DISPERSION RELATIONS FOR THE VIRTUAL COMPTON EFFECT

I. S. ZLATEV and P. S. ISAEV

Joint Institute for Nuclear Research

Submitted to JETP editor March 23, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 728-734 (September, 1959)

Dispersion relations for physical amplitudes have been derived by the Bogolyubov method in the center-of-mass system for electron bremsstrahlung and for pair production by a photon in the field of a nucleon, accurate to lowest order in e.

1. INTRODUCTION

At the present time dispersion relations (d. r.) provide one of the most effective methods for taking into account effects of strong interactions. In application to electromagnetic processes such as the nucleon Compton effect, bremsstrahlung, or pair production by photons in the field of a nucleon, etc., this method, under well defined assumptions, allows information to be obtained about the nucleon structure. It should be noted that nucleon structure has been the subject of considerable attention in recent years* since it has not only intrinsic interest but is also closely related to the study of limits of applicability of quantum electrodynamics at small distances.

A theoretically rigorous study of the influence of nucleon structure on the processes of bremsstrahlung and pair production is made possible by applying d.r. to the indicated processes. From this point of view a study of d.r. for the virtual Compton effect, which describes both of the above mentioned processes, is of certain interest.

In this work d.r. are obtained for the virtual Compton effect accurate to lowest order in e by the Bogolyubov[1] method.

The proof of d.r. for the bremsstrahlung and pair production processes was given by Vladimirov and Logunov;[3] in this paper attention is concentrated on obtaining d.r. in a form useful for practical applications.

It is shown that the cross sections agree, in the one-nucleon approximation, with those calculated by lowest order perturbation theory; however, the d.r. method allows one to introduce rigorously form factors (of the type considered by Hofstadter) into those nucleon vertices in the Feynmann graphs which contain a virtual photon line. This constitutes one of the serious advantages of the d.r. method over perturbation theory.

The resultant d.r. may be further utilized at least to estimate the contribution of the one-pion state to the processes under consideration in the same way as was done for the pion photoproduction process.[4,5]

The bremsstrahlung and pair production processes have been previously calculated by the authors[6] in lowest-order perturbation theory, taking into account only Bethe-Heitler type diagrams into which Hofstadter form factors were introduced. It is clear that for incident particle energies up to ~ 150 Mev, when the contribution from the meson cloud of the nucleon and higher order electromagnetic corrections are unimportant, these results could be used to check the validity of quantum electrodynamics at small distances, i.e., to verify the local nature of the interaction between the electromagnetic field and the current of a charged particle. Analogous results were also obtained by Bjorken et al.[7] However as the energy increases to 500 or 600 Mev the contribution from the meson cloud of the nucleon should become more and more noticeable and the one-pion approximation may introduce significant changes in the cross sections for the processes under discussion. Therefore taking into account the one-pion state will noticeably move the energy limit for the check of validity of quantum electrodynamics at small distances, namely up to an electron and photon energy of the order of 500 — 600 Mev.

In this way it becomes possible, for example, to verify quantum electrodynamics down to distances $\geq 3 \times 10^{-14}$ cm in the bremsstrahlung process where the incident electron has an energy ~ 550 Mev and the emitted photon has an energy ~ 260 Mev.

*A detailed bibliography on the nucleon structure problem may be found in the review article by Blokhintsev, Barashenkov, and Barbashov.[2]

2. DISPERSION RELATIONS FOR THE AMPLITUDE OF THE VIRTUAL COMPTON EFFECT

The S-matrix element describing the bremsstrahlung of an electron in the field of a nucleon has the form

$$\langle f\,|\,S\,|\,i\rangle = \langle p, s;\ q, \sigma;\ k, \nu\,|\,S\,|\,q_0, \sigma_0;\ p_0, s_0\rangle =$$
$$= (2\pi)^{3/2}\langle p, s\,|\,b^-(\mathbf{q}, \sigma)\, a^-(\mathbf{k}, \nu)\, Sb^{+*}(\mathbf{q}_0, \sigma_0)\,|\,p_0, s_0\rangle, \quad (2.1)$$

where p, s (p_0, s_0) are the four-momentum and spin of the final (initial) nucleon, q, σ (q_0, σ_0) are the four-momentum and spin of the final (initial) electron, k, ν are the four-momentum and polarization of the photon, $b^-(\mathbf{q}, \sigma)$ and $b^{+*}(\mathbf{q}_0, \sigma_0)$ are creation and annihilation operators for an electron in the states $(\mathbf{q}, \sigma)$ and $(\mathbf{q}_0, \sigma_0)$, and $a^-(\mathbf{k}, \nu)$ is the creation operator for a photon of momentum $\mathbf{k}$ and polarization ν; $|\,p_0, s_0\rangle = (2\pi)^{3/2} C^{+*}(\mathbf{p}_0, s_0)\,\Phi_0$ is the initial nucleon state vector.

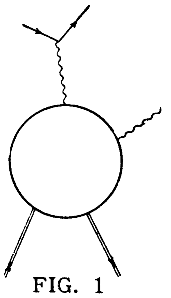


FIG. 1

Now following the method outlined in the paper by Logunov and Isaev[8] and assuming that the virtual Compton effect amplitude behaves asymptotically as $\sim 1/k_0$ (where k_0 is the photon energy)* we obtain d.r. for the diagram shown in Fig. 1, in the Breit coordinate system and to lowest order in e, as follows

$$D(k_0) = \frac{\mathrm{P}}{\pi}\int_{E_1}^{\infty}\frac{A(k_0')\,dk_0'}{k_0' - k_0} - \frac{\mathrm{P}}{\pi}\int_{E_1}^{\infty}\frac{A(-k_0')\,dk_0'}{k_0' + k_0}$$
$$+ \frac{A^{(1)}(-\lambda\mathbf{a} + \Delta\mathbf{p})}{E_p - k_0} + \frac{A^{(2)}(\lambda\mathbf{a} + \Delta\mathbf{p})}{E_p + k_0}, \quad (2.2)$$

where

$$k_0 = (k_0 + \varkappa_0)/2, \qquad \varkappa = q_0 - q,$$
$$D(k_0) = \frac{1}{2}[ST^{ret}(k_0, \mathbf{a}) + ST^{adv}(k_0, \mathbf{a})],$$
$$A(k_0) = \frac{1}{2i}[ST^{ret}(k_0, \mathbf{a}) - ST^{adv}(k_0, \mathbf{a})],$$

with S the $\boldsymbol{\lambda}$-symmetrization operator.

$$A^{(1)}(-\lambda\mathbf{a} + \Delta\mathbf{p}) = (2\pi)^3\frac{M^2 + \Delta\mathbf{p}^2}{M^2 + \mathbf{p}^2}e^n\varepsilon^l S\sum_\rho\langle\mathbf{p}, \sigma\,|\,j^l(0)\,|-\lambda\mathbf{a}$$
$$+ \Delta\mathbf{p}, \rho\rangle\langle-\lambda\mathbf{a} + \Delta\mathbf{p}, \rho\,|\,j^n(0)\,|\,\mathbf{p}_0, \sigma_0\rangle,$$
$$A^{(2)}(\lambda\mathbf{a} + \Delta\mathbf{p}) = (2\pi)^3\frac{M^2 + \Delta\mathbf{p}^2}{M^2 + \mathbf{p}^2}e^n\varepsilon^l S\sum_\rho\langle\mathbf{p}, \sigma\,|\,j^n(0)\,|\,\lambda\mathbf{a}$$
$$+ \Delta\mathbf{p}, \rho\rangle\langle\lambda\mathbf{a} + \Delta\mathbf{p}, \rho\,|\,j^l(0)\,|\,\mathbf{p}_0, \sigma_0\rangle;$$
$$\lambda\mathbf{a} = \mathbf{k} + (1 - \Delta)\,\mathbf{p} = \boldsymbol{\varkappa} - (1 + \Delta)\,\mathbf{p},$$
$$\Delta = m_\gamma^2/4\mathbf{p}^2;\ \varkappa^2 = -m_\gamma^2,\qquad \varepsilon^l = \bar{u}(\mathbf{q}, \sigma)\,\gamma^l u(\mathbf{q}_0, \sigma_0). \quad (2.3)$$

Here $\mathbf{a} = \boldsymbol{\lambda}/\lambda$ is a unit vector orthogonal to $\mathbf{p}$; $j^l(0)$, $j^n(0)$ are the electromagnetic current operators; e^n is the polarization vector of the free photon; $u(\mathbf{q}, \sigma)$ is the spinor describing an electron in the state with momentum q and spin direction σ. The spinors are normalized according to $\bar{u}u = 1$.

The d.r. for pair production by a photon in the field of a nucleon are obtained from (2.2) by the simple substitutions $k \to \kappa$, $\kappa = q + q_0$, and $m_\gamma^2 = \kappa_0^2 - \kappa^2$. The one-nucleon terms (2.3) are evaluated at the points

$$k_0 = \pm E_p = \pm(1 - \Delta)\,\mathbf{p}^2/\sqrt{M^2 + p^2}. \quad (2.4)$$

The continuous spectrum in k_0 begins at the points

$$k_0 = \mp E_1 = [\mp 2M\mu$$
$$+ \mu^2 - 2(1 - \Delta)\,\mathbf{p}^2]/2\sqrt{M^2 + \mathbf{p}^2}, \quad (2.5)$$

with μ the pion mass.

In order to avoid having the one-nucleon poles $\pm E_p$ fall in the region of the continuous spectrum $|k_0| \geq E_1$, i.e., in order that $E_p < E_1$, it is necessary to require that p^2 satisfy

$$p^2 < (2M\mu + \mu^2 + m_\gamma^2)/4..$$

It is important to note that in the d.r. under study no unphysical region appears for momenta satisfying the inequality

$$|\mathbf{p}| < [(2M\mu + \mu^2)^2 + (M + \mu)^2 m_\gamma^2 + m_\gamma^2 M^2 + (2M\mu + \mu^2)$$
$$\times\sqrt{(2M\mu + \mu^2 + m_\gamma^2)^2 + 4M^2 m_\gamma^2}\,]^{1/2}/2\sqrt{2}(M + \mu). \quad (2.6)$$

For the real Compton effect $m_\gamma = 0$ and we get

$$|\mathbf{p}| < (2M\mu + \mu^2)/2(M + \mu),$$

in agreement with the results of Bogolyubov and Shirkov.[10] The one-nucleon terms (2.3) are calculated in the same manner as that outlined by Logunov and Isaev.[8] In the Breit coordinate system

*An analogous choice for the asymptotic behavior of the amplitude for the real Compton effect was made by Gell-Mann and Mathews.[9]

they take the form

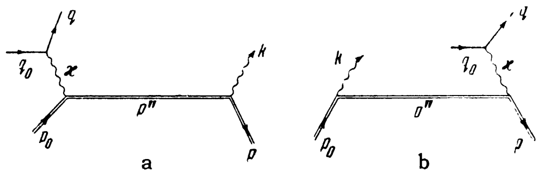
$$A_{ln}^{(1)} = -e^2 \frac{M(M^2+\Delta\mathbf{p}^2)}{2(2\pi)^3 (M^2+\mathbf{p}^2) E'' \sqrt{EE_0}} \bar{w}(\mathbf{p}, \sigma)\Big(F_1(\varkappa^2)\gamma^l + F_2(\varkappa^2)\frac{\mu_0[\hat{\varkappa}, \gamma^l]}{4M}\Big)(\hat{p}''+M)$$
$$\times\Big(F_1(k^2)\gamma^n - F_2(k^2)\frac{\mu_0[\hat{k}, \gamma^n]}{4M}\Big) w(\mathbf{p}_0, \sigma_0),$$
$$A_{nl}^{(2)} = -e^2 \frac{M(M^2+\Delta\mathbf{p}^2)}{2(2\pi)^3 (M^2+\mathbf{p}^2) E'' \sqrt{EE_0}} \bar{w}(\mathbf{p}, \sigma)\Big(F_1(k^2)\gamma^n - F_2(k^2)\frac{\mu_0[\hat{k}, \gamma^n]}{4M}\Big)(\hat{p}''+M)$$
$$\times\Big(F_1(\varkappa^2)\gamma^l + F_2(\varkappa^2)\frac{\mu_0[\hat{\varkappa}, \gamma^l]}{4M}\Big) w(\mathbf{p}_0, \sigma_0). \qquad (2.7)$$

Here F_1 and F_2 are nucleon form factors: $F_1(0) = F_2(0) = 1$ for the proton, $F_1(0) = 0$, $F_2(0) = 1$ for the neutron; μ_0 is the anomalous magnetic moment in units of nuclear magnetons.

It is important to note that in the one-nucleon approximation, i.e., when

$$\langle f|S|i\rangle = -ie\frac{me^n\varepsilon^l}{\sqrt{2k_0\varepsilon\varepsilon_0}}\frac{(2\pi)^4}{\varkappa^2}\delta(k+p-p_0-\varkappa)\times\Big(\frac{A_{ln}^{(1)}(-\lambda \mathrm{a})}{E_p-k_0}+\frac{A_{nl}^{(2)}(\lambda\mathrm{a})}{E_p+k_0}\Big) \qquad (2.8)$$

(ϵ, ϵ_0 are the electron energies in final and initial states) the expression (2.8) coincides with the sum of the matrix elements corresponding to the diagrams of Figs. 2a and 2b, in which the vertices involving the virtual photon contain $F_1(\kappa^2)$ and $F_2(\kappa^2)$ instead of the form-factor functions $\Phi_1(\kappa^2, p''^2)$ and $\Phi_2(\kappa^2, p''^2)$, and the vertices involving the real photon contain $F_1(0)$ and $F_2(0)$ instead of $\Phi_1(0, p''^2)$ and $\Phi_2(0, p''^2)$.



a b

FIG. 2.

In this way use of d.r. permits one to introduce in a rigorous manner into the bremsstrahlung and pair production processes (in lowest order in e approximation) the form factors depending on one variable which, for negative values of the argument, were studied by Hofstadter.[11]

3. STRUCTURE OF THE BREMSSTRAHLUNG AMPLITUDE

Making use of the results of Kawaguchi and Mugibayashi[12] we write T^c in the form

$$T^c = \sum_{i=1}^{12} \Omega_i T^i, \qquad (3.1)$$

where Ω_i are scalar functions of the invariants constructed of p_0, p, k, and κ, and T^i are relativistic and gauge invariant structures satisfying the condition

$$T^i = \pm (T^i)^+ \qquad (p \rightleftarrows p_0,\ \sigma \rightleftarrows \sigma_0,\ k \to -k,\ \varkappa \to -\varkappa). \qquad (3.2)$$

The resultant twelve structures are rather unwieldy (they are given explicitly in reference 13).

In going over to the real Compton effect four of these structures vanish and only eight remain. This is in agreement with the results of Ritus.[14]

We expand T^c in the frame $\mathbf{p} + \mathbf{p}_0 = 0$ in terms of twelve independent three-dimensional structures r_k, which we choose as follows

$$\begin{array}{lll} r_1 = \mathbf{e}\cdot\boldsymbol{\varepsilon}, & r_5 = i\,\boldsymbol{\varepsilon}\cdot\boldsymbol{\lambda}\,\sigma\cdot\mathbf{p}\times\mathbf{e}, & r_9 = i\,\mathbf{e}\cdot\boldsymbol{\varepsilon}\,\sigma\cdot\mathbf{p}\times\boldsymbol{\lambda}, \\ r_2 = \mathbf{e}\cdot\mathbf{p}\,\boldsymbol{\varepsilon}\cdot\boldsymbol{\lambda}, & r_6 = i\,\boldsymbol{\varepsilon}\cdot\mathbf{p}\,\mathbf{e}\cdot\mathbf{p}\,\sigma\cdot\mathbf{p}\times\boldsymbol{\lambda}, & r_{10} = i\,\boldsymbol{\varepsilon}\cdot\mathbf{p}\,\sigma\cdot\boldsymbol{\lambda}\times\mathbf{e}, \\ r_3 = \mathbf{e}\cdot\mathbf{p}\,\boldsymbol{\varepsilon}\cdot\mathbf{p}, & r_7 = \frac{i}{\lambda^2}\,\mathbf{e}\cdot\mathbf{p}\,\boldsymbol{\varepsilon}\cdot\boldsymbol{\lambda}\,\sigma\cdot\mathbf{p}\times\boldsymbol{\lambda}, & r_{11} = \frac{i}{\lambda^2}\,\boldsymbol{\varepsilon}\cdot\boldsymbol{\lambda}\,\sigma\cdot\boldsymbol{\lambda}\times\mathbf{e}, \\ r_4 = i\,\boldsymbol{\varepsilon}\cdot\mathbf{p}\,\sigma\cdot\mathbf{p}\times\mathbf{e}, & r_8 = i\,\mathbf{e}\cdot\mathbf{p}\,\sigma\cdot\mathbf{p}\times\boldsymbol{\varepsilon}, & r_{12} = i\,\mathbf{e}\cdot\mathbf{p}\,\sigma\cdot\boldsymbol{\lambda}\times\boldsymbol{\varepsilon}. \end{array} \qquad (3.3)$$

The amplitude T^c may now be written as

$$T^c = \sum_{k=1}^{12} L_k(k_0, \mathbf{p}^2)\, r_k, \qquad (3.4)$$

where $L_k(k_0, \mathbf{p}^2)$ are scalar functions of the variable k_0 and the recoil $\mathbf{p}^2$. The $\boldsymbol{\lambda}$-symmetrization operation S is now performed trivially due to the explicit dependence of the structures r_k on $\boldsymbol{\lambda}$.

4. DISPERSION RELATIONS FOR THE LORENTZ INVARIANT COEFFICIENTS

As a consequence of independence of the structures r_k it is obvious that d.r. (2.2) may be written for each coefficient L_k separately (and each of these coefficients shall behave in the complex k_0-plane no worse than the amplitude T^c, and decrease asymptotically no slower than $1/k_0$). Furthermore the symmetrization operation S will cause the structures antisymmetric in $\boldsymbol{\lambda}$ to decrease no slower than k_0^{-2}.

In order to go over from d.r. for L_k to relations for the coefficients Ω_i it is first necessary to relate Ω_i and L_k and then to investigate the behavior of $\Omega_i(k_0)$ in the complex plane.

Expanding T^i in terms of the r_k and utilizing

$$T^c = \sum_{i=1}^{12} \Omega_i T^i = \sum_{k=1}^{12} L_k r_k,$$

we find

$$\sum_{i=1}^{12} \Omega_i a_{ik} = L_k \qquad (k = 1, 2, \ldots, 12). \qquad (4.1)$$

The system (4.1) separates and can be solved for Ω_i.

An analysis of the coefficients $\Omega_i(k_0)$ shows that all Ω_i are analytic functions of k_0 in the same domain in which the function T^c is analytic. For the coefficient Ω_6 we write once-subtracted d.r. since its asymptotic behavior may be like that of a constant.

Introducing the invariant variables

$$r = k(p + p_0), \qquad t = (\varkappa - k)^2 \tag{4.2}$$

and utilizing the relations

$$\operatorname{Im}\Omega_j(k_0) = -\operatorname{Im}\Omega_j^*(-k_0) \quad (j = 1, 5, 6),$$

$$\operatorname{Im}\Omega_j(k_0) = \operatorname{Im}\Omega_j^*(-k_0) \qquad (j = 2, 3, 4, 7, \ldots, 12), \tag{4.3}$$

which follow from (3.2), we obtain d.r. for the functions $\Omega_i(r, t)$ in the frame $\mathbf{p} + \mathbf{p}_0 = 0$ in the following form

$$\operatorname{Re}\Omega_i(r, t) = \frac{P}{\pi}\int_{r_0}^{\infty}\left(\frac{1}{r'-r} + \frac{1}{r'+r}\right)\operatorname{Im}\Omega_i(r', t)\,dr' + \Omega_i^0 \quad (i = 1,5),$$

$$\operatorname{Re}\Omega_i(r, t) = \frac{P}{\pi}\int_{r_0}^{\infty}\left(\frac{1}{r'-r} - \frac{1}{r'+r}\right)\operatorname{Im}\Omega_i(r', t)\,dr' + \Omega_i^0 \quad (i = 2, 3, 4, 7-11),$$

$$\operatorname{Re}\Omega_6(r, t) = \frac{2r^2}{\pi}P\int_{r_0}^{\infty}\frac{\operatorname{Im}\Omega_6(r', t)\,dr'}{r'(r'^2 - r^2)} + \operatorname{Re}\Omega_6(0),$$

$$\operatorname{Re}\Omega_6(0) = -\frac{t + m_\gamma^2}{2m_\gamma^2}\operatorname{Re}\Omega_5(0) + \frac{1}{m_\gamma^2}\operatorname{Re}\Omega_1(0),$$

$$\operatorname{Re}\Omega_{12}(r, t) = \frac{P}{\pi}\int_{r_0}^{\infty}\left(\frac{1}{r'-r} - \frac{1}{r'+r}\right)\operatorname{Im}\Omega_{12}(r', t)\,dr' \tag{4.4}$$

[the lower limit on the integrals is $r_0 = 2M\mu + \mu^2 + \frac{1}{2}(t + m_\gamma^2)$]. The one-nucleon terms in the d.r. for Ω_6 and Ω_{12} vanish. The remaining one-nucleon terms may be obtained from (2.7) and are given explicitly in reference 13.

5. DISPERSION RELATIONS FOR PHYSICAL AMPLITUDES IN THE BARYCENTRIC FRAME

In order to obtain d.r. for the physical amplitudes in the barycentric frame we expand T^c in terms of independent three-dimensional structures ρ_k in that frame:

$$T^c = \sum_{k=1}^{12} M_k\rho_k. \tag{5.1}$$

Here M_k are physical amplitudes which depend on the total center of mass energy W and on $\cos\theta$, where θ is the angle between the directions of the virtual and real photons. We have

$$\begin{aligned}
&\rho_1 = \mathbf{e}\cdot\boldsymbol{\varepsilon}, && \rho_5 = i\,\boldsymbol{\varepsilon}\cdot\mathbf{k}\,\boldsymbol{\sigma}\cdot\boldsymbol{\kappa}\times\mathbf{e}, && \rho_9 = i\,\mathbf{e}\cdot\boldsymbol{\varepsilon}\,\boldsymbol{\sigma}\cdot\mathbf{k}\times\boldsymbol{\kappa},\\
&\rho_2 = \mathbf{e}\cdot\boldsymbol{\kappa}\,\boldsymbol{\varepsilon}\cdot\mathbf{k}, && \rho_6 = i\,\boldsymbol{\varepsilon}\cdot\boldsymbol{\kappa}\,\mathbf{e}\cdot\boldsymbol{\kappa}\,\boldsymbol{\sigma}\cdot\mathbf{k}\times\boldsymbol{\kappa}, && \rho_{10} = i\,\boldsymbol{\varepsilon}\cdot\boldsymbol{\kappa}\,\boldsymbol{\sigma}\cdot\mathbf{k}\times\mathbf{e},\\
&\rho_3 = \mathbf{e}\cdot\boldsymbol{\kappa}\,\boldsymbol{\varepsilon}\cdot\boldsymbol{\kappa}, && \rho_7 = i\,\mathbf{e}\cdot\boldsymbol{\kappa}\,\boldsymbol{\varepsilon}\cdot\mathbf{k}\,\boldsymbol{\sigma}\cdot\mathbf{k}\times\boldsymbol{\kappa}, && \rho_{11} = i\,\boldsymbol{\varepsilon}\cdot\mathbf{k}\,\boldsymbol{\sigma}\cdot\mathbf{k}\times\mathbf{e},\\
&\rho_4 = i\,\boldsymbol{\varepsilon}\cdot\boldsymbol{\kappa}\,\boldsymbol{\sigma}\cdot\boldsymbol{\kappa}\times\mathbf{e}, && \rho_8 = i\,\mathbf{e}\cdot\boldsymbol{\kappa}\,\boldsymbol{\sigma}\cdot\boldsymbol{\kappa}\times\boldsymbol{\varepsilon}, && \rho_{12} = i\,\mathbf{e}\cdot\boldsymbol{\kappa}\,\boldsymbol{\sigma}\cdot\mathbf{k}\times\boldsymbol{\varepsilon}.
\end{aligned} \tag{5.2}$$

The relation between $\Omega_i(W, \cos\theta)$ and $M_k(W, \cos\theta)$ may be obtained by expanding the relativistically invariant structures T^i in terms of ρ_k:

$$T^i = \sum_{k=1}^{12} b_{ik}\rho_k \qquad (i = 1, 2, \ldots, 12). \tag{5.3}$$

By solving the system (5.3) we find the matrix $\|c_{ik}\| = \|b_{ik}\|^{-1}$. By an appropriate choice of the structures, namely the choice (5.2), it is possible to separate the system (5.3) and reduce it to two second order and two fourth order systems. The coefficients c_{ik} are rather complicated and will not be given here. The final form of the d.r. for the physical amplitudes M_j in the barycentric frame is as follows:

$$\begin{aligned}
\operatorname{Re}M_j(W, t, m_\gamma^2) = {} & \frac{2}{\pi}P\int_{M+\mu}^{\infty}\sum_i \xi_i\Big(\frac{1}{W'^2 - W^2} \cdot \\
& + \eta_i\frac{1}{W'^2 + W^2 - 2M^2 + m_\gamma^2 + t}\Big) \times W'dW' b_{ji}(W, t, m_\gamma^2) \\
& \times\sum_k c_{ik}(W', t, m_\gamma^2)\operatorname{Im}M_k(W', t, m_\gamma^2) + \sum_i b_{ji}(W, t, m_\gamma^2)\Omega_i^0 \\
& + b_{j,6}(W, t, m_\gamma^2)\sum_k M_k\left(\sqrt{M^2 - \frac{t + m_\gamma^2}{2}}, t, m_\gamma^2\right)\Bigg[\frac{1}{m_\gamma^2}c_{1k} \\
& \times\left(\sqrt{M^2 - \frac{t + m_\gamma^2}{2}}, t, m_\gamma^2\right) \\
& - \frac{t + m_\gamma^2}{2m_\gamma^2}c_{5k}\left(\sqrt{M^2 - \frac{t + m_\gamma^2}{2}},\ t,\ m_\gamma^2\right)\Bigg],
\end{aligned}$$

where

$$\begin{aligned}
&\xi_6 = r/r',\ \xi_i = 1 \ \text{ for } i \neq 6; \quad r' = W'^2 - M^2 + \tfrac{1}{2}(t + m_\gamma^2);\\
&\eta_i = 1 \quad \text{for } i = 1,5; \qquad \eta_i = -1 \quad \text{for } i = 2, 3, 4, 6-12.
\end{aligned} \tag{5.4}$$

In conclusion we express deep gratitude to A. A. Logunov for valuable discussions and constant interest in this work, and also to D. V. Shirkov and A. N. Tavkhelidze for useful advice.

[1] Bogolyubov, Medvedev, and Polivanov, Вопросы теории дисперсионных соотношений (Problems in the Theory of Dispersion Relations), Fizmatizdat, M., 1958.

[2] Blokhintsev, Barashenkov, and Barbashov, Usp. Fiz. Nauk **68**, 417 (1959), Soviet Phys.—Uspekhi **2**, 505 (1960).

[3] V. S. Vladimirov and A. A. Logunov, Preprint, Joint Inst. Nuc. Res. R-260 (1958).

[4] Logunov, Tavkhelidze, and Solovyev, Nucl. Phys. **4**, 427 (1957).

[5] L. D. Solovyev, Nucl. Phys. **5**, 256 (1958).

[6] I. S. Zlatev and P. S. Isaev, JETP **35**, 309 (1958), Soviet Phys. JETP **8**, 213 (1959). Report at the All-Union Conference on Quantum Field Theory and Elementary Particles Theory (October 2 – 6, 1958, Uzhgorod).

[7] Bjorken, Drell, and Frautschi, Phys. Rev. **112**, 1409 (1958).

[8] A. A. Logunov and P. S. Isaev, Nuovo cimento **10**, 917 (1958).

[9] G. F. Chew, Ann. Conf. on High Energy Physics, CERN, 1958.

[10] N. N. Bogolyubov and D. V. Shirkov, Dokl. Akad. Nauk SSSR **113**, 529 (1957), Soviet Phys.–Doklady

[11] R. Hofstadter, Usp. Fiz. Nauk **13**, 693 (1957). (Russ. transl.) [sic!].

[12] M. Kawaguchi and N. Mugibayashi, Progr. Theoret. Phys. **8**, 212 (1952).

[13] I. S. Zlatev and P. S. Isaev, Preprint Joint Inst. Nucl. Res. R-321, 1959.

[14] V. I. Ritus, JETP **35**, 1485 (1958), Soviet Phys. JETP **8**, 923 (1959).

Translated by A. M. Bincer
139