RESONANCE SCATTERING OF GAMMA-QUANTA BY Mg^{24}

I. Sh. VASHAKIDZE, T. I. KOPALEĬSHVILI and G. A. CHILASHVILI

Institute of Physics, Academy of Sciences, Georgian S.S.R.

Submitted to JETP editor March 31, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 750-755 (September, 1959)

Resonance scattering of γ quanta from Mg^{24} nuclei, with excitation of the first two levels at 1.37 and 4.23 Mev has been investigated. Analysis of the correlation formula enables one to draw conclusions concerning the character of the excitation of the nucleus.

1. Until recently the problem of resonance scattering of γ quanta by nuclei was not amenable to experimental investigation because of the fact that usually $\Delta = h\nu_1 - h\nu_2 > \Gamma$, where ν_1 and ν_2 are the frequencies of the absorbed and emitted quanta, and Γ is the natural width of the excited level. As a result, the emission line $h\nu_2$ and the absorption line $h\nu_1$ hardly overlap, thus making it difficult to establish the occurrence of the resonance scattering. However this difficulty has now been essentially overcome, and there are many papers devoted to resonance scattering by various nuclei.[1-4]

The reason for the interest in this phenomenon is that by studying it one can not only determine some of the quantum numbers of excited nuclear states, but one can also decide whether the excitation occurs via a collective or a single-particle mechanism, since in many cases, as we shall show later, the expressions for the correlation function depend essentially on the assumed nuclear model.

2. The present paper gives a theoretical treatment of resonance scattering of γ quanta by Mg^{24}, with excitation of the 2^+ levels at 1.37 and 4.23 Mev (cf. Fig. 1).

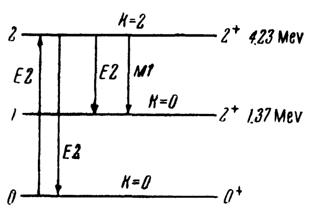

FIG. 1

Metzger[2] and Burgov and Terekhov[3] used the method of resonance scattering to determine experimentally the width of the 2^+ level at 1.37 Mev. Metzger gives a lower limit of $\Gamma_\gamma > 1.6 \times 10^{-4}$ ev, while reference 3 gives a definite value of $\Gamma_\gamma = 3.8 \times 10^{-4}$ ev. Assuming that the Mg^{24} nucleus is highly deformed,[5,6] the 2^+ level at 1.37 Mev can be treated either as a collective (rotational) level with angular momentum $I = 2$ and projection on the nuclear symmetry axis $K = 0$, or as a single-particle level resulting from the perturbed motion of a single particle in the field of the deformed nucleus. For a collective level, the reduced probability of transition from the excited state to the ground state is[7]

$$B = \tfrac{3}{5}\,(Ze\beta R_0^2 / 4\pi)^2, \tag{1}$$

where R_0 is the equilibrium radius of the sphere, and β is the equilibrium value of the nuclear deformation parameter. Formula (1) is usually used for determining the parameter β from the observed value of B. Setting the radius $R_0 = 1.45\,A^{1/3} \times 10^{-13}$ cm, one finds for Mg^{24} the value $\beta = 0.45$.

For the case of single-particle excitation, the expression for the transition probability to the ground state was given by Nilsson.[8] It is obvious that in Mg^{24} the transition from the 2^+ level at 1.37 Mev to the ground state occurs mainly by emission of E2 quanta. The corresponding level width is[8]

$$\Gamma = (4\pi / 75\hbar)\,(\omega / c)^5\, B\,(E2), \tag{2}$$

$$B\,(E2) = (5e/\sqrt{4\pi})^2 \Big| \sum_{l_1}\sum_{l_2}\sum_{\sigma} A_{l_1,\,\Omega-\sigma} A'_{l_2\Omega'-\sigma}\,\delta_{\Omega,\Omega'} \times \frac{B_{l_1 l_2}}{\sqrt{(2l_1+1)(2l_2+1)}} \times (2l_1 00\,|\,2l_1 l_2 0)\,(2l_1 0\,\Omega-\sigma\,|\,2l_1 l_2 \Omega-\sigma)\Big|^2. \tag{3}$$

Here l_1 and l_2 are the orbital angular momenta of the nucleon in the ground and excited states, having the values 0 and 2 (N=2 shell); Ω is the projection of the angular momentum of the nucleon on the nuclear symmetry axis; σ is the spin: $A_{l_1,\Omega-\sigma}$ and $A'_{l_2,\Omega'-\sigma}$ are the diagonalization coefficients corresponding to the single-particle functions in the ground and first excited states;[8]

$$B_{l_1 l_2} = \int R_{2l_1} R_{2l_2} r^4 dr, \tag{4}$$

where R_{2l_1} and R_{2l_2} are oscillator wave functions for the nucleon, which depend only on the parameter

r'_0 which determines the level spacing corresponding to the spherical oscillator potential

$$\hbar\omega = \hbar^2 / 2M_p r'^2_0. \quad (5)$$

The expression for B (E2) depends on the parameters $\delta = \beta/0.95$ and r'_0.

The nucleons in the N = 2 shell can be in states with $\Omega = \pm 1/2$, $\pm 3/2$, and $\pm 5/2$; there are three different states for $\Omega = \pm 1/2$, two for $\Omega = \pm 3/2$, and one for $\Omega = \pm 5/2$. The spacing of the levels depends on the value of $\hbar\omega$ and the deformation parameter δ. If we require that the level spacing agree with experiment, we can determine the parameter δ from the Nilsson model. In the Mg^{24} nucleus the $\pm 1/2$ and $\pm 3/2$ levels are filled if $\delta > 0$, while the $\pm 5/2$ and $\pm 1/2$ levels are filled if $\delta < 0$. The selection rules permit only $1/2 \rightarrow 1/2$ and $3/2 \rightarrow 3/2$ transitions in Mg^{24}. For the values of the parameter δ we get $\delta = \pm 0.2$ from the $1/2 \rightarrow 1/2$ transition, and $\delta = 0$ from the $3/2 \rightarrow 3/2$ transition, setting $r'_0 = 1.9 \times 10^{-13}$ cm in (5).

δ	$10^4 \Gamma_{theor}$, ev	$10^4 \Gamma_{exp}$, ev
0.2	63	3.8[4]
−0.2	0.8	
0.3	11.8	>1.6[3]

The values of the width of the excited level at 1.37 Mev are given in the table. We see that the value $\delta = -0.2$ must be discarded, since the resulting level width contradicts the experimental value. The theoretical value of the width for $\delta = 0.2$ is considerably greater than the experimental value. But if we take a somewhat greater value for δ, say 0.3, the theoretical value of the width Γ is much closer to experiment. Of course, the excitation energy is then different from the experimental value, but not very much, since with $\delta = 0.3$ we get 1.72 Mev for the excitation energy. Unfortunately, in Nilsson's paper the diagonalization coefficients are tabulated only up to $\delta = 0.3$. It is to be expected that for a somewhat greater value of δ the agreement with experiment will be more satisfactory with respect to both the width of the level and the value of the energy of excitation. As for the width of the excited level found from the collective model, nothing definite can be said since no satisfactory way has yet been found for determining the deformation parameter β independently of formula (1).

The angular distribution of the scattered γ quanta is

$$I(\theta) = I(0)\{1 - 3\cos^2\theta + 2\cos^4\theta\}. \quad (6)$$

As was to be expected, the angular distribution does not depend on the model, since we are dealing with a pure E2 transition.

3. We can get a different result concerning the dependence of the angular distribution on the nuclear model if we consider the resonance scattering of γ quanta by Mg^{24} with excitation of the second 2^+ level at 4.23 Mev. Actually γ transitions can occur from this level to the ground state as well as to the first excited 2^+ level. For the transition to the ground state, the angular distribution of the γ quanta will not depend on the model since this transition is pure E2. For the transition to the first 2^+ level, both E2 and M1 transitions are possible. If the probabilities of these transitions are of the same order of magnitude, then because of the interference term the correlation function will now depend essentially on the assumed nuclear model, and a comparison with experiment will enable us to determine the validity of the models. One must keep in mind that the probabilities of E2 transitions to the ground state and first excited state should be of the same order of magnitude. Of course the nucleus in the first excited state later makes a transition to the ground state, but this can give nothing new concerning the dependence of the angular distribution on the assumed nuclear model, since this transition is a pure E2. For this reason we shall in what follows regard the first excited state as the final state. To obtain the correlation between the quanta absorbed and emitted in the transition $2 \rightarrow 1$, we shall start from the two models: collective and single-particle. However before doing this we must check whether the conditions

$$W[E2(2 \rightarrow 1)] \sim W[E2(2 \rightarrow 0)],$$
$$W[M1(2 \rightarrow 1)] \sim W[E2(2 \rightarrow 1)],$$

are satisfied, where the numbers 0, 1, and 2 denote the ground state, the first and the second excited states, respectively.

Since both excited levels have the same angular momentum 2^+, we can use the assumption of Davydov and Filippov[9,10] that the nucleus can be represented as an asymmetric top, which has rotational levels of different energies but with the same total angular momentum. The authors cited have found the relation between these levels as a function of asymmetry parameter γ. Since the Mg^{24} nucleus is highly deformed, we apply the model of Davydov and Filippov to it, and calculate the probabilities of the radiative transitions for the same value of the asymmetry parameter which gives the observed value for the ratio of the energies of the levels ($\gamma = 22°$). We find

$$W[E2(2\rightarrow 1)]/W[E2(2\rightarrow 0)]\sim 1,$$
$$W[M1(2\rightarrow 1)]/W[E2(2\rightarrow 1)]\sim 10^{-2}. \qquad (7)$$

We see that the probability of the magnetic transition in this case is very small compared to the probability of the E2 transition. The correlation function will therefore not depend on the nuclear model, and will be given by formula (6). If however we treat the 2^+ levels at 4.23 and 1.37 Mev as single particle levels according to the Nilsson model, then as is easily shown we get for the ratio of the radiative transition probabilities

$$W[E2(2\rightarrow 1)]/W[E2(2\rightarrow 0)]$$
$$\sim W[M1(2\rightarrow 1)]/W[E2(2\rightarrow 1)]\sim 1. \qquad (8)$$

From this it follows that the transition $2\rightarrow 1$ is not a pure E2 transition, and that the correlation function on this model will differ from formula (6). To find the function we use the general correlation formula

$$I(\mathbf{n}_\gamma)\sim\sum_{pp'}\sum_{M_0}\sum_{M_2}$$
$$\times\left|\sum_{M_1}(M_0I_0K_0\,|\,\hat{H}_0\,|\,I_1M_1K_1)(I_1M_1K_1\,|\,\hat{H}_1\,|\,I_2M_2K_2)\right|^2, \qquad (9)$$

where $(I_0M_0K_0\,|\,\hat{H}_0\,|\,I_1M_1K_1)$ is the matrix element for E2 absorption of the γ quantum with transition of the nucleus from the ground state to the 2^+ state at 4.23 Mev; $(I_1M_1K_1\,|\,\hat{H}_1\,|\,I_2M_2K_2)$ is the matrix element for the transition $2\rightarrow 1$, in which (E2 + M1) quanta are emitted; $(I_0M_0K_0)$, $(I_1M_1K_1)$, and $(I_2M_2K_2)$ are the values of the total angular momentum of the nucleus, its projection on a space-fixed axis and its projection on the nuclear symmetry axis, in the initial, intermediate, and final states, respectively. The operators for absorption and emission of γ quanta have the well-known form[12]

$$\hat{H}_0=-\sqrt{\pi/15}\,ep'(kr)^2Y_{2p'},$$
$$\hat{H}_1=-\sqrt{\pi/15}\,ep\sum_{M=-2}^{2}D^2_{Mp}(\mathbf{n}_\gamma)(kr)^2Y_{2M}-(\pi e\hbar/M_pc)\sqrt{3}$$
$$\times\sum_{M\mu}D^1_{Mp}(\mathbf{n}_\gamma)f_1(kr)(11-\mu\,\mu+M\,|\,111M)Y_{1,\mu+M}\nabla_{-\mu}, \qquad (10)$$

where $f_L(kr)$ are spherical Bessel functions, multiplied by $(2\pi)^{1/2}$, and $D^l_{mm'}(\mathbf{n})$ is the transformation matrix.

For the wave function of the nuclear system in the initial, intermediate, and final states, we have[7,8]

$$\Psi_{000}=\sqrt{1/8\pi^2}\,\chi_{\Omega_0\Omega_0^0},$$
$$\Psi_{2M_12}=\sqrt{5/16\pi^2}\,\{\chi_{\Omega_1\Omega_1^0}D^2_{M_12}+\chi_{-\Omega_1,-\Omega_1^0}D^2_{M_1-2}\},$$
$$\Psi_{2M_20}=\sqrt{5/8\pi^2}\,\chi_{\Omega_2\Omega_2^0}D^2_{M_20}, \qquad (11)$$

where Ω_i $(i = 0, 1, 2)$ is the projection on the nuclear symmetry axis of the angular momentum of the nucleon excited in the N = 2 shell, in the initial, intermediate, and final states, Ω_i^0 is the sum of the projections of the angular momenta of the other nucleons in the shell.

In addition we have

$$\chi_{\Omega_i\Omega_i^0}=\chi_{\Omega_i^0}\chi_{\Omega_i}, \qquad (12)$$

where

$$\chi_{\Omega_i}=\sum_{l_i m_i\sigma_i}A^i_{l_i\Omega_i-\sigma_i}R_{2l_i}D^{l_i}_{m_i\,\Omega_i-\sigma_i}Y_{l_im_i}(0,\varphi)\chi(\sigma_i) \qquad (12')$$

is the wave function of the nucleon in the undeformed nucleus,[8] $A^i_{l_i\Omega_i-\sigma_i}$ are the diagonalization coefficients, $\chi(\sigma_i)$ are spin functions, χ_{Ω_i} is the wave function of the other nucleons in the N = 2 shell.

If we use these expressions, we find, after quite involved calculations,

$$I(\mathbf{n}_\gamma)\sim\sum_{pp'}\sum_{M_0M_2}\Big|\tfrac{5}{8}e^2(k_{20}R_0)^2\sqrt{5/3}\,N(\Omega_0\Omega_1)$$
$$\times\Big\{\sum_{M_1}B(20m_1M_2\Omega_1\Omega_2)\,[O(20m_1pM_2)\,D^1_{p'-M_2,\,p}(\mathbf{n}_\gamma)$$
$$+C(20m_1pm_2)\,D^2_{p'-M_2,\,p}(\mathbf{n}_\gamma)]$$
$$+\sum_{m_1}B(22m_1p'M_2\Omega_1\Omega_2)\,[O(22m_1pM_2)\,D^1_{p'-M_2,\,p}(\mathbf{n}_\gamma)$$
$$+C(22m_1pM_2)\,D^2_{p'-M_2,\,p}(\mathbf{n}_\gamma)]\Big\}\Big|^2, \qquad (13)$$

where

$$N(\Omega_0\Omega_1)=\sum_{l_0}\sum_{l_1}\frac{\sqrt{2l_1+1}}{(2l_0+1)^{3/2}}(l_1200\,|\,l_12l_00)\,\mathfrak{M}_2(l_1l_0)$$
$$\times\sum_{\sigma_1}\{A^1_{l_1\Omega_1-\sigma_1}A^0_{l_2\Omega_0-\sigma_1}(l_12\Omega_1-\sigma_12\,|\,l_22l_0\Omega_1-\sigma_1+2)$$
$$+A^1_{l_1,-\Omega_1-\sigma_1}A^0_{l_0,\,\Omega_0-\sigma_1}$$
$$\times(l_12-\Omega_1-\sigma_12\,|\,l_12l_0-\Omega_1-\sigma_1+2)\},$$

$$B(l_1l_2mp'M_2\Omega_1\Omega_2)=\sum_{L=l_1-2}^{l_1+2}\frac{1}{(2L+1)}(l_12m_1p'\,|\,l_12Lm_1+p')$$
$$\times(l_22m_1+p'-M_2,\,M_2\,|\,l_22Lm_1+p')$$
$$\times\sum_{\sigma'}[A^2_{l_2\Omega_2-\sigma_1},\,A^1_{l_1\Omega_1-\sigma_1}(l_12\Omega_1-\sigma_12\,|\,l_12L\Omega_1-\sigma_1+2)$$
$$\times(l_22\Omega_2-\sigma_10\,|\,l_22L\Omega_2-\sigma_1)+A^2_{l_2\Omega_2-\sigma_1}A^1_{l_1,-\Omega_1-\sigma_1}$$
$$\times(l_12-\Omega_1-\sigma_1,-2\,|\,l_12L-\Omega_1-\sigma_1-2)$$
$$\times(l_22\Omega_2-\sigma_10\,|\,l_22L\Omega_2-\sigma_1)],$$

$$O(l_1l_2m_1p'M_2)=\frac{e\hbar k_{21}}{2\sqrt{2}M_pc}\frac{1}{\sqrt{2l_2+1}}$$
$$\times\sum_{\mu=-1}^{1}(11-\mu\mu+p'-M_2\,|\,111p'-M)\{(l_11m_1-\mu\,|\,l_11l_1$$
$$+1m_1-\mu)(1l_1+1\mu+p'-M_2m_1-\mu\,|\,1l_1+1l_2m_1$$
$$+p'-M_2)\,T_1(l_1l_2)-(l_11m_1-\mu\,|\,l_11l_1-1m_1-\mu)$$

$$\times (1l_1 - 1\mu + p' - M_2 m_1 - \mu \,|\, 1l_1 - 1l_2 m_1 + p' - M_2)\, T_2(l_1 l_2)\},$$

$$T(l_1 l_2) = \sqrt{(l_1 + 1)}\,(1l_1 + 100 \,|\, 1l_1 + 1l_2 0)\,[(l_2 - l_1 - 2) \times (l_1 + l_3 + 3)\,\mathfrak{M}_0(l_1 l_2) + 2M_p c k_{21} R^2 \hbar^{-1} \mathfrak{M}_2(l_1 l_2)],$$

$$T_2(l_1 l_2) = \sqrt{l_1}\,(1l_1 - 100 \,|\, 1l_1 - 1l_2 0)\,[\mathfrak{M}_0(l_1 l_2) + 2M_p c k_{21} R^2 \hbar^{-1} \mathfrak{M}_2(l_1 l_2)],$$

$$C(l_1 l_2 m_1 M_1 M_2) = -\sqrt{\frac{2l_1 + 1}{3(2l_2 + 1)}}\,(l_1 2 m_1 M_1 - M_2 \,|\, l_1 2 l_2 m_1 + M_1 - M_2)\,(kR)^2\,\mathfrak{M}_2(l_1 l_2).$$

In these formulas M_p is the proton mass and c is the velocity of light; k_{20} and k_{21} are the wave numbers of the γ quanta emitted in the transitions $2 \rightarrow 0$ and $2 \rightarrow 1$, respectively, while the symbols $\mathfrak{M}_L(l_1 l_2)$ denote radial integrals, i.e.

$$\mathfrak{M}_L(l_1 l_2) = \int R_{2l_1} (r/R)^L R_{2l_2} r^2 dr.$$

From the selection rules we have $|\Omega_1 - \Omega_0| = 2$, $|\Omega_1 - \Omega_2| = 2$. Thus in the absorption of an E2 quantum we will have the transitions $1/2 \rightarrow 5/2$, $1/2 \rightarrow -3/2$, $3/2 \rightarrow -1/2$, if $\delta > 0$. If we require that the calculated level spacing agree with the experimental value, (4.23 Mev), we will have only the $1/2 \rightarrow -3/2$ transition. In this case the theoretical values are $\Delta E = 3.8$ Mev for $\delta = 0.3$, and $\Delta E = 2.9$ Mev for $\delta = 0.2$, (The lack of tables of the diagonalization coefficients for $\delta > 0.3$ prevents us from getting closer to the experimental value $\Delta E = 4.23$ Mev.) In this case, in the transition $2 \rightarrow 1$ to the final state only the single-particle transition $-3/2 \rightarrow 1/2$ will occur. If we substitute the computed values of k_{20} and k_{21} in formula (13), we finally get the correlation function

$$I(\theta) \sim (1 + A\cos\theta + B\cos^2\theta + C\cos^3\theta + D\cos^4\theta), \quad (14)$$

where θ is the angle between the absorbed and emitted γ quanta. For $\delta = 0.3$, the values of the coefficients in (14) are

$$A = 0.11, \quad B = -1.5, \quad C = -0.3, \quad D = 0.7.$$

The correlation function given by formula (14) is not symmetric around 90° (cf. Fig. 2), whereas in the case of collective excitation the correspond-

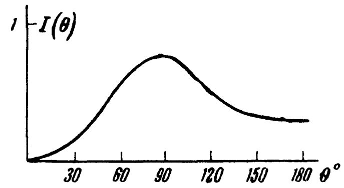

FIG. 2

ing curve determined from (6) is symmetric. Thus the experimental investigation of the correlation of γ quanta in the excitation of Mg^{24} with an energy of 4.23 Mev would enable us to draw definite conclusions concerning the nature of the excitation of this nucleus. Unfortunately, so far as we know, no one has as yet done such experiments.

In conclusion, it is our pleasant duty to express our thanks to V. I. Mamasakhlisov for his direction and continued interest in our work.

[1] P. Moon, Proc. Phys. Soc. (London) **64**, 76 (1951).

[2] F. Metzger, Phys. Rev. **83**, 842 (1951).

[3] N. A. Burgov and Yu. V. Terekhov, Атомная энергия (Atomic Energy) **2**, 514 (1957).

[4] D. G. Alkhazov et al., JETP **35**, 1056 (1958), Soviet Phys. JETP **8**, 737 (1959).

[5] G. Rakavy, Nucl. Phys. **4**, 375 (1957).

[6] V. I. Mamasakhlisov and T. I. Kopaleĭshvili, JETP **34**, 1169 (1958), Soviet Phys. JETP **7**, 809 (1958).

[7] A. Bohr and B. Mottelson, Kgl. Danske Videnskab. Selskab Mat. fys. Medd. **27**, No. 16 (1953).

[8] S. G. Nilsson, Kgl. Danske Videnskab. Selskab Mat. fys. Medd. **29**, No. 16 (1955).

[9] A. S. Davydov and G. F. Filippov, JETP **35**, 441 (1958), Soviet Phys. **8**, 303 (1959).

[10] A. S. Davydov and G. F. Filippov, JETP **35**, 703 (1958), Soviet Phys. JETP **8**, 488 (1959).

[11] V. I. Mamasakhlisov and T. I. Kopaleĭshvili, JETP **37**, 131 (1959), Soviet Phys. JETP **10**, 93 (1960).

[12] M. E. Rose, Multipole Fields, J. Wiley and Sons, New York, 1955, Russ. Transl. IIL 1957.

Translated by M. Hamermesh
142