new oscillations is shown in Fig. 3 by the dashed curve. It appears that these oscillations correspond to a group of holes which have a Fermi surface in the form of a surface of revolution, stretched out along the trigonal axis.* On the assumption that this surface is closed, a rough calculation leads to a value of the "hole" concentration $n \approx 0.5 \times 10^{18}$ cm^{-3} and an effective mass along the trigonal axis $m_3^* = (\frac{1}{2}\pi)(\partial S_m/\partial E) \approx 0.06\, m_0$ (m_0 is the free electron mass). It is most probable that the high frequency oscillations in the angular range $105° > \psi > 75°$ belong to another group of charge carriers. (Measurements in much greater magnetic fields are necessary for studying this question.) This is in agreement with the suggestion that there must be at least three charge carrier groups in bismuth.[10]

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*More exact measurements show that the part of this surface, corresponding to the angular range $180° \geqslant \psi \geqslant 105°$ and $75° \geqslant \psi \geqslant 0°$, approximates an ellipsoid.

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POLARIZATION EFFECTS IN THE $\pi^0 \to e^- + e^+ + \gamma$ *DECAY*

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A number of events were recently observed[1,2] which corresponded to a charge exchange scattering of π^- mesons on hydrogen ($\pi^- + p \to \pi^0 + n$) followed by the decay of the π^0 meson into a Dalitz pair and a photon:

$$\pi^0 \to e^- + e^+ + \gamma. \qquad (1)$$

The probability for the decay (1), summed over the polarizations of the particles in the final state, was calculated by Dalitz[3] and Kroll and Wada.[4] In this paper we give the results of a calculation of the probability for the π^0 meson to decay according to the mode (1), taking into account the spin states (longitudinal polarizations) of the electron-positron pair and of the photon.

The direct interaction Hamiltonian for process (1) is given by

$$H_{\text{int}} = eg\,\psi_{\pi^0}\,\{\psi_{e^-}^{+} O_i D^{-1}(\alpha A^{+})\,\psi_{e^+} + (\psi_{e^-}^{+}\alpha A^{+} D^{-1})\,O_i \psi_{e^+}\}. \qquad (2)$$

Here ψ_{π^0}, $\psi_{e^-}^{+}$, ψ_{e^+} and A^+ are the wave functions of the π^0 meson, electron, positron and photon respectively; D is the Dirac operator; $\boldsymbol{\alpha} = \rho_1 \boldsymbol{\sigma}$ is a Dirac matrix; if the π^0 meson is pseudoscalar $O_i = \rho_2$ and if it is scalar $O_i = \rho_3$.

The field amplitude of a circularly polarized photon is given by the formula[5]

$$a_l^{+} = (\beta - il\,[n \times \beta])/\sqrt{2}, \qquad n = \varkappa/\varkappa, \qquad (3)$$

where $\boldsymbol{\kappa}$ is the photon wave vector and $\boldsymbol{\beta} \perp \mathbf{n}$ is an arbitrary unit vector. For a right-circularly polarized photon $l = 1$ (spin parallel to $\boldsymbol{\kappa}$) and for a left-circularly polarized photon $l = -1$ (spin antiparallel to $\boldsymbol{\kappa}$). We made use of formula (21.15) in Sokolov's[5] book to calculate the matrix elements of the decay (1) with longitudinal polarization of the pair taken into account. We obtain the following expression, in the rest system of the π^0 meson (pseudoscalar), for the probability for the decay (1) with prescribed polarizations of the particles:

$$dW(s_-, s_+, l, \theta) = \frac{e^2 g^2}{\hbar^2 c 4 (2\pi)^3}\,\frac{k_+^2\, d\Omega_+\,(dk_-)}{k_{0\pi}k_+ K_-(k_{0\pi} - K_-) + k_{0\pi}K_- k_- K_+ \cos\theta}$$

$$\times \{\Phi_1 + s_- s_+ \Phi_2 + l s_- \Phi_3 + l s_+ \Phi_4\}, \qquad (4)$$

where

$$\Phi_1 = k_{0\pi}^2 q^{-2} k_-^2 k_+^2 \sin^2\theta + 2[K_-(k_{0\pi} - K_-) q^2 - (k_-^2 + \mathbf{k}_-\mathbf{k}_+)(k_-^2 + \mathbf{k}_-\mathbf{k}_+ + (2K_- - k_{0\pi}) q)],$$

$$\Phi_2 = q^{-2} k_- k_+ \sin^2\theta [k_{0\pi}^2 (K_- K_+ - k_0^2) + 2k_0^2 q^2] + (2/k_+)\{q^2 k_- (K_+ (k_{0\pi} - K_-) - k_0^2) - K_+ \times (k_- + k_+ \cos\theta)[K_-(k_-^2 + \mathbf{k}_-\mathbf{k}_+) + (2k_-^2 - k_{0\pi} K_-) q]\},$$

$$\Phi_3 = 2q^{-2} k_- k_+^2 \sin^2\theta k_{0\pi} K_- q + 2k_-(k_{0\pi} - K_-) q^2 - 2(k_- + k_+ \cos\theta)[K_-(k_-^2 + \mathbf{k}_-\mathbf{k}_+ + (2K_- - k_{0\pi}) q) - k_0^2 q],$$

$$\Phi_4 = 2q^{-2} k_-^2 k_+ \sin^2\theta k_{0\pi} K_+ q + 2(K_-/k_+) q^2 [K_+(k_{0\pi} - K_-) - k_0^2] - (2/k_+)(k_-^2 + \mathbf{k}_-\mathbf{k}_+)[K_+(k_-^2 + \mathbf{k}_-\mathbf{k}_+ + (2K_- - k_{0\pi}) q) + k_0^2 q],$$

$$\mathbf{q} \equiv -\varkappa = \mathbf{k}_- + \mathbf{k}_+, \qquad \cos\theta = \mathbf{k}_-\mathbf{k}_+/k_- k_+, \quad k_{0\pi} = m_\pi c/\hbar,$$

$$k_+ = \frac{-2bk_- \cos\theta \pm a\sqrt{b^2 - k_0^2 (a^2 - 4k_-^2 \cos^2\theta)}}{a^2 - 4k_-^2 \cos^2\theta},$$

$$a = 2(k_{0\pi} - K_-), \qquad b = k_{0\pi}(k_{0\pi} - 2K_-) + 2k_0^2,$$

$$K_\pm = \sqrt{k_0^2 + k_\pm^2}. \tag{5}$$

Here $E_\pm = c\hbar K_\pm$, $p_\pm = \hbar \mathbf{k}_\pm$ stand for the total energy and momentum of the electron and positron; $k_0 = m_0 c/\hbar$ is the rest mass; $d\Omega_+$ is the solid angle of positron emission; s_+, $s_- = \pm 1$ are the eigenvalues of the projection operator $\boldsymbol{\sigma} \cdot \mathbf{k}_\pm / k_\pm$. For $s_- = 1$ ($s_+ = 1$) the electron (positron) has right polarization and for $s_- = -1$ ($s_+ = -1$) left polarization. The corresponding expressions for Φ_i ($i = 1, 2, 3, 4$) are also easy to obtain for the case of a scalar π^0 meson. Formula (4) gives the angle and energy dependence of the degree of longitudinal polarization of the created pairs and the correlation between polarizations (the terms proportional to $s_- s_+$, ls_-, ls_+) in the decay (1); this could be of value in the determination of the properties of the π^0 meson. It follows from (4) and (5) that for extremely relativistic electrons and positrons (when k_-, $k_+ \gg k_0$ and $\Phi_1 = \Phi_2$, $\Phi_3 = \Phi_4$) the probability for the decay (1) will differ from zero only if both the electron and positron of a pair are right polarized ($s_- = s_+ = 1$) or left polarized ($s_- = s_+ = -1$). In that case the π^0-decay with the emission of a left polarized electron ($s_- = -1$) and right polarized positron ($s_+ = +1$) or vice versa ($s_- = +1$, $s_+ = -1$) is forbidden since the probability (4) vanishes.

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ATTRACTION OF SMALL PARTICLES SUSPENDED IN A LIQUID AT LARGE DISTANCES

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In the present note we derive formulas for the interaction energy connected with the Van-der-Waals forces of interaction between uncharged particles suspended in a liquid. The distance between the particles will be assumed large compared with their dimensions.

In principle this problem can be solved on the basis of the general theory of Van-der-Waals forces in dielectrics.[1] However, as shown earlier,[2] the expression for the interaction forces of arbitrary bodies in a medium can be derived by simple transformation from the corresponding expression for the interaction forces in vacuum. Indeed, the expression for the additional pressure in a medium of dielectric constant ϵ can be obtained from the expression for the pressure in vacuum by multiplying the integrand in the integral with respect to frequency* (this integral determines the pressure) by $\epsilon^{3/2}$, by replacing the dielectric constant of the interacting bodies ϵ_1 by ϵ_1/ϵ, and by increasing all the linear dimensions by a factor of $\sqrt{\epsilon}$. In accordance with this, the energy U of the interaction of the particles in the medium can be obtained from the energy of interaction in vacuum U_0 by replacing the dielectric constant of the particles ϵ_1 by ϵ_1/ϵ, their volume V by