the only sufficient reason for changing α_2^N would be an improvement in the experimental data, to which, in particular, the review of Perlman and Rasmussen is devoted.[3]

I thank A. S. Kompaneets for evaluating the results of this work, and also T. V. Novikova for help in the calculations.

[1] V. G. Nosov, Dokl. Akad. Nauk SSSR **112**, 414 (1957), Soviet Phys.–Doklady **2**, 48 (1957).

[2] V. M. Strutinskiĭ, JETP **32**, 1412 (1957), Soviet Phys. JETP **5**, 1150 (1957).

[3] I. Perlman and J. Rasmussen, Hand. d. Phys. XLII, 109 (1957).

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THE NON-ADDITIVITY OF LONDON-VAN DER WAALS FORCES

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THE so-called dispersive interaction forces between neutral atoms are not additive (notwithstanding statements which are sometimes made); the additivity occurs only in the first non-vanishing perturbation theory approximation. We consider here the next terms (of third order in the coupling constant) of the perturbation series and we shall obtain an expression for the energy of the dispersive interaction of three hydrogen atoms, in its dependence on the interatomic distances R_1, R_2, R_3 as parameters and including the retardation effect.

For this purpose it is advantageous to use the Feynman-Dyson technique, as done by Dzyaloshinskiĭ[1] for the interaction of two atoms. We are interested in processes of the kind of Fig. 1a leading to a contribution

$$S^{(6)} = -\frac{4}{3}\pi^3 \int dx_1 \ldots dx_6 D^F(1,4) D^F(3,6) D^F(2,5) \times P[j_\delta^{(3)}(5) j_\nu^{(3)}(6)] P[j_\nu^{(2)}(3) j_\mu^{(2)}(4)] P[j_\mu^{(1)}(1) j_\delta^{(1)}(2)]. \quad (1)$$

To derive this expression we eliminated the variables of the intermediate electromagnetic field, used the usual, non-Heaviside system of units [$\hbar = c = 1$ up to Eq. (5)], and took into account that, for instance, for an N-particle interaction there are in all $2^{N-1}(N-1)!$ different processes of order e^{2N} corresponding to connected diagrams of the kind shown in Fig. 1a. Generally speaking we should have started our consideration with processes of the type of Fig. 1b, leading to a non-additive correction of the second order (in the coupling constant), but in the approximation chosen by us (we are only interested in the dipole interaction) all matrix elements of $S^{(4)}$ containing "propagator functions" of three interacting atoms give zero for the ground state, since two out of the three "propagator lines" are involved in only one vertex. One must drop processes depicted in Fig. 1c for similar considerations.

In the first non-vanishing order, in which the interaction energy depends simultaneously on the coordinates of all three atoms, we therefore get precisely Eq. (1). Performing in this equation the integration over all time coordinates we find:

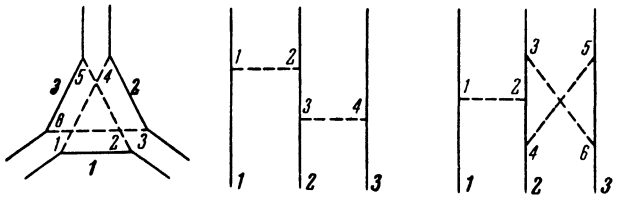
$$U = \frac{-i}{24\pi^7} \int d^3r_1 \ldots d^3r_6 \int d^3p_1 d^3p_2 d^3p_3 \exp\{ip_1(r_1 - r_4) + ip_2(r_3 - r_6) + ip_3(r_5 - r_2)\} \int_{-\infty}^{\infty} \frac{d\omega}{(p_1^2 - \omega^2)(p_2^2 - \omega^2)(p_3^2 - \omega^2)} \times \sum_{k,m,n} \left[\frac{\langle 0 | j_\delta^{(3)}(r_5) | k \rangle \langle k | j_\nu^{(3)}(r_6) | 0 \rangle}{\omega_{k0} - \omega} + \frac{\langle 0 | j_\nu^{(3)}(r_6) | k \rangle \langle k | j_\delta^{(3)}(r_5) | 0 \rangle}{\omega_{k0} + \omega} \right] \times \left[\frac{\langle 0 | j_\nu^{(2)}(r_3) | m \rangle \langle m | j_\mu^{(2)}(r_4) | 0 \rangle}{\omega_{m0} - \omega} + \frac{\langle 0 | j_\mu^{(2)}(r_4) | m \rangle \langle m | j_\nu^{(2)}(r_3) | 0 \rangle}{\omega_{m0} + \omega} \right] \times \left[\frac{\langle 0 | j_\mu^{(1)}(r_1) n \rangle \langle n | j_\delta^{(1)}(r_2) | 0 \rangle}{\omega_{n0} - \omega} + \frac{\langle 0 | j_\delta^{(1)}(r_2) | n \rangle \langle n | j_\mu^{(1)}(r_1) | 0 \rangle}{\omega_{n0} + \omega} \right]. \quad (2)$$

In the non-relativistic approximation we can put

$$\langle l | j_\mu(r_1) | m \rangle \langle m | j_\mu(r_2) | l \rangle = -e^2 [\psi_l^*(r_1) \psi_m(r_1) - \delta(r_1)\delta_{lm}] [\psi_m^*(r_2) \psi_l(r_2) - \delta(r_2)\delta_{ml}]. \quad (3)$$

and so on. If we now introduce the polarizability

$$\alpha(\omega) = \sum_n 2\omega_{n0} |d_{0n}|^2 / (\omega_{n0}^2 - \omega^2), \quad (4)$$



where d_{0n} is the dipole moment matrix element, and perform the necessary integration in (2), we get finally after elementary, but cumbersome computations

$$U(R_1, R_2, R_3) = -132\,\hbar c\alpha_1(0) \times \alpha_2(0)\,\alpha_3(0)/\pi R_1R_2R_3(R_1+R_2+R_3)^7, \quad (5)$$

and this formula is valid under the assumption that the distances between the atoms are much larger than the characteristic wave length λ_0 in the spectrum of the atom (R_1, R_2, $R_3 \gg \lambda_0$ so that exchange forces play no role whatever; as was already stated, we neglect the effects of higher multipoles).

[1] I. E. Dzyaloshinskiĭ, JETP **30**, 1152 (1956), Soviet Phys. JETP **3**, 977 (1957).

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NUMERICAL SOLUTION OF STATIC DISPERSION RELATIONS OF THE PHOTOPRODUCTION P-WAVE

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We found an exact numerical solution of the static dispersion relations obtained in references 1 and 2 for the photoproduction P-amplitudes. We used the method, proposed by Omnes[3] and based on the work of Muskhelishvili,[4] of reducing the linear singular integral equations to regular Fredholm equations. The procedure (which is not unique in the case of scattering), for the transition from the singular to the regular equations is found to be unique in this case under the following conditions: 1) the scattering phase shifts vanish at the threshold and at infinity, 2) the solution of the regular equation is bounded and has the same value at infinity as the solution of the singular equation.

The values used for the phase shifts were obtained from the Chew-Low static equations (at $f^2 = 0.08$ and a cutoff parameter $P = 7$), the solutions of which were obtained by the Salzmans[5] and repeated by Tentyukova on the "Strela" computer. The regular photoproduction equations were solved by successive approximation on the "Ural" computer of the Joint Institute for Nuclear Research.

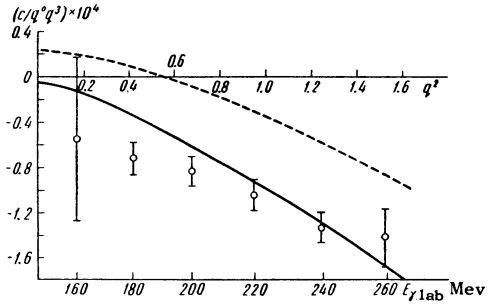
The exact solutions for the quadrupole amplitudes and the e-parts of the magnetic dipole amplitudes behave qualitatively like the corresponding Bohr terms multiplied by $\cos\delta$.[1] The μ-parts of the magnetic dipole amplitudes (including the isotope-scalar amplitudes) behave like $\sim q^{-3}\sin\delta$. This means that the meson created upon interaction of a photon with the static magnetic moment of the nucleon always experiences secondary scattering.

It is shown further that the electric dipole amplitude is independent in the static approximation of the magnetic moments. This follows from the supplementary condition and from the dispersion relations for the longitudinal amplitudes, obtained in reference 6.

As a first attempt at comparing the complete expression for the photoproduction amplitude with experiment, with allowance for the obtained corrections, we calculated the coefficient C in the photoproduction cross section of π^0 mesons at threshold:

$$d\sigma(\gamma p \to \pi^0 p)/d\Omega = A + B\cos\theta + C\cos^2\theta.$$

In the figure, C is given in $(\hbar/\mu_0 c)^2$ units, q is the meson momentum in the c.m.s. and in units of $\mu_0 c^2$, and $q^0 = \sqrt{1-q^2}$. The solid curve corresponds to the exact solution, while the dotted one corresponds to the approximate solution obtained in reference 1; the experimental points for 160 — 240 Mev are taken from reference 7, while those for 260 Mev are from reference 8. It is seen that



the exact solution leads to fair agreement with experiment near threshold. This agreement becomes somewhat worse for large energy, which can be attributed to relativistic effects. The approximate solution of reference 1 is in poor agreement with experiment, as noted by Baldin and Govorkov (private communication).

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