$$\Phi = -\frac{1}{8\pi}[\varepsilon_{\parallel}E_z^2 + \varepsilon_{\perp}(E_x^2 + E_y^2)]$$
$$-\frac{1}{8\pi}[\mu_{\parallel}H_z^2 + \mu_{\perp}(H_x^2 + H_y^2)]$$
$$-\frac{1}{4\pi}\alpha_{\parallel}E_zH_z - \frac{1}{4\pi}\alpha_{\perp}(E_xH_x + E_yH_y),$$

where $\epsilon_{\parallel}$ and $\epsilon_{\perp}$ are the longitudinal and perpendicular dielectric constants, $\mu_{\parallel}$ and $\mu_{\perp}$ the magnetic susceptibilities, and $\alpha_{\parallel}$ and $\alpha_{\perp}$ constants describing the magneto-electrical effect. Using the thermodynamic relations $4\pi\partial\Phi/\partial\mathbf{E} = -\mathbf{D}$ and $4\pi\partial\Phi/\partial\mathbf{H} = -\mathbf{B}$ we get the relations between the inductions and the field strengths:

$$D_z = \varepsilon_{\parallel}E_z + \alpha_{\parallel}H_z, \quad D_x = \varepsilon_{\perp}E_x + \alpha_{\perp}H_x,$$
$$D_y = \varepsilon_{\perp}E_y + \alpha_{\perp}H_y; \quad B_z = \mu_{\parallel}H_z + \alpha_{\parallel}E_z,$$
$$B_x = \mu_{\perp}H_x + \alpha_{\perp}E_x, \quad B_y = \mu_{\perp}H_y + \alpha_{\perp}E_y.$$

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NUCLEAR MOMENTS OF THE ODD ISOTOPES OF GADOLINIUM

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THE hyperfine structure of three lines of Gd I: 5015 A ($z''G_9$—$a''F_8$), 5103 A ($z''G_8$—$a''F_7$) and 5251 A ($z''G_8$—$a''F_8$) was investigated by means of a photoelectric spectrometer with a Fabry-Pérot interferometer.[1] The work was carried out using separated gadolinium isotopes of a high degree of enrichment (Gd^{155}—97.3%, Gd^{157}—91.4%). In agreement with paramagnetic measurements[2] it was shown unambiguously that the spin of both isotopes is $I = 3/2$. The value of the ratio of the magnetic moments $\mu_{155}/\mu_{157} = 0.79 \pm 0.02$ and the absolute values $\mu_{155} = -0.32 \pm 0.04$ and $\mu_{157} = -0.40 \pm 0.04$ agree satisfactorily within the indicated limits of error with earlier measurements.[2-4]

For the ratio of the quadrupole moments we obtain the value $Q_{155}/Q_{157} = 0.78 \pm 0.06$ which disagrees with Speck's investigation[4] in which he found that $Q_{155} \geq Q_{157}$. The absolute values of the quadrupole moments calculated from our experimental data ($Q_{155} = 1.6 \times 10^{-24}$ cm^2 and $Q_{157} = 2 \times 10^{-24}$ cm^2) are larger by almost a factor of two than those given by Speck, but it is difficult to estimate the error in the final result since the L-S coupling does not hold very well for the G-terms of gadolinium used by us, and this introduces a large indeterminacy in the estimate of the electronic matrix elements.

The values for the internal quadrupole moment $Q_0^{155} = 8 \times 10^{-24}$ cm^2 and $Q_0^{157} = 10 \times 10^{-24}$ cm^2 recalculated from the above data agree in order of magnitude with the values obtained by the method of Coulomb excitation of gadolinium nuclei.[5,6] The deformation parameters determined from our data are $\delta_{155} = 0.31$ and $\delta_{157} = 0.37$. The ratio $\delta_{155}/\delta_{157} = 0.8$ is directly obtained from the ratio of the quadrupole moments Q_{155}/Q_{157} and does not depend strongly on the possible error in the determination of the electronic wave functions. The values of the deformation parameters given above are in good agreement with the data on the variation of nuclear deformation in the series of rare earth elements obtained by the method of Coulomb excitation of even-even nuclei.[7]

By using these values of δ_{155} and δ_{157} and the values of μ_{155} and μ_{157} given above we made an estimate of g_K and g_R — the gyromagnetic ratios for the internal and the collective motions. The calculation of g_K was carried out in accordance with Nilsson's scheme, and in order to do this the expansion coefficients for the wavefunction of the unpaired nucleon tabulated by Nilsson[8] were extrapolated for the ground state of gadolinium ($N = 5$, $I = 3/2$, $h_{9/2}$) into the region of large nuclear deformations ($0.31 < \delta < 0.41$). As a result it was established that $g_{K155}/g_{K157} = 0.9 \pm 0.1$ and then $(g_R/g_K)_{157} = 1.1\,(g_R/g_K)_{155} \pm 0.2$. These relationships were obtained by utilizing only the relative values of $\delta_{155}/\delta_{157}$ and μ_{155}/μ_{157}, which are determined from the experimental data with a high degree of accuracy. The absolute values of g_K may be found with an accuracy determined by the error in the values of Q_{155} and Q_{157}. Calculations gave the values $g_{K\,155} = -0.8$ nuclear magnetons and $g_{K\,157} = -0.9$ nuclear magnetons, from which we obtain $g_{R\,155} = g_{R\,157} = 0.7$. This value differs appreciably from the approximate estimate $g_R \approx Z/A = 0.4$, and is in good agreement with the data of de Boer et al.,[6] but contradicts earlier measurements of Bjerregard and Meyer-Berkhout,[5] although the ratio $g_{K\,155}/g_{K\,157}$ determined by them experimentally is in complete agreement with the value obtained by us. This confirms to some extent the correctness of the extrapolation of Nilsson's data made by us into the region of deformations $\delta > 0.3$. Similar calculations made by Gauvin[9] for strongly deformed nuclei with an unpaired nucleon ($153 < A < 197$) have shown that for a number of nuclei such an estimate leads to $g_R > Z/A$. The values obtained by him for g_K and g_R in the case of Gd^{155} agree with our estimate. In the case of Gd^{157} the estimates of g_K and g_R differ, since we based ours on the value $\delta_{157} = 0.37$, while Gauvin adopted $\delta_{157} = 0.31$. Gauvin discusses the possibility of a modification of the evaluation of g_K which would lead to the values of $g_R \approx Z/A$, but the new experimental data[6] on the value of g_R for Gd^{157} contradict such an estimate. Therefore an additional investigation of the odd isotopes of gadolinium by the method of Coulomb excitation is highly desirable.

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RESONANCE INTERACTION OF PIONS

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In order to explain the maximum in the π^--p interaction at energy E = 1 Bev, Piccioni,[1] Dyson,[2] and Takeda[3] advanced the hypothesis of resonance interaction between π mesons. This hypothesis was also used to explain the high multiplicity of π mesons produced in nucleon-antinucleon annihilation[4,5] and to explain the inelastic (π^--p) scattering for $E \geq 1$ Bev.[6,8] However, the assumption of a resonance π-π interaction was not obligatory in all cases considered in these articles, since the experimental results could be explained in other ways.

It is of interest to consider what conclusions would follow from the assumption of a resonance π-π interaction in the case of inelastic interactions of particles at $E \gg 1$ Bev, where a large number of π mesons would be produced, and an assumption about the π-π interaction would have a considerable effect on the results of calculation. As an example, we consider inelastic π^--p collisions at E = 5 Bev. We considered this case in detail earlier[9] without taking account of a resonance π-π interaction.

We assume, as in reference 9, that statistical equilibrium is established for the K mesons in a spatial volume of radius $r_K = \hbar/m_K c$, and, for all other particles, in a spatial volume of radius $r_\pi = \hbar/m_\pi c$, where m_K and m_π are the masses of the K- and π-mesons. As we showed in reference 9, these were the best choices for explaining experimental data on multiple production of ordinary and strange particles.* We will take the same conservation laws into account and use the same method for calculating statistical weights as in our previous work.

Taking account of the resonance π-π interaction is formally equivalent to introducing a "pion isobar" of mass $\mu = 0.47$ nucleon masses,[5] spin S = 0 and isotopic spin T = 0 (variant of Dyson[2]) or T = 1 (variant of Takeda[3]) into the statistical theory of multiple production.†

In the table we show the ratio of the experimental results from reference 11 to the theoretical results