \\ \nu_{zx}+\vartheta\gamma_0^{-1}\nu_{zx}^{(1)} & \nu_{zy} & \nu_{zz} \end{pmatrix}, \quad (23)$$

and the coefficients of the thermal emf are

$$\alpha_{ik}=\frac{1}{T}\begin{pmatrix} \nu_{xx} & -\gamma_0\nu_{yx}-\vartheta\gamma_0^{-1}\nu_{yx}^{(1)} & \nu_{zx}-\vartheta\gamma_0^{-1}\nu_{zx}^{(1)} \\ -\gamma_0\nu_{xy} & \nu_{yy} & \nu_{zy} \\ -\gamma_0\nu_{xz} & -\gamma_0\nu_{yz}+\vartheta\nu_{yz}^{(1)} & \nu_{zz} \end{pmatrix}. \quad (24)$$

In the same way we have found the form of the asymptotic behavior of the thermoelectric coefficients near the singularities of all three types.

We must make a comment on the asymptotic behavior of the heat conductivity coefficients. Azbel', Kaganov, and Lifshitz[3] have considered the heat conductivity of metals with closed surfaces. In metals with open surfaces the heat conductivity tensor κ_{ik} behaves completely analogously to the electrical conductivity σ_{ik} in its dependence on **H**. This is seen immediately by considering how κ_{ik} is composed of a_{ik}, b_{ik}, and c_{ik}.

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