ON THE THEORY OF MAGNETIC-MOMENT RELAXATION IN FERRODIELECTRIC SUBSTANCES

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The relaxation of magnetic moment and the equalization of temperatures between spin waves and lattice in a ferrodielectric substance are investigated at low temperatures $T \ll \Theta_c$. For $T \gg \Theta_1$, $\Theta_1 = \Theta_c(\mu M_0/\Theta_c)^{4/7}$ a Bose distribution corresponding to the nonequilibrium magnetic moment is first set up, then the equilibrium value of the magnitude of the magnetic moment is established and, finally, a rotation of the magnetic moment to its equilibrium direction occurs. For $T \ll \Theta_1$ the magnetic moment assumes its equilibrium value simultaneously with the establishment of the Bose distribution of the spin waves, and then the magnetic moment rotates to its equilibrium direction.

Simple formulas are obtained which describe the temperature equalization of the spin waves and the lattice, and the relaxation of the magnetic moment.

1. In the paper by Akhiezer, Peletminskiĭ and the present author[1] the relaxation of magnetic moment in a ferrodielectric substance was investigated in the temperature range $\Theta_c \gg T \gg \Theta_c(\mu M_0/\Theta_c)^{4/7}$ (μ is the Bohr magneton, M_0 is the equilibrium magnetic moment per unit volume, Θ_c is the Curie temperature) and the relaxation constants were calculated. In doing this it was established that the slowest process is the rotation of the magnetic moment towards the axis of easiest magnetization.

In this paper we investigate in greater detail the establishing of the magnitude of the magnetic moment and the equalization of the lattice and the spin-wave temperature in the case when there is no deviation of the magnetic moment from the equilibrium direction. We also consider the relaxation of the magnetic moment and the leveling out of temperatures in the ferrodielectric at temperatures $T \ll \Theta_1$ [for the sake of brevity we shall use the notation $\Theta_1 = \Theta_c(\mu M_0/\Theta_c)^{4/7}$].

We start the investigation of relaxation processes in a ferrodielectric with the kinetic equation for the distribution function of the spin waves

$$\dot{n}_k = \dot{n}_k^{st} \equiv L_k\{n, N\},$$

$$L_k\{n, N\} = L_k^e\{n\} + L_k^w\{n\} + L_k^p\{n, N\} + L_k^a\{n\} \quad (1)$$

(the expressions for L_k^e, L_k^w, L_k^p, and L_k^a are given in reference 1).

In the temperature range $\Theta_c \gg T \gg \Theta_1$ the principal role in the setting up of the Bose distribution in the spin wave system is played by the exchange interaction ($L_k^e \gg L_k^w$, L_k^p, L_k^a) as a result of which during a time $\sim (\hbar/\Theta_c)(\Theta_c/T)^4$ the following distribution is established

$$n_k = \left[\exp\left(\frac{\varepsilon_k - \gamma}{T_s}\right) - 1\right]^{-1}. \quad (2)$$

This distribution is a solution of the equation $L_k^e\{n\} = 0$.

The chemical potential γ and the temperature of the spin waves T_s are determined in such a case by the absolute value of the magnetic moment of the sample

$$\mathfrak{M}^2 = \left(\int M dv\right)^2 = (M_0 V)^2 - 4\mu M_0 V \sum_{k\neq 0} n_k \quad (3)$$

and by the energy of the spin system.

The presence of the weak interactions taken into account by means of the operators L_k^w, L_k^a, L_k^p brings about the situation that γ and T_s vary slowly with time [compared to the time required for the establishment of the distribution (2)]. Variation with time of γ and T_s leads to the variation with time of the phonon temperature T_p.

On substituting into the collision integral $L_k\{n, N\}$ the distribution function for the spin waves (2) and the phonon distribution function

$$N_{fs} = [\exp(\hbar\omega_{fs}/T_p) - 1]^{-1}$$

and on expanding in terms of the quantities γ and $\Delta T = \Delta T_s - \Delta T_p$, which characterize small deviations of the system from complete thermodynamic equilibrium, we obtain

$$L_{\mathbf{k}}\{n,\ N\} = \gamma\,(\partial L_{\mathbf{k}}/\partial\gamma)_0 + \Delta T\,(\partial L_{\mathbf{k}}/\partial\Delta T)_0, \qquad (4)$$

where the subscript 0 means that the values of the derivatives of the collision integral are taken at $\gamma = 0$, $\Delta T = 0$.

The time variation of the quantities γ, $\Delta T_s = T_s - T$, $\Delta T_p = T_p - T$ (T is the equilibrium temperature) can be determined from the following equations

$$\sum_{fs}\hbar\omega_{fs}\dot{N}_{fs} = -\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}L_{\mathbf{k}}\{n,\ N\},$$

$$\sum_{\mathbf{k}}\dot{n}_{\mathbf{k}} = \sum_{\mathbf{k}}L_{\mathbf{k}}\{n,\ N\}, \qquad \sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}\dot{n}_{\mathbf{k}} + \sum_{fs}\hbar\omega_{fs}\dot{N}_{fs} = 0. \qquad (5)$$

The first equation determines the amount of heat transmitted from the lattice to the spin wave, the second equation determines the change in the total number of spin waves which determines γ, while the third relation between ΔT_s, ΔT_p, and γ represents the law of conservation of energy. By using (4) and (5) these equations can easily be put into the following form

$$c_s\Delta\dot{T}_s + c_p\Delta\dot{T}_p - \dot{\gamma}\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}\,\partial n_{\mathbf{k}}/\partial\varepsilon_{\mathbf{k}} = 0,$$

$$c_p\Delta\dot{T}_p = A_{T\gamma}\gamma + A_{TT}\,\Delta T,$$

$$-\dot{\gamma}\sum_{\mathbf{k}}\partial n_{\mathbf{k}}/\partial\varepsilon_{\mathbf{k}} + \Delta\dot{T}_s\sum_{\mathbf{k}}\partial n_{\mathbf{k}}/\partial T = A_{\gamma\gamma}\gamma + A_{\gamma T}\,\Delta T, \qquad (6)$$

where

$$A_{T\gamma} = -\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}\,(\partial L_{\mathbf{k}}/\partial\gamma)_0, \qquad A_{\gamma\gamma} = \sum_{\mathbf{k}}(\partial L_{\mathbf{k}}/\partial\gamma)_0,$$

$$A_{TT} = -\sum_{\mathbf{k}}\varepsilon_{\mathbf{k}}\,(\partial L_{\mathbf{k}}/\partial\Delta T)_0, \qquad A_{\gamma T} = \sum_{\mathbf{k}}(\partial L_{\mathbf{k}}/\partial\Delta T)_0,$$

$$c_s = \frac{15\,\zeta(^5/_2)}{32\,\pi^{^3/_2}}\frac{V}{a^3}\left(\frac{T}{\Theta_c}\right)^{^3/_2}, \qquad c_p = \frac{2\pi^2}{5}\frac{V}{a^3}\left(\frac{T}{\Theta_D}\right)^3,$$

c_s and c_p are the spin and the phonon heat capacities, a is the lattice constant, Θ_D is the Debye temperature, V is the volume of the sample. It may be shown that the quantities $A_{T\gamma}$ and $A_{\gamma T}$ are related by the equation $A_{T\gamma} = -TA_{\gamma T}$.

The values of the quantities A as functions of the temperature may be obtained by considering the specific form of the collision integrals. We shall give here only the final formulas obtained for a uniaxial ferrodielectric. If $T \gg \Theta_D^2/\Theta_c$, Θ_1, then

$$A_{\gamma\gamma} = -\frac{1}{5\pi}\frac{V}{a^3}\frac{\mu M_0}{\hbar}\frac{\mu M_0}{\varepsilon_0\Theta_c}\left(\frac{T}{\Theta_c}\right)^2,$$

$$A_{T\gamma} = \frac{1}{6\pi^2}\frac{V}{a^3}\frac{\hbar}{\rho a^5}\delta^2\left(\frac{\mu M_0}{\Theta_c}\right)^2\left(\frac{T}{\Theta_c}\right)^2,$$

$$A_{TT} = \frac{4\pi}{15}\frac{\hbar}{\rho a^5}\,[2\beta_1^2 + (2\beta_1 + \beta_2)^2]\left(\frac{T}{\Theta_c}\right)^4 \equiv \frac{4\pi}{15}\frac{\hbar}{\rho a^5}\beta_e^2\left(\frac{T}{\Theta_c}\right)^4, \qquad (7)$$

where ρ is the density of the substance, $\epsilon_0 = \mu H_0 + \mu\beta M_0$ is the energy of the spin wave with $\mathbf{k} = 0$, β is the anisotropy constant, δ is the magnetostriction constant, and β_1 and β_2 are numerical constants of the order of magnitude of unity, which appear as coefficients in the Hamiltonian of the exchange interaction between the spin waves and the phonons [the Hamiltonians of the interactions of the spin waves with each other and of the spin waves with the phonons are given in reference 1, formulas (2) – (5)].

By assuming that γ, ΔT_s, and ΔT_p vary with time as $e^{-\lambda t}$, we obtain the following values for the relaxation constants when $T \gg \Theta_D^2/\Theta_c$, Θ_1:

$$\lambda_1 \approx \frac{\mu M_0}{\hbar}\frac{\mu M_0}{\sqrt{\varepsilon_0\Theta_c}}\frac{T}{\Theta_c}, \qquad \lambda_2 \approx \frac{\hbar}{\rho a^5}\beta_e^2\left(\frac{T}{\Theta_c}\right)^{^5/_2}. \qquad (8)$$

The relaxation constant λ_1 is determined primarily by the relativistic magnetic interaction which describes the fusion of two spin waves into one and the splitting of one spin wave into two. The relaxation constant λ_2 is associated with the exchange interaction between the spin waves and the phonons describing creation and absorption of a phonon by a spin wave. Thus, if $M_0 \sim 10^3$ G, $\epsilon_0 = \mu(H + \beta M_0) \sim 10^{-16}$ erg, $\Theta_c \sim 10^3$ °K, $T \sim 10^2$ °K, then $\lambda_1 \sim 3\times 10^6\ \text{sec}^{-1}$, $\lambda_2 \sim 10^9\ \text{sec}^{-1}$. It may be seen from (8) that the inequality $\lambda_2 \gg \lambda_1$ holds in the temperature range $T \gg \Theta_c\mu M_0/\beta_e^2\sqrt{\epsilon_0\Theta_c} \sim 10$°K.

The time variation of the difference ΔT between the temperatures of the spin waves and of the phonons, and of the difference in the magnitudes of the magnetic moment $\mathfrak{M}$ is determined, according to (3), (2), and (6), by the following formulas

$$\frac{\mathfrak{M} - \overline{\mathfrak{M}}}{M_0 V} = \frac{\mathfrak{M}_0 - \overline{\mathfrak{M}}}{M_0 V}\,\frac{e^{-\lambda_1 t} + \alpha e^{-\lambda_2 t}}{1 + \alpha} + \frac{\varepsilon_0}{\Theta_c}\left(\frac{\lambda_1}{\lambda_2}\right)^2\left(\frac{T}{\Theta_c}\right)^{^1/_2}\frac{\Delta T_0}{T}\,(e^{-\lambda_2 t} - e^{-\lambda_1 t}),$$

$$\frac{\Delta T}{T} = \frac{\Delta T_0}{T}\,\frac{e^{-\lambda_2 t} + \alpha e^{-\lambda_1 t}}{1 + \alpha} + 9\left(\frac{\varepsilon_0\Theta_c}{T^2}\right)^{^1/_2}\frac{\Theta_c}{T}\,\frac{\mathfrak{M}_0 - \overline{\mathfrak{M}}}{M_0 V}\,(e^{-\lambda_1 t} - e^{-\lambda_2 t}), \qquad (9)$$

where $\alpha \approx 2(\lambda_1/\lambda_2)^2(\epsilon_0/T)^{1/2} \ll 1$, $\overline{\mathfrak{M}}$ is the equilibrium value of the magnetic moment at a given temperature, $\mathfrak{M}_0$ is the initial value of the magnetic moment, ΔT_0 is the initial temperature difference.

If these initial data are such that $\Delta T_0 = 0$, then

$$\mathfrak{M} - \overline{\mathfrak{M}} \approx (\mathfrak{M}_0 - \overline{\mathfrak{M}})\,(e^{-\lambda_1 t} + \alpha e^{-\lambda_2 t})/(1 + \alpha).$$

Since $\alpha \ll 1$ and $\lambda_2 \gg \lambda_1$, then

$$\mathfrak{M} - \overline{\mathfrak{M}} \approx (\mathfrak{M}_0 - \mathfrak{M})\,e^{-\lambda_1 t}. \qquad (10)$$

From this it can be seen that the magnetic moment relaxes during a time of the order of $\tau_{\mathfrak{M}} = 1/\lambda_1$.

If at zero time $\mathfrak{M}_0 = \overline{\mathfrak{M}}$, then the leveling out of the temperatures is described by the formula

$$\Delta T = \Delta T_0 (e^{-\lambda_2 t} + \alpha e^{-\lambda_1 t}) / (1 + \alpha). \quad (11)$$

We note that the leveling out of the temperatures proceeds at different rates during the initial and final stages of the process. Since the quantity λ_2 is sufficiently great ($\sim 10^9\ \text{sec}^{-1}$ at $T \sim 100°K$) then, apparently, experimentally it is very difficult to observe the initial stage of the leveling out of the temperatures between the spin waves and the lattice. The complex nature of the relaxation, as has been shown by Kaganov and Tsukernik,[2] leads to a complicated frequency dependence of the magnetic permeability of the ferrodielectric, by studying which one can make conclusions with respect to the interaction processes between spin waves and phonons in the ferrodielectric.

2. Let us now consider the question of the relaxation of the magnetic moment in the temperature region $\epsilon_0 < T \ll \Theta_1$. At such temperatures the average probability of the splitting of a spin wave into two and of the fusion of two spin waves into one which is given[3] by:

$$W_w = \frac{\mu M_0}{\hbar} \frac{\mu M_0}{\Theta_c} \left(\frac{T}{\Theta_c}\right)^{1/2} \ln^2 \frac{T}{\mu M_0}, \quad (12)$$

will be considerably larger than the average probability of the scattering of a spin wave by a spin wave due to the exchange interaction[1,4]

$$W_e \approx (\Theta_c / \hbar)(T / \Theta_c)^4. \quad (13)$$

This means that in the temperature region under consideration the principal role in the kinetic equation for the distribution function of spin waves (1) is played by the operator $L_{\mathbf{k}}^{W}$, while the other collision integrals $L_{\mathbf{k}}^{e}$, $L_{\mathbf{k}}^{a}$, $L_{\mathbf{k}}^{p}$ may be regarded as small perturbations.

It may be easily seen that the solution of the equation $L_{\mathbf{k}}^{W}\{n\} = 0$ is of the form

$$n_{\mathbf{k}} = \begin{cases} n_0, & \mathbf{k} = 0, \\ [\exp(\varepsilon_{\mathbf{k}} / T_s) - 1]^{-1}, & \mathbf{k} \neq 0. \end{cases} \quad (14)$$

In order of magnitude the time for setting up this distribution is equal to $\tau_W = 1/W_W$. We note that the distribution (14) also makes the collision integral $L_{\mathbf{k}}^{e}$ equal to zero.

The parameters of the distribution T_s and n_0 are determined by the energy of the spin system and by the component of the magnetic moment of the sample perpendicular to the axis of easiest magnetization

$$\mathfrak{M}_\perp^2 = 4\mu M_0 V n_0 \quad (15)$$

(in the state of complete thermodynamic equilibrium $\mathfrak{M}_\perp = 0$). The absolute value of the magnetic moment of the sample attains its equilibrium value at a given temperature simultaneously with the establishment of the distribution (14).

The establishment in a ferrodielectric of complete thermodynamic equilibrium is due to weak interactions described in the kinetic equation by the collision integrals $L_{\mathbf{k}}^{a}$ and $L_{\mathbf{k}}^{p}$. Due to these interactions the temperature of the spin waves and the lattice temperature approach their equilibrium values slowly (in comparison with τ_W), while the magnetic moment slowly rotates towards its equilibrium direction.

By utilizing the expressions for the distribution functions of the spin waves (14) and of the phonons, and also the kinetic equation (1), we can obtain the system of equations for determining the variation with time of n_0 and $\Delta T = T_s - T_p$:

$$\Delta\dot{T} + \frac{\varepsilon_0}{c_s} \dot{n}_0 = B_{T0} n_0 + B_{TT} \Delta T, \qquad \dot{n}_0 = B_{00} n_0, \quad (16)$$

where

$$B_{TT} = (1/c_s + 1/c_p) \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} (\partial L_{\mathbf{k}} / \partial \Delta T)_0, \quad B_{00} = (\partial L_0 / \partial n_0)_0,$$

$$B_{T0} = (1/c_s + 1/c_p) \sum_{\mathbf{k}} \varepsilon_{\mathbf{k}} (\partial L_{\mathbf{k}} / \partial n_0)_0 \equiv a^3 B'_{T0} / V.$$

In the case that $\epsilon_0 < T \ll \Theta_1$, Θ_D^2/Θ_c we have

$$B_{TT} \approx -\frac{\beta_1^2 \hbar}{\rho a^5} \left(\frac{T}{\Theta_c}\right)^2 \exp\left(-\frac{\Theta_t^2}{4\Theta_c T}\right), \qquad B_{00} \approx -\frac{\mu M_0}{\hbar} \frac{\mu M_0}{\Theta_c} \left(\frac{T}{\Theta_c}\right)^2,$$

$$B'_{T0} \approx -\frac{\hbar \varepsilon_0}{\pi^3 \rho a^5} \left(\frac{\gamma \mu M_0}{\Theta_c}\right)^2 \left(\frac{\varepsilon_0}{T}\right)^2 \left(\frac{\Theta_D}{T}\right)^2 \exp\left(-4\frac{\varepsilon_0 \Theta_c}{\Theta_t^2} \frac{\varepsilon_0}{T}\right),$$

where $\Theta_t = \hbar s_t / a$, and s_t is the speed of transverse sound.

By assuming that the time variation of the quantities n_0 and ΔT is given by $e^{-\lambda t}$ we shall obtain the following values for the relaxation constants:

$$\lambda' = -B_{00}, \quad \lambda'' = -B_{TT}. \quad (17)$$

In the case that $\epsilon_0 < T \ll \Theta_D^2/\Theta_c$, Θ_1 we have

$$\lambda' \approx \frac{\mu M_0}{\hbar} \frac{\mu M_0}{\Theta_c} \left(\frac{T}{\Theta_c}\right)^2, \qquad \lambda'' \approx \frac{\beta_1^2 \hbar}{\rho a^5} \left(\frac{T}{\Theta_c}\right)^2 \exp\left(-\frac{\Theta_t^2}{4\Theta_c T}\right). \quad (18)$$

From (15) and (16) we obtain

$$\Delta T = \Delta T_0 e^{-\lambda'' t} + (B'_{T0} / \lambda'') (\mathfrak{M}_{\perp 0} / 2M_0 V)^2 (e^{-\lambda' t} - e^{-\lambda'' t}),$$

$$\mathfrak{M}_\perp^2 = \mathfrak{M}_{\perp 0}^2 e^{-\lambda' t}, \quad (19)$$

where ΔT_0 and $\mathfrak{M}_{\perp 0}$ are the initial temperature difference and the initial value of the transverse component of the magnetic moment of the ferrodielectric.

These formulas show that λ' has a simple physical meaning: $2/\lambda'$ is the relaxation time for the perpendicular component of the magnetic moment.

If the deviation from the equilibrium state of the ferrodielectric is associated only with a difference in the temperatures of the spin waves and of the lattice, then the time for the leveling out of the temperatures is determined by the quantity $\tau = 1/\lambda''$. If $\Delta T_0 = 0$, then in the course of relaxation of the magnetic moment the temperatures of the spin waves and of the lattice will increase at the expense of conversion into heat of the energy associated with the deviation of the magnetic moment of the sample from its equilibrium direction. The time for the establishment of a common temperature under these conditions is equal to half the relaxation time of the transverse component of the magnetic moment.

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