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QUANTUM THEORY OF SPATIAL DISPERSION OF ELECTRIC AND MAGNETIC SUSCEPTIBILITIES

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A general expression is obtained for the electric and magnetic susceptibilities when spatial dispersion is taken into account. It is shown that electromagnetic effects in a uniform medium can be described by means of a conductivity which depends on frequency and wave vector and a magnetic susceptibility which depends only on the wave vector. A universal relation is obtained between the conductivity and the magnetic susceptibility.

In recent years a number of papers have appeared which have been devoted to an investigation of the effects of spatial dispersion in the propagation of electromagnetic waves through matter.[1-9]

In the papers by Ginzburg[2] and Agranovich and Rukhadze[3] the dielectric tensor (with spatial dispersion taken into account) was obtained on the basis of phenomenological considerations. In papers by Shafranov,[4] Drummond,[5] and Klimontovich[7] this tensor was investigated for a classical gas of charged particles. We may note that in the classical analysis it is impossible in principle to take account of the diamagnetic currents. Quantum mechanical expressions for the dielectric tensor have been considered on the basis of particular models by a number of authors.[1,8-10]

In a paper by Nakajima,[11] a method developed by Kubo[12] was used to obtain a general quantum mechanical expression for the current density in the case of a uniform medium. This expression includes the diamagnetic currents. However, Nakajima did not obtain expressions for the magnetic susceptibility and did not relate this quantity to the electrical conductivity. In the present paper we have obtained expressions for the magnetic susceptibility and the conductivity in which spatial dispersion has been taken into account. Certain properties of the magnetization current have been derived and a universal relation has been established between the conductivity and the magnetic susceptibility.

1. AVERAGING OF THE CURRENT DENSITY

We consider a system with a weak electromagnetic field which is described by the vector and scalar potentials $\mathbf{A}(\mathbf{x}, t)$ and $U(\mathbf{x}, t)$. The electromagnetic field is not quantized.

In the approximation in which the field is linear the Hamiltonian of the system can be written in the form

$$\mathcal{H} = \mathcal{H}_0 + V, \tag{1}$$

where $\mathcal{H}_0$ is the Hamiltonian of the unperturbed system and

$$V = \int \rho(\mathbf{x}) U(\mathbf{x}, t)\, d\mathbf{x} - \frac{1}{c}\int J_\mu(\mathbf{x}) A_\mu(\mathbf{x}, t)\, d\mathbf{x}. \tag{2}$$

Here

$$\rho(\mathbf{x}) = \sum_n e_n \delta(\mathbf{x} - \mathbf{x}_n) \tag{3}$$

is the charge density operator and

$$\mathbf{J}(\mathbf{x}) = \sum_n (e_n/2m_n)[\mathbf{p}_n \delta(\mathbf{x} - \mathbf{x}_n) + \delta(\mathbf{x} - \mathbf{x}_n)\mathbf{p}_n] + c \operatorname{curl} \sum_n \boldsymbol{\mu}_n \delta(\mathbf{x} - \mathbf{x}_n) \tag{4}$$

is the current density operator for the unperturbed system. $\mathbf{p}_n$ is the kinetic momentum of a particle with the perturbing field neglected and μ_n is the operator for the inherent magnetic moment of the particle. The summation is taken over all particles of the system.

The system is described by the density matrix F which obeys the equation

$$i\hbar \dot{F} = [(\mathcal{H}_0 + V), F].$$

We assume that the perturbation appears adiabatically and that $F \to F_0$ when $t \to -\infty$, where

$$F_0 = e^{-\beta\mathcal{H}_0}(\mathrm{Sp}\, e^{-\beta\mathcal{H}_0})^{-1}, \qquad \beta = 1/kT.$$

Using a method analogous to that used by Nakajima,[11] we obtain the following expression for the mean value of the change in current density proportional to the external field

$$\langle \Delta \mathbf{j}(\mathbf{x}, t)\rangle = \mathbf{j}^{(1)}(\mathbf{x}, t) + \mathbf{j}^{(2)}(\mathbf{x}, t), \tag{5}$$

where

$$j_\mu^{(1)}(\mathbf{x}, t) = \int_{-\infty}^{t} dt' \int d\mathbf{x}' \, \varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', t, t') E_\nu(\mathbf{x}', t'), \quad (6)$$

$$j_\mu^{(2)}(\mathbf{x}, t) = \int d\mathbf{x}' \, \varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', 0, 0) \frac{1}{c} A_\nu(\mathbf{x}', t) - A_\mu(\mathbf{x}, t) \, \mathrm{Sp}\left\{F_0 \sum_n \frac{e_n}{m_n} \delta(\mathbf{x} - \mathbf{x}_n)\right\}, \quad (7)$$

$$\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', t, t') = \int_0^\beta \mathrm{Sp}\{F_0 J_\nu(\mathbf{x}', t' - i\hbar\lambda) J_\mu(\mathbf{x}, t)\} d\lambda. \quad (8)$$

Equations (5) — (8) becomes Nakajima's Eqs. (7) — (10) if we assume that the medium is uniform and neglect spin terms in the expressions for the current density (4).

2. PROPERTIES OF THE RESPONSE FUNCTION

a) Symmetry Properties

In the representation in which $\mathcal{H}_0$ is diagonal $\varphi_{\mu\nu}$ has the form

$$\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', t, t') = \sum_{mk} e^{-i\omega_{mk}(t-t')} \times J_\nu(\mathbf{x}')_{mk} J_\mu(\mathbf{x})_{km} \frac{e^{-\beta E_k} - e^{-\beta E_m}}{E_m - E_k} (\mathrm{Sp}\, e^{-\beta \mathcal{H}_0})^{-1}. \quad (9)$$

The following properties of $\varphi_{\mu\nu}$ follow directly from this equation:

1) $\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}'; t, t')$

depends only on the time difference $t - t' = \tau$. In what follows we denote this dependence by $\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau)$.

2) $\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau)$ — is real,
3) $\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau) = \varphi_{\nu\mu}(\mathbf{x}', \mathbf{x}, -\tau)$,
4) $\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau, \mathbf{H}) = \varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', -\tau, -\mathbf{H})$.

Here $\mathbf{H}$ is a magnetic field. From 3) and 4) it follows that

5) $\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau, \mathbf{H}) = \varphi_{\nu\mu}(\mathbf{x}', \mathbf{x}, \tau, -\mathbf{H})$.

b) Properties of the Response Function for Coinciding Times

In what follows the following relation, which can be easily verified by direct calculation, will be found useful

$$[\rho(\mathbf{x}'), J_\mu(\mathbf{x})] = -i\hbar (\partial\delta(\mathbf{x} - \mathbf{x}')/\partial x'_\mu) \sum_n (e_n^2/m_n) \delta(\mathbf{x} - \mathbf{x}_n). \quad (10)$$

Whence we obtain

$$\partial\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', 0)/\partial x'_\nu = (\partial\delta(\mathbf{x}' - \mathbf{x})/\partial x'_\mu, \mathrm{Sp})\left\{F_0 \sum_n (e_n^2/m_n) \delta(\mathbf{x} - \mathbf{x}_n)\right\}, \quad (11)$$

so that for any function $g(\mathbf{x}')$ we have

$$\int \varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', 0) \frac{\partial g(\mathbf{x}')}{\partial x'_\nu} dx' = \frac{\partial g(\mathbf{x})}{\partial x_\mu} \mathrm{Sp}\left\{F_0 \sum_n \frac{e_n^2}{m_n} \delta(\mathbf{x} - \mathbf{x}_n)\right\}. \quad (12)$$

Further

$$\int \varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', 0) \, d\mathbf{x}' = \delta_{\mu\nu} \, \mathrm{Sp}\left\{F_0 \sum_n (e_n^2/m_n) \delta(\mathbf{x} - \mathbf{x}_n)\right\}. \quad (13)$$

c) Relation Between the Response Function and the Correlation Functions

Following references 6 and 12, we introduce the current correlation function

$$\psi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau) = \frac{1}{2} \mathrm{Sp}\{F_0 (J_\nu(\mathbf{x}', 0) J_\mu(\mathbf{x}, \tau) + J_\mu(\mathbf{x}, \tau) J_\nu(\mathbf{x}', 0))\}. \quad (14)$$

We denote the Fourier components of the functions $\psi_{\mu\nu}$ and $\varphi_{\mu\nu}$ by $\widetilde{\psi}_{\mu\nu}$ and $\widetilde{\varphi}_{\mu\nu}$, so that for example

$$\widetilde{\psi}_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega) = \frac{1}{2\pi} \int e^{i\omega\tau} \psi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau) \, d\tau.$$

Using Eq. (9) in a way similar to that given in reference 6 we obtain the relation between $\widetilde{\psi}_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega)$ and $\widetilde{\varphi}_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega)$:

$$\widetilde{\varphi}_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega) = \widetilde{\psi}_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega)/E_\beta(\omega). \quad (15)$$

Here $E_\beta(\omega)$ is the mean energy of the oscillator for a given temperature

$$E_\beta(\omega) = (\hbar\omega/2) \coth(\hbar\omega/2kT).$$

3. MAGNETIZATION CURRENT

As is apparent from Eq. (7), the current $\mathbf{j}^{(2)}(\mathbf{x}, t)$ does not contain a delay with respect to the vector potential $\mathbf{A}(\mathbf{x}, t)$. When $\omega = 0$, in the absence of an electric field the total current density of the system reduces to $\mathbf{j}^{(2)}$. Hence, when $\omega = 0$, $\mathbf{j}^{(2)}$ has the significance of a magnetization current. When $\omega \neq 0$ this simple significance no longer holds since $\mathbf{j}^{(1)}$ contains terms which are proportional to the induced electric field and, consequently, are proportional to the magnetic field. However, for convenience we will call the current $\mathbf{j}^{(2)}$ the magnetization current. We may note several of its general properties.

a) The gauge invariance; this is easily demonstrated by means of Eq. (12).

b) div $\mathbf{j}^{(2)} = 0$. This property follows directly from Eq. (11). It allows us to write $\mathbf{j}^{(2)} = c \operatorname{curl} \mathbf{M}(\mathbf{x}, t)$; $\mathbf{M}(\mathbf{x}, t)$ may be called the magnetic moment density of the system. [Actually $\mathbf{M}(\mathbf{x}, t)$ has the meaning of a magnetic moment density only when $\omega = 0$.]

c) Using Eq. (13) we can write $\mathbf{j}^{(2)}$ in the form

$$j_\mu^{(2)}(\mathbf{x}, t) = \frac{1}{c}\int d\mathbf{x}' \cdot \varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', 0)\,[A_\nu(\mathbf{x}', t) - A_\nu(\mathbf{x}, t)]. \quad (16)$$

From Eq. (13) it follows that $\int \mathbf{j}^{(2)}(\mathbf{x}, t)\, d\mathbf{x} = 0$.

d) From Eq. (12) we have

$$\partial j_\mu^{(2)}(\mathbf{x}, t)/\partial t = -\int d\mathbf{x}' \cdot \varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', 0)\,[E_\nu(\mathbf{x}', t) - E_\nu(\mathbf{x}, t)]. \quad (17)$$

e) As is apparent from Eq. (15), in the classical limit

$$\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau) = \beta \langle J_\nu(\mathbf{x}', 0)\, J_\mu(\mathbf{x}, \tau)\rangle.$$

Here the angular brackets denote averages over the classical canonical distribution. In this case a direct calculation yields $\mathbf{j}^{(2)} = c \operatorname{curl} \mathbf{M}_{\mathrm{cl}}$, where

$$\mathbf{M}_{\mathrm{cl}} = \langle \sum_n \boldsymbol{\mu}_n \delta(\mathbf{x} - \mathbf{x}_n) \frac{1}{kT} \sum_m (\boldsymbol{\mu}_m, \mathbf{H}(\mathbf{x}_m))\rangle.$$

$\mathbf{M}_{\mathrm{cl}}$ is the classical paramagnetic moment density of the system.

Thus we obtain a well-known result: the diamagnetic moment computed from classical mechanics is zero.

4. CASE OF AN EXTERNAL FIELD THAT VARIES HARMONICALLY IN TIME

Let $E_\mu(\mathbf{x}, t) = E_\mu(\mathbf{x})e^{-i\omega t}$. Then, using Eq. (11) we have

$$j_\mu^{(1)}(\mathbf{x}, t) = e^{-i\omega t}\int K_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega) E_\nu(\mathbf{x}')\, d\mathbf{x}',$$

$$K_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega) = \int_0^\infty e^{i\omega t}\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau)\, d\tau. \quad (18)$$

Using Eqs. (11), (17), and (18) (with $\omega \neq 0$) we can express the total current density in terms of the electric field:

$$\langle \Delta j_\mu(\mathbf{x}, t)\rangle = e^{-i\omega t}\Big\{E_\nu(\mathbf{x})\int K_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega)\, d\mathbf{x}' + \int K'_{\mu\nu}(\mathbf{x}, \mathbf{x}'\ \omega)\,[E_\nu(\mathbf{x}') - E_\nu(\mathbf{x})]\, d\mathbf{x}'\Big\}. \quad (19)$$

Here

$$K'_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega) = -\frac{1}{i\omega}\int_0^\infty e^{i\omega\tau}\frac{\partial\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau)}{\partial\tau}\, d\tau.$$

The quantity $K'_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega)$ is given by the relation

$$K'_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega) = K_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega) - (i/\omega)\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', 0). \quad (20)$$

The divergence in $K'_{\mu\nu}(\mathbf{x}, \mathbf{x}', \omega)$ when $\omega = 0$ is due to the fact that it is impossible to express the total current density in terms of the electric field alone when $\omega = 0$. In this case it is necessary to use Eqs. (16) and (18).

5. CASE OF A UNIFORM MEDIUM

In the case of a uniform medium $\varphi_{\mu\nu}(\mathbf{x}, \mathbf{x}', \tau)$ depends only on the difference $\mathbf{x} - \mathbf{x}' = \mathbf{r}$ and, for a medium which has a center of inversion, is an even function of $\mathbf{r}$. In what follows, for the case of a uniform medium we will designate the response function by $\varphi_{\mu\nu}(\mathbf{r}, \tau)$:

$$\varphi_{\mu\nu}(\mathbf{r}, \tau) = \int_0^\beta \mathrm{Sp}\,\{F_0 J_\nu(0, -i\hbar\lambda)\, J_\mu(\mathbf{r}, \tau)\}\, d\lambda. \quad (21)$$

We list the symmetry properties of this function which follow from the results of Sec. 2:

1) $\varphi_{\mu\nu}(\mathbf{r}, \tau) = \varphi_{\nu\mu}(-\mathbf{r}, -\tau)$,

2) $\varphi_{\mu\nu}(\mathbf{r}, \tau, \mathbf{H}) = \varphi_{\mu\nu}(\mathbf{r}, -\tau, -\mathbf{H})$,

3) $\varphi_{\mu\nu}(\mathbf{r}, \tau, \mathbf{H}) = \varphi_{\nu\mu}(-\mathbf{r}, \tau, -\mathbf{H})$. (22)

Let

$$E_\mu(\mathbf{x}, t) = E_\mu(\mathbf{k}, \omega)e^{i(\mathbf{kx} - \omega t)} \quad \text{etc.}$$

Then Eq. (6) yields:

$$j_\mu^{(1)}(\mathbf{x}, t) = \sigma_{\mu\nu}(\mathbf{k}, \omega) E_\nu(\mathbf{x}, t). \quad (23)$$

Here

$$\sigma_{\mu\nu}(\mathbf{k}, \omega) = \int_0^\infty e^{i\omega\tau}\int e^{-i\mathbf{kr}}\varphi_{\mu\nu}(\mathbf{r}, \tau)\, d\mathbf{r}\, d\tau. \quad (24)$$

Equation (16) can be written in the form

$$j_\mu^{(2)} = (1/c)\int \varphi_{\mu\nu}(\mathbf{r}, 0)\,[A_\nu(\mathbf{x} - \mathbf{r}) - A_\nu(\mathbf{x})]\, d\mathbf{r},$$

whence, by means of Eq. (12) and the relation $\mathbf{H} = \operatorname{curl} \mathbf{A}$, we have

$$\operatorname{curl}_\mu \mathbf{j}^{(2)}(\mathbf{x}, t) = (1/c)\int \varkappa_{\mu\nu}(\mathbf{r})\,[H_\nu(\mathbf{x} - \mathbf{r}, t) - H_\nu(\mathbf{x}, t)]\, d\mathbf{r}, \quad (25)$$

where

$$\varkappa_{\mu\nu}(\mathbf{r}) = \delta_{\mu\nu}\sum_\alpha \varphi_{\alpha\alpha}(\mathbf{r}, 0) - \varphi_{\mu\nu}(\mathbf{r}, 0).$$

From Eq. (16) and property b) of the current $\mathbf{j}^{(2)}$ (cf. Sec. 3) we have

$$\mathbf{j}^{(2)} = c \operatorname{curl} \mathbf{M}, \qquad M_\mu(x, t) = \chi_{\mu\nu}(\mathbf{k}) H_\nu(\mathbf{x}, t),$$

where

$$\chi_{\mu\nu}(\mathbf{k}) = (kc)^{-2}\int \varkappa_{\mu\nu}(\mathbf{r})(e^{-i\mathbf{kr}}-1)\,d\mathbf{r}. \quad (26)$$

It is clear from these relations that to describe electromagnetic effects in uniform media it is sufficient to introduce a conductivity $\sigma_{\mu\nu}(\mathbf{k}, \omega)$ which depends on ω and a magnetic susceptibility $\chi_{\mu\nu}(\mathbf{k})$ which is independent of ω.

When $\omega \neq 0$, in general it is not possible to introduce a magnetic susceptibility. From Eq. (19) we obtain the following expression for the total current density:

$$\langle \Delta j_{\mu}(\mathbf{x}, t)\rangle = [\sigma_{\mu\nu}(0, \omega) + \sigma'_{\mu\nu}(\mathbf{k}, \omega)]\, E_{\nu}(\mathbf{x}, t).$$

Here

$$\sigma_{\mu\nu}(0, \omega) = \int_0^\infty e^{i\omega\tau}\,d\tau \int \varphi_{\mu\nu}(\mathbf{r}, \tau)\,d\mathbf{r},$$

$$\sigma'_{\mu\nu}(\mathbf{k}, \omega) = -\frac{1}{i\omega}\int_0^\infty e^{i\omega\tau}\,d\tau \int \frac{\partial\varphi_{\mu\nu}(\mathbf{r}, \tau)}{\partial\tau}(e^{-i\mathbf{kr}}-1)\,d\mathbf{r}. \quad (27)$$

The Kramers-Kronig relation and the fluctuation-dissipation theorem can be obtained from the following formula

$$\sigma_{\mu\nu}(\mathbf{k}, \omega) = \frac{\pi}{E_{\beta}(\omega)}\widetilde{\Phi}_{\mu\nu}(\mathbf{k}, \omega) + i\int_{-\infty}^{\infty}\frac{1}{E_{\beta}(\omega')}\frac{\widetilde{\Phi}_{\mu\nu}(\mathbf{k}, \omega')}{\omega - \omega'}\,d\omega'. \quad (28)$$

This follows directly from Eqs. (15) and (24) where

$$\widetilde{\Phi}_{\mu\nu}(\mathbf{k}, \omega) \equiv \int e^{-i\mathbf{kr}}\widetilde{\Phi}_{\mu\nu}(\mathbf{r}, \omega)\,d\mathbf{r}.$$

We may note the symmetry properties of $\sigma_{\mu\nu}$ and $\chi_{\mu\nu}$ which follow from Eq. (19) [the same properties are possessed by $\sigma'_{\mu\nu}(\mathbf{k}, \omega)$]:

$$\sigma^{*}_{\mu\nu}(\mathbf{k}, \omega) = \sigma_{\mu\nu}(-\mathbf{k}, -\omega),\ \ \sigma_{\mu\nu}(\mathbf{k}, \omega, \mathbf{H}) = \sigma_{\nu\mu}(-\mathbf{k}, \omega, -\mathbf{H}). \quad (29)$$

The last relation is the Onsager relation. If there is a center of inversion

$$\varphi_{\mu\nu}(\mathbf{r}, \tau) = \varphi_{\mu\nu}(-\mathbf{r}, \tau),\ \ \sigma_{\mu\nu}(\mathbf{k}, \omega) = \sigma_{\mu\nu}(-\mathbf{k}, \omega). \quad (30)$$

Similar properties are possessed by $\chi_{\mu\nu}(\mathbf{k})$.

The properties in (29) are a particular case of the general symmetry properties of the kinetic coefficients.[6]

The tensor $\chi_{\mu\nu}$ is related to the tensor $\sigma_{\mu\nu}$ by the following relation:

$$\chi_{\mu\nu}(\mathbf{k}) = \frac{1}{2\pi k^2c^2}\int_{-\infty}^{\infty} d\omega\Big\{\delta_{\mu\nu}\sum_{\alpha}[\sigma^{s}_{\alpha\alpha}(\mathbf{k}, \omega) - \sigma^{s}_{\alpha\alpha}(0, \omega)] - [\sigma^{s}_{\mu\nu}(\mathbf{k}, \omega) - \sigma^{s}_{\mu\nu}(0, \omega)]\Big\}. \quad (31)$$

Here

$$\sigma^{s}_{\mu\nu}(\mathbf{k}, \omega) = \sigma_{\mu\nu}(\mathbf{k}, \omega, \mathbf{H}) + \sigma_{\mu\nu}(\mathbf{k}, -\omega, -\mathbf{H}),$$

H is the magnetic field in the unperturbed Hamiltonian.

Thus the magnetic susceptibility is closely related to the spatial dispersion of the conductivity.

In the expression $j_{\mu} = ic\epsilon_{\mu\nu l}k_{\nu}\chi_{ln}H_n$ the components χ_{ln} do not appear when l or n are equal to z if **k** is directed along the z-axis. Hence the longitudinal part of $\chi_{ln}(\mathbf{k})$ has no physical significance.

For a given direction of the vector **k**, when $k \to 0$, we can determine from Eq. (26) only four (transverse) components of the magnetic susceptibility tensor $\chi_{ln}(0)$ for a uniform magnetic field. In order to obtain all ten components of this tensor it is necessary to use Eq. (26) with $k \to 0$ for three mutually perpendicular directions of **k**. In this case the diagonal components of the tensor χ_{ln} appear twice; for example the component χ_{xx} is obtained when **k** is along the y axis or along the z axis. In the first case

$$\chi_{xx} = -\frac{1}{2c^2}\int \varkappa_{xx}y^2 d\mathbf{r} = -\frac{1}{2c^2}\int \varphi_{zz}y^2\,d\mathbf{r},$$

and in the second case

$$\chi_{xx} = -\frac{1}{2c^2}\int \varkappa_{xx}z^2 d\mathbf{r} = -\frac{1}{2c^2}\int \varphi_{yy}z^2 d\mathbf{r}.$$

These integrals are equal because of the relation

$$\sum_{\mu}\int \varphi_{x\mu}(\mathbf{r}, 0)\nabla_{\mu}(xy^2)\,d\mathbf{r} = \sum_{\mu}\int \varphi_{y\mu}(\mathbf{r}, 0)\nabla_{\mu}(y^2x)\,d\mathbf{r} = 0,$$

which follows from Eq. (12).

A comparison of the results obtained for $k \to 0$ for three mutually perpendicular directions of **k** leads to the following expression for the complete tensor $\chi_{\mu\nu}(0)$:

$$\chi_{\mu\nu}(0) = -\frac{1}{2c^2}\int\Big[\frac{1}{2}\delta_{\mu\nu}\sum_{\alpha}\varphi_{\alpha\alpha}(\mathbf{r}, 0) - \varphi_{\mu\nu}(\mathbf{r}, 0)\Big]r^2 d\mathbf{r}. \quad (32)$$

In the case of a fixed uniform (i.e., varying only slightly in a distance of the order of a correlation radius) field the magnetic susceptibility $\chi_{ln}(0)$ is related to the specific magnetic moment by the relation $\chi_{ln}(0) = \partial M_l(\mathbf{H})/\partial H_n$, where **H** is the magnetic field in the unperturbed Hamiltonian.

In this work we have used the conventional scheme of a self-consistent field in which the remote interactions are included in the macroscopic electromagnetic field which satisfy Maxwell's equations; only the near interactions appear in the unperturbed Hamiltonian. In this scheme the electromagnetic fields **E** and **H** are the resultants of the fields of the external sources and the fields produced by the charges and currents of the system. However, it is possible to include all interactions between the particles of a system in the unperturbed Hamiltonian; in this case, by **E** and **H** we are to understand the fields of the external sources.

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[1] S. I. Pekar, JETP **33**, 1022 (1957), Soviet Phys. JETP **6**, 785 (1958).

[2] V. L. Ginzburg, JETP **34**, 1593 (1958), Soviet Phys. JETP **7**, 1096 (1958).

[3] V. M. Agranovich and A. A. Rukhadze, JETP **35**, 982, 1171 (1953), Soviet Phys. JETP **8**, 819 (1959).

[4] V. D. Shafranov, JETP **34**, 1475 (1958), Soviet Phys. JETP **7**, 1019 (1958).

[5] J. E. Drummond, Phys. Rev. **110**, 293 (1958).

[6] L. D. Landau and E. M. Lifshitz, Электродинамика сплошных сред, (Electrodynamics of Continuous Media) Gostekhizdat, 1957.

[7] Yu. L. Klimontovich, JETP **34**, 173 (1958), Soviet Phys. JETP **7**, 119 (1958).

[8] G. E. H. Reuter and E. H. Sondheimer, Proc. Roy. Soc. **A195**, 336 (1948).

[9] F. G. Bass and M. I. Kaganov, JETP **34**, 1154 (1958), Soviet Phys. JETP **7**, 799 (1958).

[10] J. Lindhard, Det. Kong. Dan. Vid. Mat.-fys. Medd. **28**, 8 (1954).

[11] S. Nakajima, Proc. Phys. Soc. **69A**, 441 (1956).

[12] R. Kubo, J. Phys. Soc. (Japan) **12**, 570 (1957).

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