rays exhibits the same anomaly[8] as the chemical composition of certain types of non-stationary stars.

The author is deeply indebted to Academician V. A. Ambartsumyan for a fruitful discussion of this problem and for valuable advice.

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THE LANDAU CORRECTION COEFFICIENT IN THE DETERMINATION OF THE VISCOSITY OF A LIQUID

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THE solution of the Navier-Stokes equation for a circular disc undergoing torsional vibrations in its own plane in an unbounded liquid yields the following expression for the coefficient of viscosity of a liquid

$$\eta = 4I^2(\gamma - \gamma_0)^2 \theta / \pi^3 R^8 \rho N^2, \quad (1)$$

where I is the moment of inertia of the disc, R is its radius, θ is the period of vibration of the disc in the liquid, ρ is the density of the liquid, N is the number of discs entering into the system, and γ and γ_0 are the damping coefficients of the disc in the liquid under study and in a vacuum. Equation (1) was obtained under the approximations $\gamma/\omega \ll 1$, $R/\lambda \gg 1$, $\theta_0/\theta \approx 1$, where θ_0 is the period of rotation of the disc in a vacuum. The difference $\gamma - \gamma_0$ is the proper absorption coefficient, whose presence is brought about by the action of the liquid on the upper and lower surface of the disc only. In view of the fact that in the derivation of (1) edge effects were not taken into account (in particular, the effects of the liquid on the lateral surface of the disc), they should be excluded in some way or other.

In the determination of the viscosity by means of (1), L. D. Landau introduced a correction coefficient and the equation for η was written in the form

$$\eta = 4I^2(\gamma - \gamma_0)^2 \theta / \pi^3 R^8 \rho N^2 (1 + 2d/R + 2\lambda/R)^2. \quad (2)$$

Here d is the thickness of the disc used in the systems, and λ is the penetration depth of the viscous wave.

In this paper, an experimental method is described for the measurement of the viscosity of a liquid by means of rotating discs which excludes the action of viscous forces on the lateral surface of the disc without the introduction of any correction coefficients. A test of the method was carried out on measurements in helium II.

The essence of the experiment is as follows. A compound disc of thickness D was divided parallel to the plane of the characteristic oscillations into two, three, etc., parts which formed a rather complicated but nonetheless single oscillatory system. Depending on into how many parts the disc was divided, several oscillatory systems with the same moment of inertia and the same lateral surfaces were obtained; however, in each individual case there was a different number of discs (N = 1, 2, 3, 6), in the system. The discs in this case were separated from each other by distances $l \gg \lambda$, where λ is the penetration depth.

If we determine the damping coefficients γ_N and γ_0 for these systems, both in helium II and in vacuo, and compute the expression $(\gamma_N - \gamma_0)/N$ for each system, then this expression, in view of the additivity of the damping, should give the total value of the damping (brought about both by the front surfaces and the lateral surfaces) of the individual disc entering into the various systems. Since the thickness of the individual discs $d_N = D/N$ is different in different systems, the ratios obtained from their values must differ from one another.

If we plot $(\gamma_N - \gamma_0)/N$ vs. d_N for a given temperature, and extrapolate this curve to $d_N = 0$, we obtain the damping brought about by the action of the liquid alone on the upper and lower surfaces of a single disc, i.e., the difference $(\gamma - \gamma_0)_{d=0}$. Knowing $\gamma - \gamma_0$, and making use of Eq. (1), it is possible to construct the curve for the temperature dependence of the viscosity.

Values are given in Fig. 1 for the viscosity coefficient η_n of the normal component of helium II for different temperatures, obtained from (1) upon substitution in it of our measured values of $(\gamma - \gamma_0)_{d=0}$ and the values of ρ_n taken from the work of Andronikashvili.[2] The values of η_n, taken from the work of Andronikashvili,[1] which were obtained by him according to (2), are plotted in the same figure (the solid curve in the drawing was drawn through the mean values).

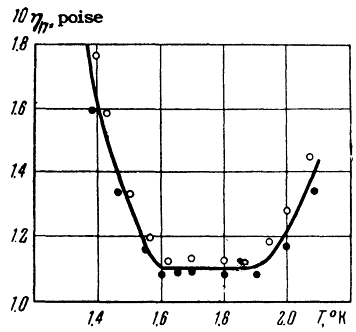
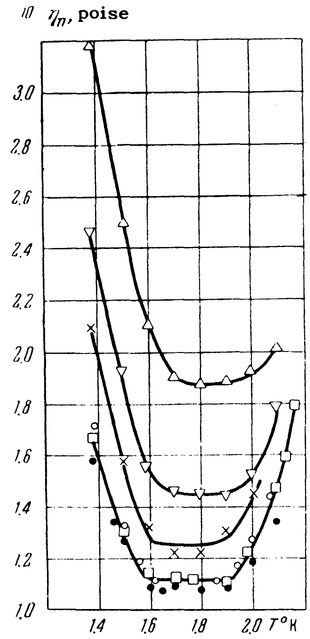


FIG. 1. Temperature dependence of the viscosity of the normal component of helium II. o – data of Andronikashvili, • – data obtained according to Eq. (1).

It is seen from Fig. 1 that within the limits of error of the experiment (5 — 7%), the values of the coefficient of viscosity obtained by the usual method are in excellent agreement with one another, which also indicates the validity of the application of the Landau correction coefficient for thin discs. At the same time this circumstance again confirms the reliability of the temperature curve $\eta_n(T)$ given by Andronikashvili.

We now investigate up to what upper limit of the ratio $(d+\lambda)/R$ does the correction coefficient remain applicable. For this purpose we find the dependence $\eta_n(T)$ from the measured values of γ_N for all four systems, by means of (2), making use of the method of successive approximations (see Fig. 2). In the calculations it was taken into consideration that the number of discs in the first system $N = 1$ and the thickness $d_1 = 0.276$ cm; in the second case, $N = 2$ and $d_2 = 0.138$ cm; in the third $N = 3$ and $d_3 = 0.092$ cm; in the fourth $N = 6$ and $d_6 = 0.046$ cm.

FIG. 2. The temperature dependence of the viscosity of the normal component of helium II, computed by Eq. (2) for different values of $(d+\lambda)/R$; Δ – 0.19 to 0.25, ∇ – 0.10 to 0.16, × – 0.07 to 0.13, □ – 0.05 to 0.1, o – data of Andronikashvili, • – data obtained from Eq. (1).



As is seen from Fig. 2, the temperature dependence of the coefficient of viscosity is strongly dependent on the value of $(d+\lambda)/R$. In this case the correction coefficient gives an excellent result only for the fourth system; therefore the concept of the thin disc must be limited to the values $(d+\lambda)/R < 0.05$ to 0.1. It is necessary to note that the experimental method of measurement of η_n just described does not include that part of the damping brought about by the presence of "angles" of the disc, which are formed by the front and lateral surfaces, i.e., the corner effect. But, since at all temperatures the values of the viscosity coefficient determined by the method of elimination of the effect of lateral surface are in excellent agreement with data obtained by Eq. (2) for thin discs, one must regard the correction for the corner effect to be negligibly small.

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