and to the purely gravitational field. In particular, if ordinary matter is absent, we obtain a noteworthy relation which establishes the equality (except for the sign) of the spin part of the energy-momentum of the gravitational field and the canonical quasi-tensor of the energy-momentum density of gravitation, introduced by us and also obtained by Møller.

It seems more natural to us to regard as the energy-momentum density of the total system of fields, the sum of the canonical (unsymmetric) quasi-tensors of all fields, and not the sum of the symmetric tensor of the ordinary matter field and the canonical quasi-tensor of the gravitational field, as proposed by Møller. This is based, first of all, on the desirability of having a uniform definition of the physical quantities for all fields. On the other hand, from Møller's point of view a quantity describing the total system of fields is replaced by one which is characteristic only of the gravitational field. Our point of view corresponds also to the covariant principles of second quantization.[5] We note, however, that both methods coincide completely in the consideration of the free gravitational field.

Møller concludes from the vanishing of the energy carried by the two known forms of gravitational waves in the absence of ordinary matter, that the usual quantum theories of gravitation are not useful. It should be noted in this connection that even if we are not concerned with real, energy carrying radiation, the calculation of vacuum effects may force us to accept the quantization of gravitation and the idea of gravitons. On the other hand, if the existence of energy carrying gravitational waves were definitely established, our earlier conclusion that the gravitons can be transformed into ordinary matter would in some sense undoubtedly be true in the general case as well as in the linear weak field approximation.[6]

*These papers were presented at the Colloquium on Gravitation in Paris and at the 9th High Energy Conference in Kiev in 1959 by Møller and also by Geiniot, who independently arrived at similar results.

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ON LINEAR THEORIES OF GRAVITATION

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DESPITE the fact that the general theory of relativity has now found wide recognition, attempts are still being made to approach the problem of gravitation by a somewhat different method. Here we have in mind mainly the various linear theories of gravitation based on the usual pseudo-euclidean space-time metric.[1,2] It is here essential that the linear theories yield, in first approximation, the same values for the so-called three critical effects as the general theory of relativity (see, e.g., references 1 to 4).

The linear theories involve serious theoretical difficulties. One of these is that the energy density of the gravitational field is not positive definite.[5,6] However, attempts are being made to bypass this difficulty (see, e.g., reference 7). Notwithstanding the clear superiority of the theory of Einstein, it is therefore of definite interest to find those differences between the general theory of relativity and the linear theories which can, in principle, be observed in experiment.

There is no point in looking for discrepancies in the effects of the gravitational red shift and the deflection of light in the gravitational field of the sun: these are solely determined by the field equations, which are the same as in the linear approximation of the general theory of relativity. There remains the possibility to search for discrepancies in those effects which depend on the equations of motion in addition to the field equations.

In the general theory of relativity, one of the first integrals of the equations of motion, corresponding to the second Kepler law, has the form[8]

$$(1 - 2\varkappa m/c^2 r)^{-1} r^2 d\varphi/dt = \text{const.} \qquad (1)$$

Similar expressions can easily be obtained in the linear theories as well. In the theories of Birkhoff[1] and Belinfante[2] we have, respectively,

$$\exp(2\varkappa m/c^2 r)\, r^2 d\varphi/dt = \text{const}, \tag{2}$$

$$[1 + (\eta + 1)\varkappa m/2c^2 r + v^2/2c^2 - K(F - 4C_1)\varkappa m/c^2 r]\, r^2 d\varphi/dt = \text{const}. \tag{3}$$

Here $\kappa = 6.67 \times 10^{-8}\, g^{-1}\, sec^{-2}\, cm^3$ is the gravitational constant, m is the mass of the central body, $r = p/(1 + e \cos\varphi)$ is the distance from the planet or the satellite to the central body, p is a parameter, e is the eccentricity of the orbit, c is the velocity of light in vacuum, and $\eta \approx 6$ is a constant to be determined by experiment (see the table of constants in reference 2). We note that the constants entering in the last term of the left hand side of (3) are such that

$$v^2/2c^2 - K(F - 4C_1)\varkappa m/c^2 r \geqslant 0. \tag{4}$$

Comparing the relations (1) and (3), we see that we can find a discrepancy between the Einstein theory and the theory of Belinfante by measuring the dependence of $d\varphi/dt$ on φ (say for a satellite with a large e). The maximal difference between (1) and (3) is

$$\Delta\left(\frac{d\varphi}{dt}\right)_{\max} = \left(\frac{d\varphi}{dt}\right)_{\varphi=0} \frac{\eta - 3}{2} \frac{\varkappa m (1+e)^3}{c^2 p} \approx \frac{\eta - 3}{2} \frac{(\varkappa m)^{3/2}(1+e)^3}{c^2 p^{5/2}}. \tag{5}$$

For an earth satellite with $e \approx 0.9$ and $p \approx 10^8$ cm, this difference is 5×10^{-12} rad sec^{-1}.

From (1), (2), and (3) one easily finds the expressions for the third Kepler law:

$$\frac{S_1 + \lambda(\varkappa m/c^2 r_1) L_1}{S_2 + \lambda(\varkappa m/c^2 r_2) L_2} = \frac{T_1}{T_2} \sqrt{\frac{a_1}{a_2}}, \tag{6}$$

where S is the area, and L, the perimeter of the orbit, a is the major half axis, and T is the period of revolution. The constant λ takes the value $\lambda = 1$ for the general theory of relativity and for the theory of Birkhoff, and $\lambda = (\eta + 1)/4$ for the theory of Belinfante ($\lambda = 0$ corresponds to the classical Kepler law in the theory of Newton).

We see from a comparison of formulas (1) and (2) that they lead to Kepler laws which differ only in second approximation, so that it is at present impossible to detect this difference in experiment. One may distinguish between the general theory of relativity and the theory of Birkhoff, however, by considering the rotation effect.

The rotation of the central body leads, according to the Einstein theory, to an additional displacement of the perihelion of the planet, which is equal to

$$\Delta\tilde{\omega} = -\frac{16\pi\sqrt{\varkappa m}\,\Omega l^2}{5c^2 p^{3/2}(1 - e^2)^3}\left(1 - 3\sin^2\frac{i}{2}\right). \tag{7}$$

per revolution.[9,10] In the theories of Birkhoff and Belinfante we obtain, respectively,

$$\Delta\tilde{\omega} = -\frac{3\pi\sqrt{\varkappa m}\,\Omega l^2}{5c^2 p^{3/2}(1 - e^2)^3}(3\cos i - 1), \tag{8}$$

$$\Delta\omega = \frac{4\pi\sqrt{\varkappa m}\,\Omega l^2}{5c^2 p^{3/2}(1 - e_0^2)^3}[(7 - 3\cos 2\omega_0 - 10.5\sin^2 i \cos 2\omega_0)\cos i + 6\cos^2 i - 1]. \tag{9}$$

In the theory of Belinfante we also obtain, besides the effect (9), an increase in the major half axis and in the eccentricity of the orbit, given by

$$\Delta a = \frac{36\pi e_0\sqrt{\varkappa m}\,\Omega l^2 \sin^2 i \cos i}{5c^2(1 - e_0^2)^{5/2}\sqrt{a_0}}(2\sin 2\omega_0 + 1), \tag{10}$$

$$\Delta e = \frac{4\pi\sqrt{\varkappa m}\,\Omega l^2}{5c^2 a_0^{3/2}}(6 + 4{,}5 e_0 \sin 2\omega_0). \tag{11}$$

per revolution. Here i is the inclination of the orbit, ω_0 is the longitude of the perihelion, Ω is the angular velocity of the rotation, and l is the radius of the central body (the index 0 characterizes the initial values of the corresponding constants). Although the quantity (10) is also very small (for Mercury, $e \approx 0.2$; $a \approx 5.8 \times 10^{12}$ cm, $i = 7°$, $\Delta a = 1$ cm), it does have fundamental importance in that it indicates that the planetary orbits are not stationary in the theory of Belinfante.

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