

electron in a magnetic field executes a finite motion (its trajectory in momentum space being a closed curve), whereas in crossed fields it executes an infinite one, since the trajectory (2) is an open curve.

The explicit dependence of the period on the electric field can be obtained only for a definite law of dispersion. However, if  $E/H \ll 1$ , it is possible to obtain the result

$$T^* \approx T \left\{ 1 - (c v_0 / e H T) \oint v_{\perp}^{-2} dl (\mathbf{n} + \mathbf{p}/R) \right\}. \quad (6)$$

Here  $T$  is the period of revolution in a magnetic field,  $\mathbf{n} = \mathbf{v}_{\perp} / |\mathbf{v}_{\perp}|$  is the normal to the trajectory of the electron in a magnetic field, and  $R$  is the radius of curvature of the trajectory. The integration extends over the trajectory in a magnetic field. Thus  $\Delta T/T \sim (c/v)(E/H)$ .

Once we know the frequency of revolution of the electron ( $\omega^* = 2\pi/T^*$ ), it is easy to write down the distance between quantum energy levels in the classical approximation:<sup>4,5</sup>

$$\Delta \varepsilon^* = \hbar \omega^* = 2\pi |e| \hbar H / c (\partial S^* / \partial \varepsilon^*).$$

In connection with the dependence of the frequency of revolution of an electron in crossed fields on the size of the electric field, an interesting peculiarity should apparently occur in diamagnetic resonance in those semiconductors in which the dependence of the energy of the current carriers on the quasimomentum is appreciably nonquadratic: the resonance frequency should depend on the electric current passed through the specimen.

A nonquadratic dependence of the energy on the components of the quasi-momentum occurs not infrequently near the edge of the conduction band. Often it is a consequence of the crystal symmetry. Here the quadratic dependence on the magnitude of the momentum is retained near the edge of the band, but the angular dependence becomes complicated. Thus the energy spectrum of "holes" in Ge and Si crystals has the form<sup>6</sup>

$$\varepsilon = A p^2 \pm [B^2 p^4 + C^2 (p_x^2 p_y^2 + p_x^2 p_z^2 + p_y^2 p_z^2)]^{1/2},$$

where  $A$ ,  $B$ , and  $C$  are constants.

To observe such effects in metals is in all probability impossible, since in a metal (in consequence of the large electrical conductivity) it is impossible to produce any appreciable electric field. To estimate the order of magnitude of the effect, we must start from formula (6), remembering however that the resonance frequencies are determined not by all the electrons but by those that have extremal effective masses.<sup>7</sup> It can be shown that for these electrons no effect

linear in the electric field is present because of the symmetry of the trajectory. Therefore, apparently,  $\Delta\omega/\omega \sim (c/v)^2 (E/H)^2$ .

\*We have in mind the mean velocity in a plane perpendicular to the magnetic field.

<sup>†</sup>Except for an unimportant constant,  $\varepsilon^*$  coincides with the total energy of the particle.

<sup>1</sup>L. D. Landau and E. M. Lifshitz, *Теория поля (Field Theory)*, Gostekhizdat, 1948 [Transl: Addison-Wesley, 1951].

<sup>2</sup>Lifshitz, Azbel', and Kaganov, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **31**, 63 (1956), *Soviet Phys. JETP* **4**, 41 (1957).

<sup>3</sup>W. Shockley, *Phys. Rev.* **79**, 191 (1950).

<sup>4</sup>I. M. Lifshitz, Report at a session of the physics-mathematics section, Academy of Sciences, Ukr. S.S.R., 1951; I. M. Lifshitz and A. M. Kosevich, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **29**, 730 (1955), *Soviet Phys. JETP* **2**, 636 (1956).

<sup>5</sup>L. Onsager, *Phil. Mag.* **43**, 1006 (1952).

<sup>6</sup>Dresselhaus, Kip, and Kittel, *Phys. Rev.* **98**, 368 (1955).

<sup>7</sup>M. Ya. Azbel' and É. A. Kaner, *J. Exptl. Theoret. Phys. (U.S.S.R.)* **32**, 896 (1957), *Soviet Phys. JETP* **5**, 730 (1957).

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## IDENTIFICATION OF PARTICLES IN HIGH ENERGY STARS

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THE identification of particles in high energy stars\* is often made by comparing the measurements of the momentum  $p_1$  of one of the particles with its possible limiting values under predetermined assumptions about the mass and the number of remaining particles  $2, 3, \dots, n$ . These last are united into one composite particle having some effective mass  $m_{\text{eff}}$ . The formula for the momentum of particle 1 at an angle of observation  $\theta_1$  under the assumption that the other particle has a mass  $m_{\text{eff}}$  gives the limiting pos-

sible values of the momentum of particle 1.<sup>1,2,3</sup> Ordinarily the value  $\bar{m} = m_2 + m_3 + \dots + m_n$  is taken for  $m_{\text{eff}}$ , a result which considers the velocities of particles 2, 3, ..., n to be equal in magnitude and direction.<sup>2</sup>

We shall show that the solution for the bounds  $p_{1\text{max}}$ ,  $p_{1\text{min}}$  of the momentum  $p_1$  of particle 1 can be restricted if one takes into account the angles  $\theta_{ij}$  between the other charged particles  $i$  and  $j$  ( $i, j = 2, 3, \dots, n'$ ) and if the lower limit  $\tilde{p}_i$  of their momenta  $p_i$  is estimated.

An attempt has been made earlier<sup>4</sup> to take into account information about the angles and momenta of the particles for purposes of identification. Unlike reference 4, the present work includes this information directly in  $m_{\text{eff}}$ . In this way only knowledge of the lower bounds of the momenta is required (in reference 4, knowledge of the values of  $p_i$  themselves is required, which is difficult for large  $p_i$  and leads to an indeterminacy in the limiting values for  $p_1$ ).

To deduce a necessary formula, we note that the equation for the momentum  $p_1$  (or energy  $E_1$ ) of one of the secondary particles having the total energy  $E$  and momentum  $P$  coincides with the equation for  $p_1$  in the decay of a particle with energy  $E$  and momentum  $P$  into two particles with masses  $m_1$  and  $m_{\text{eff}}$ , if we take

$$m_{\text{eff}}^2 = (E_2 + \dots + E_n)^2 - (\mathbf{p}_2 + \dots + \mathbf{p}_n)^2. \quad (1)$$

It is easy to show that the roots of the equation above for  $p_1$  have the characteristic

$$dp_{1\text{max}}/dm_{\text{eff}} < 0, \quad dp_{1\text{min}}/dm_{\text{eff}} \geq 0.$$

This means that increasing the estimate for  $m_{\text{eff}}$  shrinks the region of solutions for the value  $p_1$ .

To increase this estimate, we write (1) as three positive terms

$$m_{\text{eff}}^2 \equiv \sum_{i=2}^n m_i^2 + 2 \sum_{i<j}^n (E_i E_j - p_i p_j) + 2 \sum_{i<j}^n p_i p_j (1 - \cos \theta_{ij}). \quad (2)$$

Taking into account  $E_i E_j - p_i p_j \geq m_i m_j$  and  $p_i > \tilde{p}_i$  (where  $\tilde{p}_i$  is the lower bound of  $p_i$ ),<sup>†</sup> we immediately get the following estimate:

$$m_{\text{eff}}^2 \geq \tilde{m}^2 \equiv \bar{m}^2 + \Delta^2 \equiv \bar{m}^2 + 2 \sum_{2 \leq i < j}^{n'} \tilde{p}_i \tilde{p}_j (1 - \cos \theta_{ij}). \quad (3)$$

Here the sum is carried out over all pairs of charged particles, except particle 1. The masses of the neutral particles are included in  $\bar{m}$ .

Thus, if we take  $\tilde{m}$  instead of  $\bar{m}$  for the value of  $m_{\text{eff}}$ ,  $p_{1\text{min}}$  and  $p_{1\text{max}}$  come closer and closer together as  $\tilde{p}_i$  increases and  $\theta_{ij}$  becomes

larger. For narrow beams of secondary particles the use of formula (3) gives no effect.

If for some particles  $i$  and  $j$  not only  $\tilde{p}$  but also  $p$  is known,  $\tilde{p}$  can be changed to  $p$  in the equations and the term  $E_i E_j - p_i p_j - m_i m_j$  can be added. This makes  $p_{1\text{max}}$  and  $p_{1\text{min}}$  converge even more.

The most probable contribution from neutral particles to  $m_{\text{eff}}$  can be taken into account by adding to  $\Delta^2$  the term  $\frac{1}{2}n'(\frac{1}{2}n' - 1)\bar{p}^2(1 - \cos \bar{\theta})$ , where  $\bar{p}$  and  $\bar{\theta}$  are the average values of  $p_i$  and  $\theta_{ij}$  in the given interaction.

The results of this work are given in more detail in reference 5.

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\*We consider high energy stars to be those in which there are tracks of relativistic particles.

<sup>†</sup>For gray tracks, for example, one can take  $\tilde{p}_i = m_i$ ; for neutral particles,  $\tilde{p}_i = 0$  is taken.

<sup>1</sup>I. L. Rozental', Usp. Fiz. Nauk **54**, 405 (1954).

<sup>2</sup>R. M. Sternheimer, Phys. Rev. **93**, 642 (1954).

<sup>3</sup>G. I. Kopylov, "On Estimating the Number of Secondary Particles Near Limiting Angles," preprint, Joint Inst. Nuc. Res. R-166 (1958).

<sup>4</sup>Birger, Grigorov, Guseva, Zhdanov, Slavatskiĭ, and Stashkov, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 971 (1956), Soviet Phys. JETP **4**, 872 (1957).

<sup>5</sup>G. I. Kopylov, preprint, Joint Inst. Nuc. Res., R-341 (1959).

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## NOTE ON A BARYON SCHEME

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LET us assume that each of the eight known baryons is described by a four-component wave function. The general equation for all the baryons is in this case an equation for a 32-component spinor. A 32-dimensional spinor space can be treated as a rep-