The appearance of E_1 is entirely due to the fact that the "interiors" of the two different particles p and H are identical. If the meson cloud surrounding the proton in the H atom is in a state with $l_\pi = 1$ then a repulsion will appear not only in the ^3S but also in the ^1S state. The average value of the coefficient of the term $\hbar^2/2\mu R^2$ should be of the order of unity.

In the study of two identical particles (e.g., two atoms H) with meson clouds having $l_\pi = 1$ there also appears a repulsion in the ^1S state. However the average Fermi energy in this case remains unchanged; that is, the repulsion in the ^1S state is compensated for by an attraction in the ^3P state (reduced centrifugal potential in that state).

Let us return from models to baryons. The hypothesis of one common "core" leads to the conclusion that in the interaction of different or identical baryons in S states there should appear a strong repulsion at small distances with a potential $\sim \hbar^2/2\mu R^2$. The present-day data[1] on the p-p and p-n interactions at small distances are in agreement with this estimate. No such repulsion should be observed in the interaction of any baryons with any antibaryons.

In the interaction of identical particles the short range interaction, averaged appropriately over the various angular momentum states, vanishes. The study of short range forces between different particles in various spin and angular momentum states could replace the "gedanken" experiment on the determination of the number ν of elementary particles from the density dependence of the energy considered at the beginning of this note, and would make it possible to establish whether or not the different pairs of particles under study have a common "interior."

I take this opportunity to express my gratitude to A. D. Sakharov; a discussion with him on the state of matter in superdense stars served as the origin of this work.

*This example was discussed by S. S. Gershteĭn in connection with the theory of hydrogen mesic molecules.

†The total orbital angular momentum of the system equals zero, however the meson also carries one unit of angular momentum.

[1] P. S. Signell and R. E. Marshak, Phys. Rev. 109, 1229 (1958).

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POLARIZATION EFFECTS IN THE DIRECT TRANSITION OF $\mu^+\mu^-$ INTO AN ELECTRON-POSITRON PAIR

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RECENTLY Zel'dovich[1] called attention to the possibility of a direct transition of a $\mu^+\mu^-$ pair through a virtual photon into an electron-positron pair. It is of interest to study this process in more detail, in particular using not only the nonrelativistic approximation employed by Zel'dovich.

The matrix element describing this process can be obtained directly from the exchange part of the matrix element for Bhabha scattering (see, e.g., reference 2, formula 49,49) by replacing the initial state electron and positron wave functions in it by μ-meson wave functions. Keeping this remark in mind it is easy to write down the expression for the probability of the transition $\mu^+\mu^- \rightarrow e^+e^-$. In the center-of-mass system, neglecting the rest masses of the electron and positron in comparison to their energies, we obtain

$$dw = (e^4 d\Omega / 8c\hbar^2 L^3 K_\mu^2)\, S^+ S, \qquad (1)$$

where the spin part of the matrix element is

$$S = b_\mu'^+ \alpha_\nu b_\mu b_e'^+ \alpha_\nu b_e. \qquad (2)$$

Here α_ν is a four-vector composed of Dirac matrices and the b's are the spinor amplitudes of the wave functions of the corresponding particles. Further calculations dealing with the spin states of the particles are considerably simplified if use is made of a table given in the monograph by Sokolov[2] (formulas 21,17 and 21,18). Applying these formulas to Eq. (1) and summing over the electron and positron spins we find

$$dw(s_\mu, s_\mu') = \frac{e^4 d\Omega}{8\, c\hbar^2 L^3 K_\mu^2}\left(1 - s_\mu s_\mu' \frac{k_\mu^2}{K_\mu^2} + \frac{k_{0\mu}^2}{K_\mu^2}\right)(1 - s_\mu s_\mu' \cos^2\theta). \qquad (3)$$

Here $\hbar\mathbf{k}$ is the momentum of the particle, $c\hbar K = c\hbar\sqrt{k^2 + k_0^2}$ is its energy, s is the projection of the particle's spin onto its direction of motion, and θ is the angle between the meson and electron momenta. In Eq. (3) $s_\mu s_\mu'$ can take on the values ± 1, where the value -1 corresponds to the ortho-state of $\mu^+\mu^-$ with total spin parallel or antiparallel to the direction of motion of the mesons. Setting $s_\mu s'_\mu = -1$ in Eq. (3) we find

$$dw_1^s = dw_{-1}^s = (e^4 d\Omega / 4c\hbar^2 L^3 K_\mu^2)(1 + \cos^2\theta). \qquad (4)$$

In order to find the transition probability $\mu^+\mu^- \rightarrow e^+e^-$ in the third ortho-state where the projection of the total spin of the mesons onto their direction of motion is zero, we introduce the appropriate symmetric combination of the spinor amplitudes directly into Eq. (1) and write it in the form

$$dw_0^s = (e^4 d\Omega / 8c\hbar^2 L^3 K_\mu^2)(b_e^+ \alpha_{1\nu} b'_e b'^+_e \alpha_{2\nu} b_e) \cdot 2^{-1/2} \{b_\mu^+(1)\alpha_{1\nu} b'_\mu(1) + b_\mu^+(-1)\alpha_{1\nu} b'_\mu(-1)\} \cdot 2^{-1/2} \{b'^+_\mu(1)\alpha_{2\nu} b_\mu(1) + b'^+_\mu(-1)\alpha_{2\nu} b_\mu(-1)\}. \qquad (5)$$

For simplicity let us choose the z axis along the direction of motion of the mesons. One can then show that $b(s) = \rho_3\sigma_1 b(-s)$. With this fact in mind we obtain from Eq. (5), after summing over the electron and positron spins, the following expression for the transition probability in the third ortho-state:

$$dw_0^s = (e^4 d\Omega / 4c\hbar^2 L^3 K_\mu^2) \times (1 - k_\mu^2 / K_\mu + k_{0\mu}^2 / K_\mu^2)(1 - \cos^2\theta). \qquad (6)$$

A comparison of Eqs. (4) and (6) shows that the transition probability in the orthostate depends strongly on the value of the projection of the total spin of the mesons onto their direction of motion. When this projection equals ± 1 the electrons are emitted mainly along the direction of motion of the mesons, whereas when this projection equals 0 the electrons are emitted mainly in a direction perpendicular to the line of motion of the mesons.

In the nonrelativistic approximation ($k_\mu \rightarrow 0$, $\cos^2\theta \rightarrow 1/3$) the $\mu^+\mu^- \rightarrow e^+e^-$ transition probability is the same in all three ortho-states and is equal to

$$w^s = 4\pi e^4 / 3c\hbar^2 L^3 k_{0\mu}, \qquad (7)$$

which agrees with the result of Zel'dovich. However as the meson energy increases w_0^S decreases much faster then w_1^S, w_{-1}^S and in the extreme relativistic limit (when $K_\mu \gg k_{0\mu}$) the probability w_0^S vanishes.

As can be seen from Eqs. (3), (4), and (6) the $\mu^+\mu^- \rightarrow e^+e^-$ transition probability in the para-state is zero not only in the nonrelativistic approximation but in general.

I am indebted to Prof. A. A. Sokolov for his help and advice.

[1] Ya. Zel'dovich, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 646 (1959), Soviet Phys. JETP **9**, 450 (1959).

[2] A. A. Sokolov, Введение в квантовую электродинамику (Introduction to Quantum Electrodynamics), M., Fizmatgiz, 1958, Ch. 4.

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AN INVESTIGATION OF THE QUANTIZED OSCILLATIONS IN THE MAGNETIC SUSCEPTIBILITY OF BISMUTH AT EXTREMELY LOW TEMPERATURES

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It can be deduced from galvanomagnetic measurements that bismuth belongs to the group of metals having equal numbers of electrons and holes. We can now regard it as established that a portion of the Fermi surface for groups of electrons is well described by Shoenberg's three-ellipsoid model,[1] proposed on the basis of the measurement of quantized oscillations of the magnetic susceptibility of bismuth at helium temperatures.

Experiments by other authors on oscillations of magnetic susceptibility,[2,3] electrical resistance and Hall emf[4,5,6] in high magnetic fields, related to another part of the Fermi surface, have not until now yielded positive results. We thought that these oscillations were not observed at helium temperatures because of their small amplitude, which would be sufficiently enhanced for observation at much lower temperatures. For this purpose we developed the apparatus[7] and measured the anisotropy of magnetic susceptibility of bismuth at extremely low temperatures. These experiments are of interest in themselves since, as far as we know, the magnetic susceptibility of metals and semiconductors has not before been studied at these low temperatures.

The apparatus, which is a torsion balance, is shown in Fig. 1. The salt pill, 1, with the heat link,[8] 2, and the sample holder, 3, are fixed to the balance suspension system. The glass sleeve for the balance, 4, consists of two tubes separated