[9] W. Swiatecki, Phys. Rev. 100, 936 (1955).

[10] S. Thompson, Report to the Eighth Mendeleev Congress, Moscow, March 1959.

[11] D. D. Ivanenko, Сб. Менделеев, (Mendeleev Anthology), U.S.S.R. Acad. Sci. 1956.

[12] J. A. Wheeler, in Niels Bohr and the Development of Physics (Pergamon, London, 1955); F. G. Werner and J. A. Wheeler, Phys. Rev. 109, 126 (1958).

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ON THE DISTRIBUTION FUNCTION FOR DISSIPATIVE PROCESSES IN A RAREFIED RELATIVISTIC GAS

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Submitted to JETP editor April 4, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) 37, 553-554 (August, 1959)

THE motion of a rarefied gas or of a gas with a large flow gradient, cannot, generally speaking, be treated as the motion of a continuous medium, and additional consequences of kinetic theory must be used.

As is known, Grad[1] considered such a flow by adding to the independent parameters (in addition to the mean velocity, density, and temperature) also the heat flow and the tensor of viscous stresses. Using a specific form of nonrelativistic Maxwellian distribution (which can be considered as a weighting function for three-dimensional Hermite polynomials in velocity space), Grad expanded the distribution function in Hermite polynomials. Retaining only the first three terms of the series, he obtained a distribution function over the coordinates and velocities, describing the processes of viscosity and heat conduction in the nonrelativistic approximation.

It is easy to obtain a distribution function for rarefied relativistic gas, with allowance for the phenomena of viscosity and heat conduction, by introducing orthogonal polynomials with a weight $\exp(-\sigma\sqrt{1+u^2})$. Here $\sigma = mc^2/kT$, where T is the temperature in the proper reference system of the given gas element; $u^2 = u_\alpha^2$, where u_α are the spatial components of the four-velocity of the gas particles.* By way of an example we cite the first two polynomials of this type

$$g^{(0)} = 1/2\sqrt{\pi}\,\sigma^{1/4}K_2^{1/2}(\sigma), \qquad g_\alpha^{(1)} = \xi_\alpha/2\sqrt{\pi}\,\sigma^{1/4}K_3^{1/2}(\sigma),$$
$$\xi_\alpha = \sigma^{1/2}u_\alpha. \tag{1}$$

Here $K_\nu(\sigma)$ is the MacDonald function.

As is known,[2] the scalar distribution can be written in the form

$$F = icf(x, p)\,\delta(H + mc). \tag{2}$$

Here H is the invariant Hamiltonian function, while x and p are the 4-coordinates and 4-momentum of the particle. The scalar f(x, p) coincides with the ordinary distribution function and its expression in the proper coordinate system of the gas in equilibrium differs from $\exp(-\sigma\sqrt{1+u^2})$ only by a multiplicative factor. If we now expand $f\,[\exp(-\sigma\sqrt{1+u^2})]^{-1/2}$ in terms of the functions $[\exp(-\sigma\sqrt{1+u^2})]^{1/2} g_\alpha^{(n)}$ (the expansion is valid in the sense of convergence in the mean) and confine ourselves to the first three terms of the expansion, we obtain after simple calculations an expression for f(x, p) in the proper system of reference of the given element of gas

$$f(x, p) = \exp(-\sigma\sqrt{1+u^2})\left\{\frac{n\sigma}{4\pi(mc)^3K_2(\sigma)} + \frac{\sigma^2\tau_{\alpha\beta}\xi_\alpha\xi_\beta}{8\pi m^4c^5K_3(\sigma)} + \frac{\sigma^{5/2}}{24\pi m^5c^6K_4(\sigma)}\left[T_{\alpha\beta\gamma}\xi_\alpha\xi_\beta\xi_\gamma - 3\frac{K_4(\sigma)}{K_3(\sigma)}T_{\alpha\beta\beta}\xi_\alpha\right]\right\}. \tag{3}$$

Here n is the density of the particles in a proper system of the given element of gas, $\tau_{\alpha\beta}$ is the additional term in the three-dimensional portion of the energy-momentum tensor due to the dissipative processes, and $T_{\alpha\beta\gamma}$ are the spatial components of the tensor $T_{ikl} = \int p_i p_k F_l d^4p$.

The expression for T_{ikl} in any system of reference can be obtained from its components in the proper system of reference. As a result we obtain

$$T_{ikl} = \frac{nm^2c^3}{\sigma}\frac{K_3(\sigma)}{K_2(\sigma)}(U_i\delta_{kl} + U_k\delta_{il} + U_l\delta_{ik}) + \frac{nm^2c^3K_4(\sigma)}{K_2(\sigma)}U_iU_kU_l + \frac{mcK_4(\sigma)}{K_3(\sigma)}(U_i\tau_{kl} + U_k\tau_{il} + U_l\tau_{ik}) + R_{ikl}, \tag{4}$$

where U_i is the 4-velocity corresponding to the average motion, and R_{ikl} is a tensor, whose components $R_{\alpha\beta4}$ and R_{444} vanish in the proper system and whose remaining components coincide with the components of T_{ikl} in the same system. The tensor T_{ikl} can be used in the study of transport phenomena, and also to investigate the structure of a shock wave. We note that for large values of σ, Eqs. (3) and (4) go into the corresponding nonrelativistic expressions.

Inasmuch as the function f is a scalar, its form for any system of reference should be

$$f(x,p) = \exp(p_l q_l)\left\{\frac{n\sigma}{4\pi(mc)^3 K_2(\sigma)} + \frac{\sigma^2 \tau_{ik}\xi_i\xi_k}{8\pi m^4 c^5 K_3(\sigma)} + \frac{\sigma^{5/2}}{24\pi m^5 c^6 K_4(\sigma)}\left[R_{ikl} + \frac{(mc)^2[K_4^2(\sigma) - K_5(\sigma)K_3(\sigma)]}{[K_3^2(\sigma) - K_4(\sigma)K_2(\sigma)]} \times (\nu_i U_k U_l + \nu_k U_l U_l + \nu_l U_i U_k)\right] \times \left[\xi_i\xi_k\xi_l - \frac{K_4(\sigma)}{K_3(\sigma)}(\xi_i\delta_{kl} + \xi_k\delta_{il} + \xi_l\delta_{ik})\right]\right\},$$

where the q_i have the same meaning as in reference 2, and ν_i is an additional term in the vector of material flow density, due to the dissipative process.

Inasmuch as $T_{iil} = -m^2c^2(cnU_l + \nu_l)$, the parameters that determine the state of the gas are n, T, U_i, τ_{ik}, and R_{ikl}. Using Eq. (3) and the requirement that the mean energy be expressed only in terms of the Maxwellian portion of the distribution function, it is easy to show that $\tau_{ii} = 0$.

In conclusion, I consider it my pleasant duty to thank Prof. V. L. German for interest in the work and for valuable advice, and to G. I. Budker and S. I. Braginskiĭ for very valuable discussions.

*The Greek indices run through three values, and the Latin ones through four; repeated indices imply summation.

[1] H. Grad, Commun. on Pure and Appl. Math. **2**, 331 (1949).

[2] S. T. Belyaev and G. I. Budker, Dokl. Akad. Nauk SSSR **107**, 807 (1956), Soviet Phys.-Doklady **1**, 218 (1956).

Translated by J. G. Adashko
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ELECTRON RESONANCE IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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Submitted to JETP editor May 21, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 555-556 (August, 1959)

It is known that a free electron placed in crossed electric and magnetic fields drifts in the direction perpendicular to the electric and magnetic fields. The drift velocity, i.e., the mean velocity of the electron's motion* (apart from a part dependent on its initial velocity), is equal to $\overline{\mathbf{v}} = cH^{-2}[\mathbf{E}\times\mathbf{H}]$, where **E** and **H** are the intensities of the electric and magnetic fields. In addition, the electron executes an oscillatory motion along the electric field, at a frequency eH/mc; that is, the frequency in crossed fields does not depend on the electric field.[1]

The situation is different for an electron in a metal or semiconductor. The complicated dispersion law has a pronounced effect on the character of the motion of a conduction electron. We shall start from the classical equation of motion, the Lorentz generalized equation,

$$d\mathbf{p}/dt = e\{\mathbf{E} + c^{-1}[\mathbf{v}\times\mathbf{H}]\}, \quad \mathbf{v} = \partial\varepsilon/\partial\mathbf{p}. \tag{1}$$

It is easy to show that the integrals of the motion in this case are

$$\varepsilon^*(\mathbf{p}) \equiv \varepsilon(\mathbf{p}) - \mathbf{v}_0\mathbf{p} = \text{const},$$

$$\mathbf{v}_0 = cH^{-2}[\mathbf{E}\times\mathbf{H}], \; p_z = \text{const}. \tag{2}$$

The z axis is taken along the magnetic field; $\mathbf{v}_0$ coincides with the mean velocity of the electron's motion, i.e., with the drift velocity, only in case the trajectory of the electron in momentum space, as determined by Eq. (2), is closed. In fact, on introducing the velocity $\mathbf{v}_0$ in Eq. (1) we get

$$d\mathbf{p}/dt = (e/c)[\mathbf{v} - \mathbf{v}_0]\times\mathbf{H}. \tag{3}$$

From this it is clear that $\overline{\mathbf{v}} = \mathbf{v}_0$ if $\overline{d\mathbf{p}/dt} = 0$. This happens in the case of closed trajectories.[2]

Equation (3) shows that the motion in crossed fields of a particle with the dispersion law $\epsilon = \epsilon(\mathbf{p})$ can be treated as motion in a magnetic field alone of a particle with the dispersion law†

$$\varepsilon^*(\mathbf{p}) = \varepsilon(\mathbf{p}) - \mathbf{v}_0\mathbf{p}. \tag{4}$$

Therefore the results obtained before are easily transferred to this case. In particular, the period T* of revolution of an electron around a closed orbit is[2,3]

$$T^* = -(c/eH)\,\partial S^*/\partial\varepsilon^*. \tag{5}$$

Here S* is the area bounded by the curve determined by Eq. (2); it depends, naturally, on the electric field. It is interesting to note that this dependence disappears in the case of a quadratic dispersion law: the presence of the term $-\mathbf{v}_0\mathbf{p}$ in the Eq. (2) merely perturbs the trajectory without changing its area. Thus the dependence of the period of revolution on the electric field is an effect specific to an electron with a complicated (non-quadratic) dispersion law. It should be noticed that cases are possible in which a conduction