ELECTRON-ELECTRON SCATTERING AND QUANTUM ELECTRODYNAMICS AT SMALL DISTANCES

G. V. AVAKOV

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THE most direct experiments that could give us information about the electron would be experiments on the scattering of electrons by electrons. Assuming that the interaction is of vector type and gauge invariant up to sufficiently high energies, we can determine the electric and magnetic form-factors of the electron by a comparison of the theoretical formula with the experimental data. We note that when we take the electric form-factor into account we must also regard the magnetic form-factor of the electron as nonvanishing. Then the vertex operator Γ_μ corresponding to a vertex in the Feynman diagram for the scattering will in the most general case have the well known form[1-3]

$$\Gamma_\mu = a(q^2)\gamma_\mu + \gamma_\mu\gamma_\nu \frac{q_\nu}{\sqrt{-q^2}} b(q^2), \tag{1}$$

where the invariant functions $a(q^2)$ and $b(q^2)$ of the square of the four-momentum transfer ($q^2 = q_0^2 - \mathbf{q}^2$) are respectively the electric and magnetic form-factors.

Omitting the cumbersome calculations, we present the formula for the scattering cross section in the center-of-mass system of the colliding electrons:

$$\begin{aligned}\frac{d\sigma}{\sigma_0 d\Omega} &= f^2(q^2)\frac{2+2x+x^2}{8x^2} + f(q^2)f(p^2)\frac{(1+x)^2}{4x} \\ &+ f^2(p^2)\frac{1+2x+2x^2}{8} + \varepsilon^4\frac{\varphi^2(q^2)}{q^4}\frac{(2+x)^2}{(1+x)^2} \\ &+ \varepsilon^4\frac{\varphi(q^2)\varphi(p^2)}{q^2p^2}\frac{2+2x^2+5x}{(1+x)^2} + \varepsilon^4\frac{\varphi^2(p^2)}{p^4}\frac{(2x+1)^2}{(1+x)^2} \\ &+ 2\varepsilon^2 f(q^2)\frac{\varphi(q^2)}{-q^2}\frac{1}{x} + 2\varepsilon^2 f(p^2)\frac{\varphi(p^2)}{-p^2}x \\ &+ \varepsilon^2 f(q^2)\frac{\varphi(p^2)}{-p^2}\frac{2+6x+3x^2}{2x(1+x)^2} \\ &+ \varepsilon^2 f(p^2)\frac{\varphi(q^2)}{-q^2}\frac{x(2x^2+6x+3)}{2(1+x)^2},\end{aligned} \tag{2}$$

where $\sigma_0 = e^4/16\varepsilon^2$; $e^2 = 1/137$; $f = a^2$, $\varphi = b^2$; $x = \tan^2(\vartheta/2)$,

$$q^2 = -4\varepsilon^2 x/(1+x) = -4\varepsilon^2\sin^2(\vartheta/2),$$

$$p^2 = -4\varepsilon^2/(1+x) = -4\varepsilon^2\cos^2(\vartheta/2),$$

ϵ is the energy of the electron, and ϑ is the scattering angle.

In (2) we can make the replacements $x = q^2/p^2$, $\epsilon^2 = -(q^2+p^2)/4$. Then on the right side we have a function of q^2 and p^2 only. Equation (2) actually contains four unknown functions (since a single form-factor for different values of the argument amounts to different unknowns). By varying x and ϵ we can get six quadratic equations with the six unknowns $f(p_i)$ and $\varphi(p_i)$. In fact, by changing x and ϵ in a special way through six pairs of values, we get the table of values of q^2 and p^2:

p_1^2, p_1^2	p_1^2, p_2^2	p_1^2, p_3^2
	p_2^2, p_2^2	p_2^2, p_3^2
		p_3^2, p_3^2.

Here the values on the left are those of q^2, and those on the right, of p^2. The first element in the table, p_1^2, p_1^2 (which means a choice of x and ϵ such that $q^2 = p^2 = p_1^2$), gives an equation with two unknowns. The second element, p_1^2, p_2^2, is obtained by changing p^2 with constant q^2. This adds one more equation and increases the number of unknowns by two. The element p_1^2, p_3^2 does the same. It is easy to see that the remaining elements of the table give new equations but do not form new unknowns.

After finding the functions φ and f, we can compare their ratio with the value of μ obtained from the scattering of electrons by α particles.[4]

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