CIRCULAR POLARIZATION OF γ QUANTA EMITTED BY A NUCLEUS AFTER μ^- CAPTURE

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Formulas for the circular polarization of γ quanta emitted by a nucleus after μ^- capture are deduced. The hyperfine splitting of the mesic atom levels is taken into account.

SINCE the μ^- mesons emitted in the decay of π^- mesons are longitudinally polarized, the nuclear μ^- capture leads to the formation of polarized nuclei. If, therefore, the daughter nucleus is formed in an excited state, the γ rays emitted by it will, in general, be polarized (the angular distribution will here be isotropic). In the present paper we consider the circular polarization of the γ rays for the case when the nucleus goes into a discrete state in the μ^- capture, i.e., when no neutron is emitted. The process under consideration is the following: a nucleus A_Z with spin j_1 captures in the K shell a polarized μ^- meson and goes over into an excited nuclear state A_{Z-1} with spin j_2,* which, under emission of a γ quantum with multipolarity J (the formulas obtained can easily be generalized for the case of mixed multipoles), goes over into the ground state with spin j_3.

We took the Hamiltonian of the four-fermion interaction in the form of a superposition of vector (v), axial-vector (a), and pseudoscalar (p) coupling with the coupling constants g_v, g_a, and g_p. The presence of v and a coupling follows from the theory of the universal Fermi interaction, as proposed by Feynman and Gell-Mann and by Sudarshan and Marshak.[1] The p coupling was added in view of the fact that the effects connected with the emission of virtual π mesons lead to the appearance of terms analogous to the pseudoscalar coupling in the S matrix describing the weak interaction.[2] The effective coupling constant g_p is proportional to the mass of the lepton, and while it is negligibly small in β decay, it does have a considerable magnitude in μ^- capture: according to the estimates of Goldberger and Treiman[2] $g_p \approx 8 g_a$, so that the inclusion of the pseudoscalar variant becomes inevitable, despite the fact that the constant g_p enters in the expressions for the probabilities of the processes under consideration with the factor v/c, where v is the velocity of the nucleons.

The circular polarization of the γ quanta, C_γ, is given by

$$C_\gamma = (W_+ - W_-)/(W_+ + W_-), \tag{1}$$

where W_+ and W_- are the probabilities for the emission of γ quanta whose spin is oriented in the direction of the momentum (right polarization) and in the opposite direction of the momentum (left polarization), respectively. The calculation leads to the following expression for C_γ in the case of the longitudinal neutrino:

$$C_\gamma = P_\mu \alpha \cos\theta, \qquad \alpha = B/A, \tag{2}$$

where P_μ is the polarization of the μ^- meson at the instant of its entrance in the K orbit of the mesic atom,* and θ is the angle between the direction of the polarization vector of the μ^- meson and the direction of emission of the γ quantum;

$$\begin{aligned} A = \mathrm{Re}\,\Big(&|g_v|^2\langle Y_\Lambda\rangle|^2 + |g_a|^2|\langle i\gamma_5 Y_{\Lambda-1}\rangle|^2 \\ &+ |g_p|^2\langle i\gamma_4\gamma_5 Y_{\Lambda-1}\rangle|^2 + 2g_p^* g_a\langle i\gamma_4\gamma_5 Y_{\Lambda-1}\rangle^*\langle i\gamma_5 Y_{\Lambda-1}\rangle \\ &+ \sum_l \{|g_a|^2|\langle \mathcal{T}_{l\Lambda}\rangle|^2 + |g_v|^2|\langle i\gamma_5\mathcal{T}_{l\Lambda-1}\rangle|^2\} \\ &+ 2\sum_{\Lambda'=\Lambda\pm1}\Big\{\Big(-|g_v|^2\langle Y_\Lambda\rangle^*\langle i\gamma_5\mathcal{T}_{\Lambda\Lambda'}\rangle \\ &+ |g_a|^2\langle i\gamma_5 Y_{\Lambda'}\rangle^*\langle\mathcal{T}_{\Lambda'\Lambda}\rangle \\ &+ g_p^* g_a\langle i\gamma_4\gamma_5 Y_{\Lambda'}\rangle^*\langle\mathcal{T}_{\Lambda'\Lambda}\rangle\Big)a_1(\Lambda,\Lambda') \\ &+ \sum_l g_v^* g_a\langle i\gamma_5\mathcal{T}_{l\Lambda'}\rangle^*\langle\mathcal{T}_{l\Lambda}\rangle a_2(\Lambda,\Lambda')\Big)\Big\}, \end{aligned} \tag{3a}$$

$$\begin{aligned} B = \mathrm{Re}\,\big(&|g_v|^2|\langle Y_\Lambda\rangle|^2 b_1(\Lambda) + [|g_a|^2|\langle i\gamma_5 Y_{\Lambda-1}\rangle|^2 \\ &+ |g_p|^2|\langle i\gamma_4\gamma_5 Y_{\Lambda-1}\rangle|^2 + 2g_p^* g_a\langle i\gamma_4\gamma_5 Y_{\Lambda-1}\rangle^*\langle i\gamma_5\mathcal{T}_{\Lambda-1}\rangle] \end{aligned}$$

*For $j_2 = 0$ the γ quanta are, of course, not circularly polarized.

*For $j_1 = 0$ P_μ coincides with the polarization of the μ^- meson at the instant of its capture by the nucleus, P_μ^0, which is experimentally observed in measurements of the asymmetry of electrons from μ^- decay in matter; in the general case P_μ and P_μ^0 are connected by the relation

$$P_\mu^0 = {}^1/_3\, P_\mu\,[1 + 2(2j_1 + 1)^{-2}].$$

$$\times b_1(\Lambda-1)+2\sum_I\Big\{g_v^*g_a\langle Y_\Lambda\rangle^*\langle\mathcal{Y}_{I\Lambda}\rangle b_2(\Lambda)$$
$$+[g_v^*g_a\langle i\gamma_5\mathcal{Y}_{I\Lambda-1}\rangle^*\langle i\gamma_5\mathcal{Y}_{\Lambda-1}\rangle$$
$$+g_p^*g_v\langle i\gamma_4\gamma_5Y_{\Lambda-1}\rangle^*\langle i\gamma_5\mathcal{Y}_{I\Lambda-1}\rangle]\,b_2(\Lambda-1)$$
$$+\sum_{II'}\{|g_a|^2\langle\mathcal{Y}_{I\Lambda}\rangle^*\langle\mathcal{Y}_{I'\Lambda}\rangle b_3(\Lambda)$$
$$+|g_v|^2\langle i\gamma_5\mathcal{Y}_{I\Lambda-1}\rangle^*\langle i\gamma_5\mathcal{Y}_{I'\Lambda-1}\rangle b_3(\Lambda-1)\}$$
$$+2\sum_{\Lambda'=\Lambda\pm1}\Big\{[g_v^*g_a\langle Y_\Lambda\rangle^*\langle i\gamma_5Y_{\Lambda'}\rangle$$
$$+g_p^*g_v\langle i\gamma_4\gamma_5Y_{\Lambda'}\rangle^*\langle Y_\Lambda\rangle]\,b_4(\Lambda,\ \Lambda')$$
$$+\sum_I[-|g_v|^2\langle Y_\Lambda\rangle^*\langle i\gamma_5\mathcal{Y}_{I\Lambda'}\rangle+|g_a|^2\langle i\gamma_5Y_{\Lambda'}\rangle^*\langle\mathcal{Y}_{I\Lambda}\rangle$$
$$+g_p^*g_a\langle i\gamma_4\gamma_5Y_{\Lambda'}\rangle^*\langle\mathcal{Y}_{I\Lambda}\rangle]\,b_5(\Lambda,\Lambda')$$
$$+\sum_{II'}g_v^*g_a\langle i\gamma_5\mathcal{Y}_{I\Lambda'}\rangle^*\langle\mathcal{Y}_{I'\Lambda}\rangle b_6(\Lambda,\ \Lambda')\Big\}\Big),\qquad(3b)$$

where Λ is the orbital angular momentum of the neutrino, and the index I takes the values Λ, $\Lambda\pm1$.

The quantity A is, up to a constant factor, the total probability for the process under consideration. The abovementioned nuclear matrix elements and the coefficients a_n, b_n are given in the Appendix.

Formulas (3) apply to the case when the neutrino is emitted with the definite angular momentum $\Lambda=\Lambda_{min}$, the smallest possible according to the selection rules. Here the values of Λ_{min} for relativistic transitions (whose matrix elements contain γ_5) differ by ±1 from the values of Λ_{min} for nonrelativistic transitions between the same states. Strictly speaking, the expressions for A and B should be summed over $\Lambda\ge\Lambda_{min}$, since the condition $k_\nu R\ll1$ is not satisfied in the μ^- capture owing to the large energy released (k_ν is the wave number of the emitted neutrino, and R is the radius of the nucleus). However, various estimates and actual calculations show[3] that the probability for the emission of the neutrino with $\Lambda=\Lambda_{min}$ is considerably higher than the probability for the emission of the neutrino with larger angular momenta.

It should be noted that formulas (3) contain Gell-Mann's[4] correction to the allowed transitions on account of the "weak magnetism." In order to see this, we consider the transition $\Delta j\equiv j_2-j_1=\pm1$ (no). It is mainly due to the forbidden axial-vector interaction (matrix element $\langle\mathcal{Y}_{10}\rangle\sim\langle\sigma\rangle$). The correction caused by the "weak magnetism" is determined by the matrix element $\langle i\gamma_5\mathcal{Y}_{11}\rangle$. Indeed, the total matrix element for the μ transition, M_V, corresponding to the nuclear matrix element $\langle i\gamma_5\mathcal{Y}_{11}\rangle$ in the first approximation in $(v/c)_{nucl}$ and neglecting terms containing $(k_\nu r)^3$, can be written in the form (up to constant factor)

$$M_v\sim g_v\sum_m\langle i\gamma_5\mathcal{Y}^*_{11m}\rangle(\bar{u}_\nu i[1-\gamma_5]\mathcal{Y}_{11m}(\mathbf{k}_\nu/k_\nu)u_\mu)$$
$$\sim\frac{\mu g_v\hbar}{2Mc}\int dV\psi^*_{j_2}\{(-i[\mathbf{r}\times\nabla]+\sigma)\,\mathbf{curl\ A}\}\psi_{j_1},$$
$$\mathbf{A}=(\mathbf{k}_\nu\mathbf{r})(\bar{u}_\nu[1-\gamma_5]\sigma u_\mu),\qquad(4)$$

where M is the mass of the nucleon, and u_ν and u_μ are the Dirac bispinors for the neutrino and the μ^- meson. We see that the structure of M_V is analogous to that of the energy operator for the interaction of a magnetic moment with the magnetic field. The quantity μ, the total magnetic moment for the transition in units of a nuclear magneton, takes, according to Gell-Mann,[4] account of the virtual π mesons. It is, however, easy to show that for transitions of the type $\Delta j=\pm1$ (no) the corrections for "weak magnetism" and other relativistic corrections of the same order of smallness [of first order in $(v/c)_{nucl}$] do not affect the polarization of the γ rays, although they contribute to the total probability of the process. The problem of such corrections in the μ^- capture was treated in more detail by Ioffe.[5]

In deriving the expression for C_γ we took into account the effect of the hyperfine splitting of the levels of the mesic atom, which plays an essential role in the process under consideration. In particular, in transitions obeying the Fermi selection rules the circular polarization of the γ quanta is entirely due to the presence of the hyperfine interaction which leads to the polarization of the nucleus in the intermediate state. In this case the circular polarization may have a considerable magnitude. For example, for $j_1=\frac{1}{2}$, $\Lambda=0$ ("allowed" transition), and $J=1$ (dipole γ quantum) we have for a pure Fermi transition

$$\alpha=\begin{cases}{}^1/_2, & j_3={}^1/_2\\ -{}^1/_4, & j_3={}^3/_2.\end{cases}$$

Without account of the hyperfine interaction we would have obtained $\alpha=0$. As another example we consider an allowed transition followed by dipole radiation ($\Lambda=0$, $J=1$) with $j_1=j_2=j_3=\frac{1}{2}$ and Gamow-Teller coupling. Neglecting the hyperfine structure we obtain $\alpha={}^2/_3$, while its inclusion leads to $\alpha={}^1/_6$. The maximal value of α is $\alpha_{max}=1$, corresponding to $|(C_\gamma)_{max}|=P_\mu\approx15$ to 20%.[6] A polarization of such magnitude occurs, for example, in the transition $0 \to 1 \to 0$ with $\Lambda = 0$ and $J = 1$. The formulas for A and B become much simpler for $j_1 = 0$, when there is no hyperfine splitting. The expressions for the quantities a_n and b_n for $j_1 = 0$ and also for $\Lambda = 0$ ("allowed" transitions) are given in the Appendix.

The occurrence of the circular polarization of the γ rays in the process under consideration is not connected with the nonconservation of spatial parity in the interaction $(\bar{n}p)(\bar{\nu}\mu)$. The measurement of C_γ cannot, therefore, serve to determine the handedness of the neutrino emitted in the μ^- capture. On the other hand, it is precisely this circumstance which makes possible the independent determination of the sign of the longitudinal polarization of the μ^- meson via the quantity C_γ.

The measurement of C_γ is best performed for nuclei with zero spin, since the hyperfine interaction leads to an additional depolarization of the μ^- mesons. Besides this, the lifetime of the μ^- mesons in mesoatomic orbits is larger than $\hbar/E_{hyp}$, where E_{hyp} is the shift of the mesoatomic levels due to the hyperfine interaction. The μ^- meson, therefore, will interact with the spin of the nucleus until its transition to the K orbit (unless, of course, it is captured immediately in the K orbit). This implies that at the instant of the entrance of the μ^- meson in the K orbit the nucleus may be partly polarized, which was not taken into account in the derivation of (3). It is very difficult to take this effect into account consistently (see reference 7).

It should be noted that the isotopic spin selection rules ($\Delta T = 0$ for Fermi coupling, $\Delta T = 0, \pm 1$ for Gamow-Teller coupling) are important for the μ^- capture by light nuclei. Since stable nuclei in the ground state have the lowest possible isotopic spin, and the number of neutrons in them is greater or equal to the number of protons, the μ^- capture, which increases the number of neutrons, will lead to an increase in the isotopic spin. The transitions obeying the Gamow-Teller selection rules will therefore predominate in the μ^- capture by light nuclei with excitation of discrete nuclear levels.

We note that the μ^- capture leads in the majority of cases to the emission of a neutron by the nucleus. Nevertheless, the transition of the nucleus into a discrete state may have a considerable probability. For example, the μ^- capture in C^{12} leads, with a probability of 13%, to the formation of a bound state of B^{12}.[3] The theoretical calculations of the probability for the transition of the nucleus into a particular state are not sufficiently reliable owing to our scarce knowledge of the nuclear wave functions. The observation of the spectrum of the γ rays emitted by the nucleus after the μ^- capture, which must precede the measurement of their circular polarization, is therefore of interest in itself.

APPENDIX

The aforementioned nuclear matrix elements are determined by the following formulas:

$$\langle Y_\Lambda \rangle \equiv \langle j_2 m_2 | j_\Lambda (k_\nu r) R_\mu (r) Y_{\Lambda m} | j_1 m_1 \rangle / C^{j_2 m_2}_{j_1 m_1 \Lambda m};$$

$$\langle \mathcal{Y}_{I\Lambda} \rangle \equiv \langle j_2 m_2 | j_\Lambda (k_\nu r) R_\mu (r) \mathcal{Y}_{I\Lambda m} | j_1 m_1 \rangle | C^{j_2 m_2}_{j_1 m_1 I m}.$$

Here $j_\Lambda(x)$ is the spherical Bessel function;* $R_\mu(r)$ is the normalized wave function of the ground state of the mesic atom; for nuclei with not too large Z we can set $R_\mu(r) \approx \text{const} = R_\mu(0)$; $Y_{\Lambda m}$ is the associated Legendre function,

$$\mathcal{Y}_{I\Lambda m} = \sum_{m_\Lambda, \lambda} C^{Im}_{\Lambda m_\Lambda 1 \lambda} Y_{\Lambda m \Lambda} \sigma_\lambda,$$

$$\sigma_{\pm 1} = \mp (\sigma_x \pm i\sigma_y)/\sqrt{2}, \quad \sigma_0 = \sigma_z,$$

σ_x, σ_y, and σ_z are the Pauli matrices, and $C^{c\gamma}_{a\alpha b\beta}$ are the Clebsch-Gordan coefficients.

The matrix elements for the relativistic transitions, containing the matrix $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, are defined in a similar manner:

$$a_1(\Lambda, \Lambda') = (-)^{(\Lambda+\Lambda'+1)/2} \sqrt{(2\Lambda+1)/3}\, C^{10}_{\Lambda 0 \Lambda' 0};$$

$$a_2(\Lambda, \Lambda') = (-)^{(\Lambda-\Lambda'+1)/2+I} \sqrt{2} \sqrt{(2\Lambda+1)(2\Lambda'+1)}$$

$$\times C^{10}_{\Lambda 0 \Lambda' 0} W(\Lambda\Lambda' 11;\ 1I).$$

$$b_1(\Lambda) = (-)^{-j_1+j_3+\Lambda+J+1} \sqrt{3/2} \sum_j C_1(j) C_2(j) \delta_{I\Lambda}.$$

$$b_2(\Lambda) = 3(-)^{j_1+j_3+\Lambda+J+1} \sum_j \sqrt{2I+1}\, W$$

$$\times (jj\,{}^1\!/_2\,{}^1\!/_2;\ 1j_1) C_1(j) C_3(j) \delta_{I'\Lambda};$$

$$b_3(\Lambda) = \sqrt{3/2}\,(-)^{-j_1+j_3+I+J+1} \sum_j C_1(j) \{\delta_{II'} C_2(j)$$

$$+ 6(-)^{\Lambda+I+2j_1} \sqrt{(2I+1)(2I'+1)}$$

$$\times W(II'11;\ 1\Lambda) W(jj\,{}^1\!/_2\,{}^1\!/_2;\ 1j_1) C_3(j)\},$$

$$b_4(\Lambda, \Lambda') = (-)^{(\Lambda+\Lambda'-1)/2} \sqrt{(2\Lambda'+1)/3}\, C^{10}_{\Lambda 0 \Lambda' 0} \delta_{I\Lambda} b_2(\Lambda);$$

$$b_5(\Lambda, \Lambda') = (-)^{(\Lambda+\Lambda'+1)/2} \sqrt{(2\Lambda+1)/3}\, C^{10}_{\Lambda 0 \Lambda' 0} \delta_{I'\Lambda'} b_3(\Lambda);$$

*In view of the great magnitude of the released energy in the μ^- capture it is impossible to replace the Bessel function $j_\Lambda(k_\nu r)$ in the nuclear matrix elements by the first term in its expansion in powers of $k_\nu r$, as is usually done in β decay.

$$b_6(\Lambda, \Lambda') = (-)^{(\Lambda+\Lambda'-1)/2}\sqrt{3}\sqrt{(2\Lambda+1)(2\Lambda'+1)}$$

$$\times C^{10}_{\Lambda 0 \Lambda' 0}(-)^{J+j_1+j_3}$$

$$\sum_j C_1(j)\Big\{(-)^{\Lambda+2j_1+1}C_2(j)W(\Lambda\Lambda'11;\ 1I)\delta_{II'}$$

$$+(-)^{I'}\frac{\sqrt{(2I+1)(2I'+1)}}{2\Lambda'+1}$$

$$\times[\delta_{I'\Lambda'}+6(2\Lambda'+1)W(\Lambda\Lambda'11;\ 1I')W(II'11;\ 1\Lambda')]$$

$$\times C_3(j)W(jj\,{}^1\!/_2\,{}^1\!/_2;\ 1j_1)\Big\}.$$

$$C_1(j) = (2j_2+1)(2j+1)^2\sqrt{(2J+1)/J(J+1)}$$

$$\times W(JJj_2j_2;\ 1j_3)$$

$$C_2(j) = W(jjj_1j_1;\ 1\,{}^1\!/_2)W(jj\,{}^1\!/_2\,{}^1\!/_2;\ 1j_1)W(j_1j_1j_2j_2;\ 1I),$$

$$C_3(j) = \sum_f(2f+1)W(II'\,{}^1\!/_2\,{}^1\!/_2;\ 1f)W(jj_1fI,\ {}^1\!/_2 j_2)$$

$$\times W(jj_1fI';\ {}^1\!/_2 j_2)W(j_2j_2jj;\ 1f).$$

In these formulas W(abcd; ef) is a Racah coefficient. If $j_1 = 0$, we have (a_n^0, b_n^0 are the values of a_n, b_n for $j_1 = 0$):

$$a_1^0(\Lambda, \Lambda') = a_1(\Lambda, \Lambda'); \quad a_2^0(\Lambda, \Lambda') = a_2(\Lambda, \Lambda'),$$

$$b_1^0(\Lambda) = b_4^0(\Lambda) = 0;$$

$$b_2^0(\Lambda) = (-)^{j_2+j_3+J+1}\sqrt{2j_2+1}\sqrt{\frac{2J+1}{J(J+1)}}$$

$$\times W(JJj_2j_2;\ 1j_3)\delta_{\Lambda j_2}\delta_{Ij_2};$$

$$b_3^0(\Lambda) = \sqrt{6}(-)^{\Lambda+J+j_3+1}(2j_2+1)\sqrt{\frac{2J+1}{J(J+1)}}W(j_2j_211;\ 1\Lambda)$$

$$\times W(JJj_2j_2;\ 1j_3)\delta_{Ij_2}\delta_{I'j_2};$$

$$b_5^0(\Lambda, \Lambda') = (-)^{(\Lambda+j_2+1)/2}\sqrt{(2\Lambda+1)/3}\,C^{10}_{\Lambda 0 j_2 0}\delta_{I'j_2}b_3^0(\Lambda);$$

$$b_6^0(\Lambda, \Lambda') = (-)^{(\Lambda+\Lambda'-1)/2}\sqrt{\frac{2\Lambda+1}{3(2\Lambda'+1)}}$$

$$\times C^{10}_{\Lambda 0 \Lambda' 0}(-)^{J+j_2+j_3}(2j_2+1)\sqrt{\frac{2J+1}{J(J+1)}}W(JJj_2j_2;\ 1j_3)$$

$$\times[\delta_{j_2\Lambda'}+6(2\Lambda'+1)W(\Lambda\Lambda'11;\ 1j_2)\ W(j_2j_211;\ 1\Lambda')].$$

If simultaneously $j_1 = 0$ and $j_2 = 0$, then

$$a_1^0(0,1) = -1/\sqrt{3}, \quad a_2^0(0,1) = -\sqrt{\tfrac{2}{3}},$$

$$b_1^0(0) = b_2^0(0) = b_4^0(0) = 0;$$

$$b_3^0(0) = \sqrt{6}(-)^{j_3+J}\sqrt{\frac{2J+1}{J(J+1)}}W(JJ11;\ 1j_3)\delta_{I1}\delta_{I'1};$$

$$b_5^0(0,1) = b_3^0(0)/\sqrt{3},$$

$$b_6^0(0,1) = 2(-)^{J+j_3+1}\sqrt{\frac{2J+1}{J(J+1)}}W(JJ11;\ 1j_3)\delta_{I1}\delta_{I'1}.$$

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