THE ANISOTROPY OF MAGNETORESISTANCE AND THE TOPOLOGY OF FERMI SURFACES OF METALS

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The anisotropy of magnetoresistance has been studied for single crystals of Ag, Au, Cu, Sn, Pb, Tl, and Ga. Stereographic projections of the singular field directions have been constructed, and some considerations about the topology of the Fermi surface of metals are discussed on the basis of these.

In this paper we describe some* results of a further study of the anisotropy of magnetoresistance in single crystals of Au, Cu, Sn, Pb, Tl and Ga, which we examined earlier,[1] and also of Ag single crystals investigated for the first time.

For most of the metals the measurements were made on 10 — 15 specimens with different orientations of the crystallographic axes relative to the specimen axis. The orientation was determined by back-scattering Laue photography or by an optical method.†

The purity of the specimens used can be expressed as the ratio $\rho(300°)/\rho(4.2°)$ (the ratio of the resistance at room temperature to that at 4.2°K), which was about 10,000 for Sn, Pb and Ga, 3000 for Tl, and 1000 for Au, Cu and Ag. The specimen diameters were large enough for size effects not to affect the results.

All measurements were made at T = 4.2°K, since lowering the temperature further would not have produced any significant reduction in conductivity. The residual resistances in zero field, ρ_0, were very low and necessitated the determinations of emf's of the order of 10^{-8} v. For this a potentiometric system was used with a photo-multiplier connected to the output with negative feedback. The sensitivity of this arrangement was a few times 10^{-10} v.

The rotation diagrams for Cu and Ag are shown in Figs. 1 and 2; these give the dependence of relative change of resistance, $\Delta\rho_H/\rho_0 = \rho_H/\rho_0 - 1$, for constant field, H = 23,500 oe, on the angle ϑ through which H is turned in the plane perpendicular to the specimen axis. The angular dependence of the resistance of Pb and Ga is shown in polar coordinates in Figs. 3 and 4.

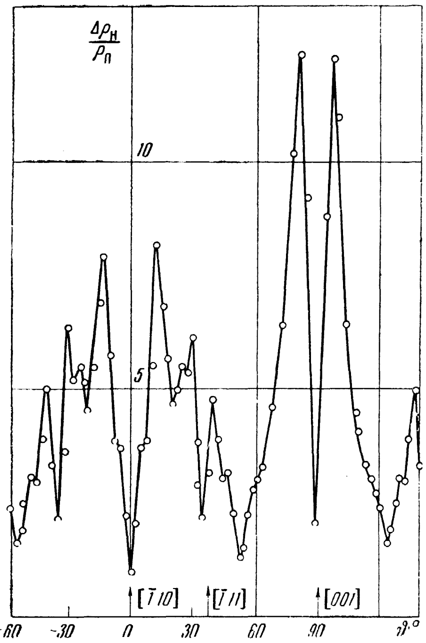

FIG. 1. Copper single crystal. [110] axis along the specimen axis. T = 4.2°K, $\rho(300°)/\rho(4.2°) = 728$; H = 23,500 oe.

From a study of the anisotropy in the resistance of silver it was found that as for other metals,[1] the resistance reaches complete saturation for directions of the minima in the resistance rotation diagram, and in the directions of the maxima increases indefinitely almost as the square of the field (Fig. 5). From this observation we may conclude[2] that silver has an open Fermi surface.

As for gold, the resistance of some specimens of Cu and Ag were averaged over ϑ (from 0 to 180°) at several values of the field (from 8000 to 24,000 oe). These average values are plotted in Fig. 6 and $\Delta\rho_H/\rho_0$ is seen to vary linearly with field. We may then assume that Kapitza's law (linear increase of resistance with field[3]) for

*The authors hope soon to describe in full all the experiments and the results.

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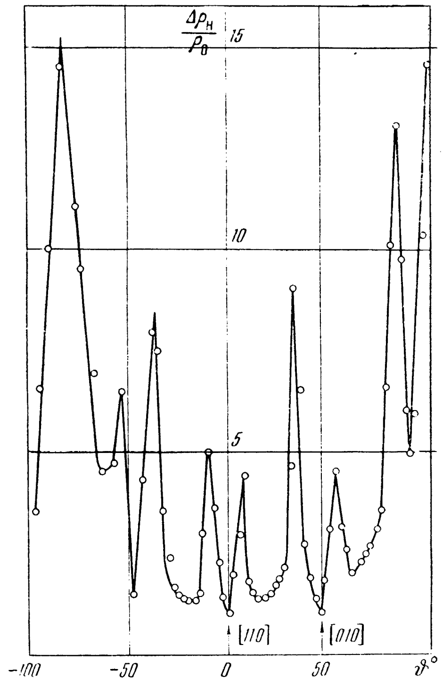

FIG. 2. Silver single crystal. [001] axis approximately (± 5°) along the specimen axis. T = 4.2°K, $\rho(300°)/\rho(4.2°) = 741$; H = 23,500 oe.

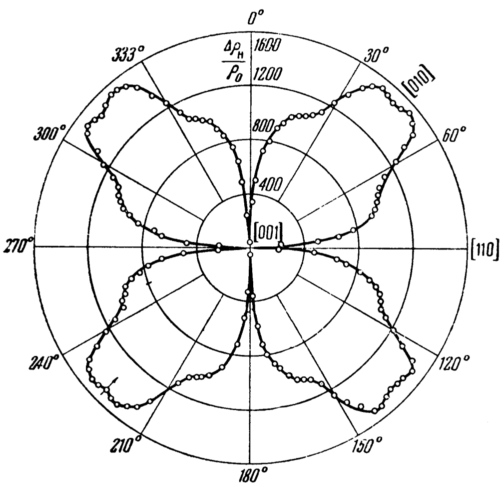

FIG. 3. Lead single crystal. [001] axis coincident with specimen axis. T = 4.2°K, $\rho(300°)/\rho(4.2°) = 8{,}750$; H = 23,500 oe.

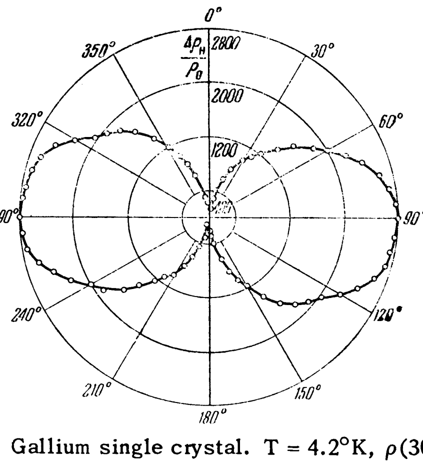

FIG. 4. Gallium single crystal. T = 4.2°K, $\rho(300°)/\rho(4.2°) = 28{,}000$; H = 23,500 oe.

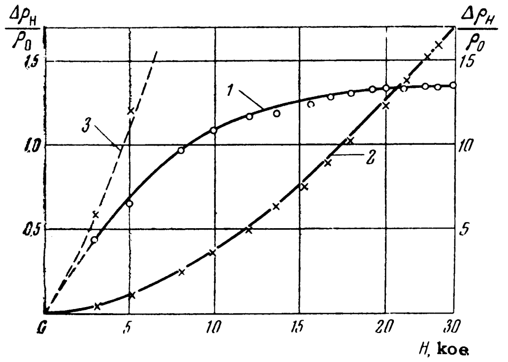

FIG. 5. Silver single crystal (see Fig. 2). Magnetoresistance for fixed angle ϑ: curve 1 for $\vartheta = 0$ (min), curves 2 and 3 for $\vartheta = 80°$(max); T = 4.2°K. The left hand ordinate scale refers to curves 1 and 3, the right hand to curve 2.

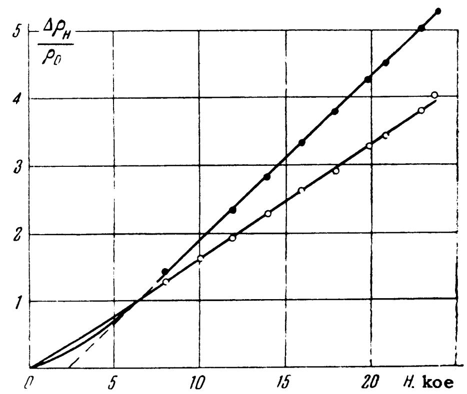

FIG. 6. The resistance averaged over ϑ for several values of magnetic field. ● – Cu [110] single crystal (see Fig. 1), o – Ag [001] single crystal (see Fig. 2).

polycrystalline Cu and Ag is a consequence of the averaging of various laws of resistance increase, as for Au.[1]

The variation of half-widths of the narrow maxima and minima in the rotation diagrams of Au, Cu, Pb and Sn were investigated. It was found that the half widths of the maxima decrease with increasing magnetic field (as 1/H), while those of the minima stay the same (Fig. 7). The results are in good agreement with the theory of Lifshitz and Peschanskiĭ.[4]

An interesting transition of a minimum into a maximum can be seen in the rotation diagram for lead and tin: for one and the same direction (for example, the [010] direction in Sn, shown in Fig. 8) the depth of the minimum progressively decreases as the current direction relative to the crystallographic axes varies. This change in the minimum can be regarded as confirming the relation

$$\rho = B\,(H/H_0)^2 \cos^2\alpha + A, \qquad (1)$$

where α is the angle between the direction of open trajectories and the current direction.[4]

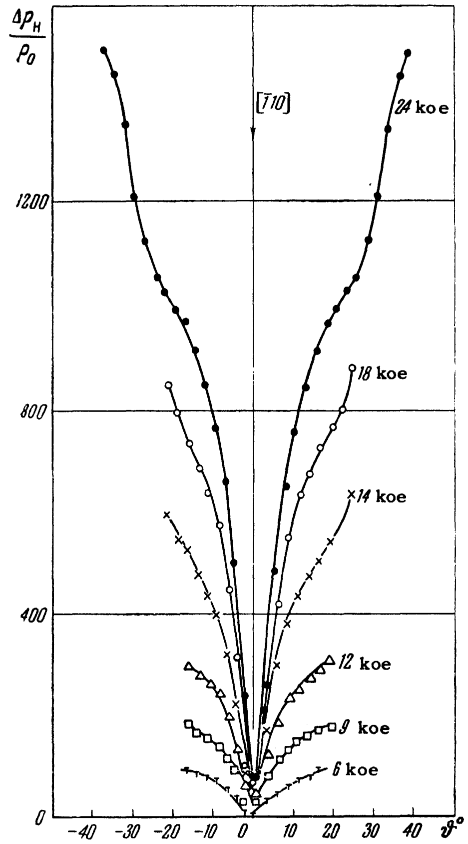

FIG. 7. The change in shape of the narrow minima in the rotation diagram for lead (Pb [001], see Fig. 3); T = 4.2°K.

The appearance of the rotation diagrams of Tl and Ga depends little on the specimen orientation and the dependence of $\Delta\rho/\rho_0$ on ϑ agrees well with (1). We may deduce from this that Tl and Ga have Fermi surfaces in the form of corrugated planes.[5]

From the data for Au, Cu and Ag specimens of various orientations, stereographic projections can be constructed of the singular magnetic field directions (directions of the maxima and of narrow minima etc.). Such a projection (for Ag) is shown in Fig. 9. The shaded regions and the thick lines joining them correspond to the field directions for which the resistance increases quadratically with field. The stereographic projections for Au and Cu are sufficiently close to that for Ag.

By comparison with the theory of Lifshitz and Peschanskiĭ,[4,5] an analysis of the stereographic projections points to the Fermi surfaces of these metals — "the space net" — being formed from "corrugated cylinders" with axes along the body and face diagonals of the reciprocal lattice. The data available[4,6] indicate that the Fermi surface of copper also has open directions along the edges of the reciprocal lattice.

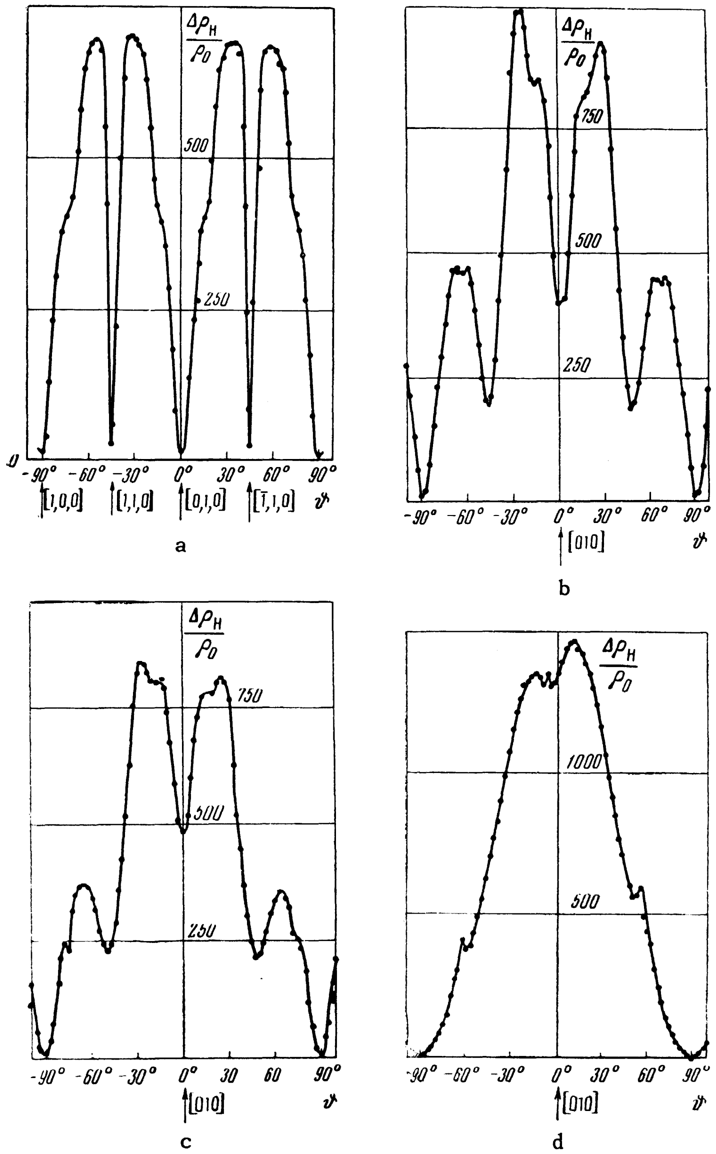

FIG. 8. Tin single crystals. The change in the depth of the minimum for fixed direction of the field H = 23,500 oe and varying current direction (specimen orientation); a – Sn I, $\varphi = 0°$; b – Sn II, $\varphi = 26°$; c – Sn III, $\varphi = 36°$; d – Sn IV, $\varphi = 51°$. (φ is the angle between the specimen axis, in the (100) plane, and the [001] axis.)

The stereographic projections of lead and tin cannot be interpreted so unambiguously. Nevertheless, we can say that a Fermi surface of lead formed by broad cylinders in the [111] direction, and in tin by broad cylinders in the [010] and [110] directions of the reciprocal lattice do not contradict the experimental data.* It is possible that these surfaces would appear more complicated if studied in more detail. It would seem likely that metals with the same crystal lattice will have nearly the

*In our previous paper[1] the direction [001] for Sn should read [100].

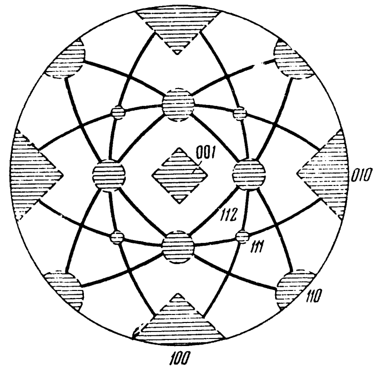

FIG. 9. Stereographic projection of the singular field directions for silver.

same form of Fermi surface. This is apparently true for Au, Cu, and Ag, which have body centered cubic lattices. However, if we compare the galvanomagnetic characteristics of these metals with the results for lead, which is also body centered cubic, it is easily seen that there is a considerable difference (Figs. 1, 2, and 3). While the resistance of Au, Cu, and Ag only varies quadratically over a narrow range of angles, the quadratic region in Pb occupies almost the whole rotation diagram, and saturation is found only in a narrow interval. An analogy can be found between the rotation diagrams of Pb and Sn, which have different lattices and belong to the same column of the periodic table. The diagrams for Tl and Ga are also similar and they are in the same column of the table and have different lattices.

These facts suggest a possible connection between the form of the angular dependence of resistance (for metals with open Fermi surfaces) and the position in the periodic table. It is probable that further study will clarify the reasons for metals having similar galvanomagnetic properties. All the available data show that, contrary to previous ideas, a large number of metals have open Fermi surfaces.

The results presented here, together with data from earlier work,[7,8] indicate that Au, Cu, Ag, Pb, Sn, Tl, Ga, Zn, and Cd have open Fermi surfaces. Bi and In[9,10] probably have closed surfaces, as do possibly Al, Be and Na,[8,11] for which a small anisotropy in resistance is characteristic. It may be that Al has a Fermi surface with very narrow regions of open directions.[1,12] The old division into metals with $n_1 = n_2$ and metals with $n_1 \neq n_2$ (n_1 and n_2 are the numbers of electrons and holes) still holds for metals with closed surfaces. There is no strong resistance anisotropy, dependent on magnetic field, for these metals; for the first case the resistance increases quadratically and in the second the resistance is independent of field for large fields. This classification is meaningless for metals with open Fermi surfaces.

It is a pleasure to express our thanks to Academician P. L. Kapitza for his constant interest in the work. We also thank Professor I. M. Lifshitz and V. G. Peschanskiĭ for discussion of the results.

[1] N. E. Alekseevskiĭ and Yu. P. Gaĭdukov, JETP **35**, 554 (1958), Soviet Phys. JETP **8**, 383 (1959); JETP **36**, 447 (1959), Soviet Phys. JETP **9**, 311 (1959).

[2] Lifshitz, Azbel', and Kaganov, JETP **31**, 63 (1956), Soviet Phys. JETP **4**, 41 (1957).

[3] P. L. Kapitza, Proc. Roy. Soc. **A123**, 292 (1929).

[4] I. M. Lifshitz and V. G. Peschanskiĭ, JETP **35**, 1251 (1958), Soviet Phys. JETP **8**, 875 (1959).

[5] V. G. Peschanskiĭ, Thesis, Khar'kov State University (1959).

[6] A. B. Pippard, Phil. Trans. Roy. Soc. **A250**, 325 (1957).

[7] Lazarev, Nakhimovich, and Parfenova, JETP **9**, 1169 (1939).

[8] E. S. Borovik, Thesis, Physico-technical Institute, Academy of Sciences, Ukrainian S.S.R., Khar'kov (1954).

[9] L. Schubnikow and W. J. de Haas, Leiden Comm. 210a (1930).

[10] E. S. Borovik, Dokl. Akad. Nauk SSSR **69**, 767 (1949).

[11] E. Justi and H. Scheffers, Z. Physik **39**, 105 (1938).

[12] B. Lüthi and J. L. Olsen, Nuovo cimento **3**, 840 (1956).

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