TAKING ACCOUNT OF THE GRAVITATIONAL ENERGY

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EVER since the formulation of the general theory of relativity physicists have encountered serious difficulties in defining the energy and momentum of the gravitational field, needed, in particular, to take account of the transformation of these quantities into the energy and momentum of other fields. This problem has to do with the circumstance that in the formulation of the theory the equation of continuity acquires the physical meaning of a conservation law through the vanishing of the usual, not the covariant divergence. Here we call the conservation law corresponding to the vanishing of the usual divergence, the exact conservation law. The exactly conserved "pseudo-tensor" of the energy-momentum density of the gravitational field, introduced by Einstein, is, on one hand, not a generally covariant quantity; on the other hand, the energy defined in terms of it depends in an essential way on the choice of the reference systems connected with the purely spatial coordinate transformations with no change in the time coordinate. Definitions of the energy-momentum density of the gravitational field different from this have therefore been proposed, in particular by Lorentz and Levi-Civita. However, the exactly conserved tensor for all fields including the gravitational, proposed by these authors, vanishes at all points of space-time and cannot, according to Einstein's equations, have any deep physical meaning (see, for example, reference 1).

The problem of the definition of energy and momentum in the general theory of relativity has begun to be discussed again recently in the literature. Møller, in his recent papers,[2,3] has derived a new expression* for the total energy of a system of gravitational and other fields, avoiding the abovementioned difficulty concerning the critical dependence of the total energy on the spatial reference systems. At the same time, the non-tensor character of the energy-momentum density (it is an affine tensor density) turns out to be essential to the problem if only for the fact that even from a tensor quantity it is impossible to obtain a genuine vector by integration. Møller showed the uniqueness of his results from the requirement that the energy of the system be covariant with respect to purely spatial coordinate transformations and from the postulate that it can be expressed in terms of the metric tensor and its first and second derivatives.

On the other hand, our earlier formulation of Noether's theorem, when applied to the gravitational field,[4] leads to conserved quantities different from those of Einstein; as we shall show, these are closely connected with the "pseudo-tensor" derived later by Møller. For this we use a unified treatment of the gravitational and other fields to determine, for example, the conserved quantities, in particular, the canonical quasi-tensor of the energy-momentum density and the spin part of the energy-momentum. The latter is particularly important in the case of gravitation. Indeed, the gravitational field is described by a metric tensor, so that there must be particles with spin corresponding to it; it is known that in the weak field approximation the spin of these is equal to two. With the help of the expressions obtained by Møller[2] and Mitskevich[4] one easily establishes the relation

$$\mathfrak{T}^{\alpha}_{\beta} = -\,\mathfrak{U}^{\alpha}_{\beta}(\mathrm{grav}), \tag{1}$$

where $\mathfrak{T}^{\alpha}_{\beta}$ is the "pseudo-tensor" of energy-momentum of the total system of fields, introduced by Møller, and $\mathfrak{U}^{\alpha}_{\beta}(\mathrm{grav})$ is the expression for the spin part of the energy of the gravitational field, as found by us. It is also easily seen from the general derivation of the spin part of the energy-momentum[4] that this quantity has the necessary transformation properties (invariance of the integrated energy under purely spatial transformations which leave the time coordinate unchanged) also for other fields. It should be noted that the symmetric tensor found by us coincides with the expressions of Lorentz and Levi-Civita and is, owing to Einstein's equations, identically zero for the total system of fields. We therefore have the following relation

$$\mathfrak{T}^{\alpha}_{\beta}(\mathrm{sym})(\mathrm{tot}) = \mathfrak{t}^{\alpha}_{\beta}(\mathrm{tot}) + \mathfrak{U}^{\alpha}_{\beta}(\mathrm{tot}) = 0. \tag{2}$$

The canonical quasi-tensor of energy-momentum for the total system of fields, $\mathfrak{t}^{\alpha}_{\beta}$ (derived in reference 4), therefore also leads to the solution of the aforementioned problem of the determination of the total energy in the presence, and with account, of gravitation, for we can write, according to (2),

$$\mathfrak{t}^{\alpha}_{\beta}(\mathrm{f}) + \mathfrak{t}^{\alpha}_{\beta}(\mathrm{grav}) = -\,(\mathfrak{U}^{\alpha}_{\beta}(\mathrm{f}) + \mathfrak{U}^{\alpha}_{\beta}(\mathrm{grav})), \tag{3}$$

where $\mathfrak{t}^{\alpha}_{\beta}(\mathrm{f})$ and $\mathfrak{t}^{\alpha}_{\beta}(\mathrm{grav})$ refer, respectively, to the ordinary fields in the presence of gravitation and to the purely gravitational field. In particular, if ordinary matter is absent, we obtain a noteworthy relation which establishes the equality (except for the sign) of the spin part of the energy-momentum of the gravitational field and the canonical quasi-tensor of the energy-momentum density of gravitation, introduced by us and also obtained by Møller.

It seems more natural to us to regard as the energy-momentum density of the total system of fields, the sum of the canonical (unsymmetric) quasi-tensors of all fields, and not the sum of the symmetric tensor of the ordinary matter field and the canonical quasi-tensor of the gravitational field, as proposed by Møller. This is based, first of all, on the desirability of having a uniform definition of the physical quantities for all fields. On the other hand, from Møller's point of view a quantity describing the total system of fields is replaced by one which is characteristic only of the gravitational field. Our point of view corresponds also to the covariant principles of second quantization.[5] We note, however, that both methods coincide completely in the consideration of the free gravitational field.

Møller concludes from the vanishing of the energy carried by the two known forms of gravitational waves in the absence of ordinary matter, that the usual quantum theories of gravitation are not useful. It should be noted in this connection that even if we are not concerned with real, energy carrying radiation, the calculation of vacuum effects may force us to accept the quantization of gravitation and the idea of gravitons. On the other hand, if the existence of energy carrying gravitational waves were definitely established, our earlier conclusion that the gravitons can be transformed into ordinary matter would in some sense undoubtedly be true in the general case as well as in the linear weak field approximation.[6]

*These papers were presented at the Colloquium on Gravitation in Paris and at the 9th High Energy Conference in Kiev in 1959 by Møller and also by Geiniot, who independently arrived at similar results.

[1] W. Pauli, Theory of Relativity, Pergamon Press, New York (1958).

[2] C. Møller, Ann. of Phys. **4**, 347 (1958).

[3] C. Møller, Paper in the Max-Planck-Festschrift, Berlin (1958).

[4] N. V. Mitskevich, Ann. Physik **1**, 319 (1958).

[5] N. V. Mitskevich, Доклады Болгарской АН (Trans. Bulgarian Academy of Sciences) **11**, 376 (1958).

[6] A. Sokolov and D. Ivanenko, Квантовая теория поля (Quantum Theory of Fields), pt. 2, GITTL, M. (1952); D. D. Ivanenko, Paper in the Max-Planck-Festschrift, Berlin (1958); see also D. R. Brill and J. A. Wheeler, Revs. Modern Phys. **29**, 465 (1957).

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ON LINEAR THEORIES OF GRAVITATION

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DESPITE the fact that the general theory of relativity has now found wide recognition, attempts are still being made to approach the problem of gravitation by a somewhat different method. Here we have in mind mainly the various linear theories of gravitation based on the usual pseudo-euclidean space-time metric.[1,2] It is here essential that the linear theories yield, in first approximation, the same values for the so-called three critical effects as the general theory of relativity (see, e.g., references 1 to 4).

The linear theories involve serious theoretical difficulties. One of these is that the energy density of the gravitational field is not positive definite.[5,6] However, attempts are being made to bypass this difficulty (see, e.g., reference 7). Notwithstanding the clear superiority of the theory of Einstein, it is therefore of definite interest to find those differences between the general theory of relativity and the linear theories which can, in principle, be observed in experiment.

There is no point in looking for discrepancies in the effects of the gravitational red shift and the deflection of light in the gravitational field of the sun: these are solely determined by the field equations, which are the same as in the linear approximation of the general theory of relativity. There remains the possibility to search for discrepancies in those effects which depend on the equations of motion in addition to the field equations.

In the general theory of relativity, one of the first integrals of the equations of motion, corresponding to the second Kepler law, has the form[8]

$$(1 - 2\varkappa m/c^2r)^{-1}r^2d\varphi/dt = \text{const.} \tag{1}$$

Similar expressions can easily be obtained in the