THE NUMBER OF ELEMENTARY BARYONS AND THE UNIVERSAL BARYON REPULSION HYPOTHESIS

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CONSIDER a hypothetical experiment in which neutrons are compressed to a density such that the energy at the Fermi surface exceeds a few Mc^2. In that case a partial transformation of neutrons into other baryons — protons and hyperons — will be thermodynamically favored, and it will be necessary to consider as many independent Fermi distributions as there are elementary particles. The problem of the number of elementary particles may be approached in this way since if some particle is in reality not elementary it would not give rise to a separate Fermi distribution. If, for example, the Σ^+ is a bound p and K^0 complex, then at high densities it will turn out that "inside" the Σ^+ there is a p identical to other free protons (the Σ^+ will be "crushed").

In the asymptotic expression for the energy of a relativistic Fermi gas $\epsilon = AN^{4/3}$, where N is the total density of all baryons, the coefficient A is given by

$$A = [3(3\pi^2)^{1/3}/4]\,\nu^{-1/3}\hbar c = a\nu^{-1/3},$$

where ν is the number of the kinds of truly elementary particles and a is the coefficient for $\nu = 1$. Thus there exists in principle, a possibility of determining ν; it was assumed in the above that in the limit of large densities all interactions are small compared to the Fermi energy.

If one were to insist on designating as different particles (index 1 and 2) that are actually made up of identical fermions with different surrounding boson clouds, then the increase in the total energy of a system consisting of N_1 particles of one type and N_2 particles of the second type, in comparison with the Fermi energy of each group separately, will give the appearance of a repulsion of these particles:

$$\varepsilon = a(N_1 + N_2)^{4/3} = aN_1^{4/3} + aN_2^{4/3} + V,$$

where V is the energy of the apparent interaction.

The question arises whether the experimentally observed nucleon repulsion ("hard core") at small distances is not precisely such an "apparent" interaction, due to the fact that all baryons contain "inside" them one common fermion — the carrier of the conserved baryon charge. Such a repulsion may be investigated in an elementary example.* Let us consider the collision between a proton p and a mesic atom H consisting of p and π^-. We may treat p and H as two different spin $\frac{1}{2}$ particles. Then the ^3S state of the system with parallel spins and zero orbital angular momentum is allowed. The wave function ψ for the system, after separating out the center of mass motion and the spin function, may be written as

$$\psi(\mathbf{R}, \boldsymbol{\rho}) = \Phi(\mathbf{R})\chi(\mathbf{R}, \boldsymbol{\rho}), \quad \mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2.$$
$$\boldsymbol{\rho} = \mathbf{r}_\pi - (\mathbf{r}_1 + \mathbf{r}_2)/2.$$

For the ^3S state χ should be an odd function and may be written approximately as

$$\chi = B[\varphi(\boldsymbol{\rho} - \mathbf{R}/2) - \varphi(\boldsymbol{\rho} + \mathbf{R}/2)],$$

where φ stands for the ground state function of the π^- in the atom:

$$\varphi(\boldsymbol{\rho} - \mathbf{R}/2) = \varphi(\mathbf{r}_\pi - \mathbf{r}_1), \quad \varphi(\boldsymbol{\rho} + \mathbf{R}/2) = \varphi(\mathbf{r}_\pi - \mathbf{r}_2).$$

As the protons approach each other χ goes over into the function describing a P-state meson ($l_\pi = 1$) in the field of the two protons with a projection of the angular momentum onto the direction **R** equal to $l_{\mathbf{R}} = 0$. In deriving the Schrödinger equation for Φ it is customary to add to $V(\mathbf{r}_1 - \mathbf{r}_2) = V(\mathbf{R})$ the meson energy $E_\pi(R)$ calculated from the function χ as $\langle\chi^* H_\pi \chi\rangle$ where

$$H_\pi = -(\hbar^2/2m_\pi)\Delta_\rho + V(|\mathbf{r}_\pi - \mathbf{r}_1|) + V(|\mathbf{r}_\pi - \mathbf{r}_2|).$$

Actually it is also necessary to consider the effect of the operator $(-\hbar/2\mu)\Delta_R$ on $\chi(\mathbf{R}, \boldsymbol{\rho})$ (μ is reduced mass of the two protons). For small R the contribution of this term is (due to the angular part of Δ_R)

$$E_1 = \langle\chi^* | -(\hbar^2/2\mu)\Delta_R | \chi\rangle = (\hbar^2/2\mu)\,2R^{-2}.$$

The Schrödinger equation for Φ

$$-(\hbar^2/2\mu)\Delta_R\Phi + [V(R) + E_\pi(R) + \hbar^2/\mu R^2]\Phi = E\Phi,$$

insures the vanishing for $R = 0$ of a spherically symmetric function Φ only if the term E_1 is included. Such a result is self-explanatory: two protons with parallel spins must at small distances be in a state† $L = 1$ and E_1 is the centrifugal potential. However E_1 appears as the effective potential in the equation for the function Φ describing the spherically symmetric (S) state.

In the study of the scattering of p on H after elimination of the ^3S component the contribution fo E_1 will manifest itself as a strong repulsion at small distances. In contrast to the usual term E_π the expression for E_1 does not depend on the properties (mass, charge) of the meson.

The appearance of E_1 is entirely due to the fact that the "interiors" of the two different particles p and H are identical. If the meson cloud surrounding the proton in the H atom is in a state with $l_\pi = 1$ then a repulsion will appear not only in the ^3S but also in the ^1S state. The average value of the coefficient of the term $\hbar^2/2\mu R^2$ should be of the order of unity.

In the study of two identical particles (e.g., two atoms H) with meson clouds having $l_\pi = 1$ there also appears a repulsion in the ^1S state. However the average Fermi energy in this case remains unchanged; that is, the repulsion in the ^1S state is compensated for by an attraction in the ^3P state (reduced centrifugal potential in that state).

Let us return from models to baryons. The hypothesis of one common "core" leads to the conclusion that in the interaction of different or identical baryons in S states there should appear a strong repulsion at small distances with a potential $\sim \hbar^2/2\mu R^2$. The present-day data[1] on the p-p and p-n interactions at small distances are in agreement with this estimate. No such repulsion should be observed in the interaction of any baryons with any antibaryons.

In the interaction of identical particles the short range interaction, averaged appropriately over the various angular momentum states, vanishes. The study of short range forces between different particles in various spin and angular momentum states could replace the "gedanken" experiment on the determination of the number ν of elementary particles from the density dependence of the energy considered at the beginning of this note, and would make it possible to establish whether or not the different pairs of particles under study have a common "interior."

I take this opportunity to express my gratitude to A. D. Sakharov; a discussion with him on the state of matter in superdense stars served as the origin of this work.

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*This example was discussed by S. S. Gershteĭn in connection with the theory of hydrogen mesic molecules.

†The total orbital angular momentum of the system equals zero, however the meson also carries one unit of angular momentum.

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[1] P. S. Signell and R. E. Marshak, Phys. Rev. 109, 1229 (1958).

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POLARIZATION EFFECTS IN THE DIRECT TRANSITION OF $\mu^+\mu^-$ INTO AN ELECTRON-POSITRON PAIR

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RECENTLY Zel'dovich[1] called attention to the possibility of a direct transition of a $\mu^+\mu^-$ pair through a virtual photon into an electron-positron pair. It is of interest to study this process in more detail, in particular using not only the nonrelativistic approximation employed by Zel'dovich.

The matrix element describing this process can be obtained directly from the exchange part of the matrix element for Bhabha scattering (see, e.g., reference 2, formula 49,49) by replacing the initial state electron and positron wave functions in it by μ-meson wave functions. Keeping this remark in mind it is easy to write down the expression for the probability of the transition $\mu^+\mu^- \rightarrow e^+e^-$. In the center-of-mass system, neglecting the rest masses of the electron and positron in comparison to their energies, we obtain

$$dw = (e^4 d\Omega / 8c\hbar^2 L^3 K_\mu^2)\, S^+ S, \tag{1}$$

where the spin part of the matrix element is

$$S = b_\mu^{'+} \alpha_\nu b_\mu b_e^{'+} \alpha_\nu b_e. \tag{2}$$

Here α_ν is a four-vector composed of Dirac matrices and the b's are the spinor amplitudes of the wave functions of the corresponding particles. Further calculations dealing with the spin states of the particles are considerably simplified if use is made of a table given in the monograph by Sokolov[2] (formulas 21,17 and 21,18). Applying these formulas to Eq. (1) and summing over the electron and positron spins we find

$$dw(s_\mu, s_\mu') = \frac{e^4 d\Omega}{8c\hbar^2 L^3 K_\mu^2}\left(1 - s_\mu s_\mu' \frac{k_\mu^2}{K_\mu^2} + \frac{k_{0\mu}^2}{K_\mu^2}\right)(1 - s_\mu s_\mu' \cos^2\theta). \tag{3}$$

Here $\hbar\mathbf{k}$ is the momentum of the particle, $c\hbar K = c\hbar\sqrt{k^2 + k_0^2}$ is its energy, s is the projection of the particle's spin onto its direction of motion, and θ is the angle between the meson and electron momenta. In Eq. (3) $s_\mu s_\mu'$ can take on the values ± 1, where the value -1 corresponds to the ortho-state of $\mu^+\mu^-$ with total spin parallel or antipar-