EFFECT OF VISCOSITY IN MULTIPLE PRODUCTION ON THE ENERGY DISTRIBUTION OF SECONDARY PARTICLES

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The effect of viscosity on processes taking place in a simple wave is considered in the hydrodynamical theory of multiple production of particles. It is shown that the effect of viscosity on the energy distribution of the fastest particles may be significant at sufficiently high energies.

In most researches on the hydrodynamical theory of multiple production of particles, the equations of a relativistic ideal liquid are used without consideration of viscosity. The effect of viscosity can be of a two-fold nature. In the first place, the viscosity increases the energy dissipation, raises the entropy, and, consequently, the number of secondary particles. In the second place, the appearance of new particles can bring about a significant change in the energy distribution, particularly in that region in which the number of particles is small while the energy possessed by them is large (for example in a simple wave[1]).

The problem of the role of viscosity was considered by Emel'yanov.[2] It was found that in the region of the fundamental solution (see reference 3) in the case in which the viscosity coefficient is not large, the number of secondary particles that owe their origin to the viscosity is small in comparison with N_0 and increases slightly with increase in the primary energy E_L (while the number of secondary particles N_0 formed in the initial stage upon passage of the shock waves increases significantly with the energy $N_0 \sim E_L^{1/4}$).

In the present work we compute the change ΔN in the number of particles which arise as a result of the viscosity in the region of the simple wave. This region is of interest, first, because even an increase that is small in absolute value can appreciably change the energy distribution, and second, because the velocity gradients in the region of the simple wave are larger than in the region of the fundamental solution, and therefore the role of the viscous terms is much more significant.

It should be noted that it is not immediately possible to determine the coefficient of viscosity of a relativistic liquid, in view of which the results here are of a qualitative character. Much more important from our point of view is the character of the dependence of ΔN on the energy of the primary particle E_L.

To estimate the number of particles formed in a simple wave because of viscosity, we make use of the expression for the 4-divergence of the entropy flux:[4]

$$\frac{\partial}{\partial x^i}(\sigma u^i) = -\frac{\tau_i^k}{T}\frac{\partial u^i}{\partial x^k}, \tag{1}$$

where σ is the entropy density, u^i is the velocity of an element of volume, T is the temperature, τ_{ik} is the viscous part of the energy momentum tensor, equal[4] to

$$\tau_{ik} = -\eta'\left(\frac{\partial u_i}{\partial x^k} + \frac{\partial u_k}{\partial x^i} + u_i u^l \frac{\partial u_k}{\partial x^l} + u_k u^l \frac{\partial u_i}{\partial x^l}\right) + \left(\frac{2}{3}\eta' - \zeta\right)\frac{\partial u^l}{\partial x^l}(g_{ik} + u_i u_k). \tag{2}$$

Equation (1) takes the form

$$\frac{\partial}{\partial x^i}(\sigma u^i) = \frac{4}{3}\frac{\eta}{T}\left(\frac{\partial u^i}{\partial x^i}\right)^2, \tag{3}$$

after substitution of the expression for τ_{ik}. Here $\eta = \eta' + {}^3\!/_4\,\zeta$. Further, we shall carry out the calculation under the assumption that the coefficient of viscosity η is small. Then quantities entering into the right hand side (T and u^i), we express in the form

$$T = T_0 + \eta T', \qquad u^i = u_0^i + \eta u'^i, \tag{3a}$$

where we shall assume that

$$\eta T' \ll T_0, \qquad \eta u^i \ll u_0^i. \tag{4}$$

Taking into account the assumptions made above, we can find the total increase in the entropy in first order in the coefficient of viscosity, making use of the one-dimensional solution for the simple wave

$$T_0 = T_0^*[(t-x)(1-c_0)/(t+x)(1+c_0)]^{c_0/2},$$
$$u_0^0 = (t+c_0x)/\sqrt{(1-c_0^2)(t^2-x^2)},$$
$$u_0^1 = (x+c_0t)/\sqrt{(1-c_0^2)(t^2-x^2)},$$
$$u_0^2 = u_0^3 = 0, \qquad (5)$$

where $c_0 = 1/\sqrt{3}$ is the velocity of sound.

The increase in momentum in the simple wave will be

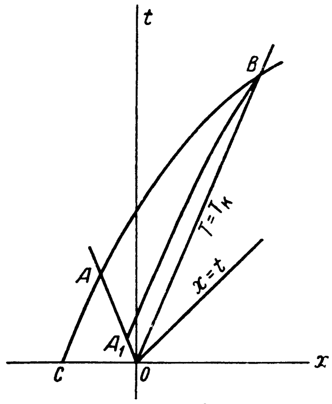
$$\Delta S = \int_\Omega dx^4 \frac{\partial \sigma u^i}{\partial x^i} = \frac{4\eta}{3}\int_\Omega dx^4 \left(\frac{\partial u^i}{\partial x^i}\right)^2 \frac{1}{T}. \qquad (6)$$

The region of integration Ω here encloses the entire simple wave with the exception of those of its parts where the temperature is less than critical $T_k = \mu$. Relative to these parts, we have assumed (as is usually done in hydrodynamical theory) that the interaction between particles is completely absent. Furthermore, a region of the order of the mean free path of the particles λ around the point $t = x = 0$ has been discarded. This point represents the state of the system at the moment when the shock wave reaches the edge of the nucleon, and therefore a discontinuity of temperature takes place in it. The region of integration Ω is shown in the drawing. Integration has resulted in*

$$\Delta S = \frac{2\sqrt{3}\,\pi a^2\eta}{\mu}\left\{\ln\frac{l(1-c_0)}{2c_0\lambda}\left(\frac{T_0^*}{\mu}\right)^{(1+c_0)^2/2c_0^2} - \frac{\mu}{T_0^*}\ln\frac{l(\sqrt{3}-1)}{2\lambda} - \frac{(1+c_0)^2}{2c_0^2}\left(1-\frac{\mu}{T_0^*}\right)\right\}, \qquad (7)$$

where πa^2 is the cross section of the interaction and l is the initial longitudinal dimension of the system. It is evident from this expression that the additional number of particles, $\Delta N \sim \Delta S$, formed in a simple wave increases logarithmically with the energy. However, the ratio of the increase of entropy to its initial value $\Delta S/S_0$ remains very small and falls off with the energy (inasmuch as $S_0 \sim E_L^{1/4}$). Thus the effect of viscosity on the total number of secondary particles is not large.

We now compute the number of secondary particles which are formed in the simple wave and remain in it to the moment of its decay. For this purpose we need to take the integral of (6) not over the entire region Ω, but over a flow tube of particles remaining in the simple wave, Ω_1. The region of integration Ω_1 is shown in the drawing.*



AB is the boundary with the region of the general solution, COA is the region at rest, OA_1B is the region Ω_1, OAB is the region Ω.

As a result of the integration, we obtain

$$\Delta S_1 = \frac{2\sqrt{3}\,\pi a^2\eta}{\mu}\left\{\ln\frac{l(1-c_0)}{2\lambda c_0}\left(\frac{T_0^*}{\mu}\right)^{(1+c_0)^2/2c_0^2} - \frac{\mu}{T_0^*}\ln\frac{l(1+c_0)}{2\lambda c_0} - \frac{1+c_0}{c_0^2}\left(1-\frac{\mu}{T_0^*}\right)\right\}. \qquad (8)$$

It is seen here that the quantity ΔS_1 differs from ΔS only by terms which are independent of the energy of the primary particle; the role of these terms decreases with energy. Consequently, at sufficiently high energy, practically none of the particles created in the simple wave emerge from it. This means on the other hand that the new formation of the particles takes place at the boundary $T = \mu$, which is natural, since the effect of viscosity is greater the lower the temperature.

*In the integration it is convenient to transform to the new variables $\tau = t + x$ and $\xi = t - x$. In these variables, Eq. (6) has the form

$$\Delta S = \frac{2\pi}{3T_0^*}\iiiint [\tau(1+c_0)]^{(c_0-2)/2}[\xi(1-c_0)]^{-(c_0+2)/2}\, d\tau\, d\xi\, dy dz.$$

The equations for the limits of integration are the following: For the boundary $T = \mu$:

$$\tau = [(1-c_0)/(1+c_0)]\,(T_0^*/\mu)^{2c_0}\xi;$$

the boundary of the rarefaction wave with the general solution:

$$\tau = \tau_k\,(\xi/\xi_k)^{(1+c_0)^2/(1-c_0)^2},$$

where τ_k and ξ_k are the critical values of the variables τ and ξ corresponding to the temperature $T = \mu$. They are equal to

$$\tau_k = {}^1\!/_2\, l(\sqrt{3}-1)\,(T_0^*/\mu)^{(1+c_0)^2/2c_0^2},$$
$$\xi_k = {}^1\!/_2\, l(\sqrt{3}+1)\,(T_0^*/\mu)^{(1-c_0)^2/2c_0^2}.$$

*The region Ω_1 differs from Ω only in that the boundary with the fundamental solution is replaced by the equation of the flow line passing through the point (τ_k, ξ_k). In the coordinates τ, ξ, this equation has the form

$$\tau = \tau_k\,(\xi/\xi_k)^{(1+c_0)/(1-c_0)}.$$

We estimate the absolute number of particles remaining in the simple wave under the assumption that these are pions. Then, in accord with reference 3,

$$\Delta N_1' = 0.2\Delta S_1 \approx 0.4\pi a^2\eta\mu^{-1}[\ln(E_L/\mu) - (1+c)/c_0^2]. \quad (9)$$

In spite of the fact that $\Delta N_1 \ll N_0$ always, the value of ΔN_1 can, with increase in energy, be compared with the number of particles remaining in the simple wave without account of viscosity (according to reference 1, this number does not depend on the energy and is equal to unity in order of magnitude), or can even surpass it. This can materially change the character of the interaction, since in the region where the greatest amount of energy is concentrated there will be found not a single particle but several, and the fraction of the energy remaining with a single particle will be much less. The energy for which $\Delta N_1 \approx 2$ and this phenomenon sets in can be estimated from (9):

$$E_k \sim \exp\{\mu\Delta N_1/2\sqrt{3}\pi a^2\eta\}. \quad (10)$$

It must be noted, however, that it is not possible to estimate this value of the energy E_k at all precisely, since the coefficient of viscosity η enters into (10). The value of this coefficient can be estimated very roughly; at the same time E_k depends very strongly on it (exponentially). For example, if the ideal gas model is taken, then

$$\eta \approx \mu. \quad (11)$$

this value for η for a given density and temperature is an upper estimate.*

*Comparison of the kinetic coefficients of viscosity for gases and liquids shows that they are much smaller in liquids than in gases (see reference 4).

Consequently, by substituting (11) in (10), we obtain a reduced value of the critical energy $E_k \sim 10^{11}$ ev. Even if we take the coefficient of viscosity to be one third of this, then the corresponding value will be $E_k \sim 10^{14}$ ev.

These examples show that it is not possible to give a value for the critical energy E_k at the present time.

The calculations carried out above give grounds in support of the idea that the character of the elementary act must change with increase in energy. That is, beginning with a certain energy E_k, the effect of the reservation of a large fraction of the energy (~50 per cent) to a single particle[2] should no longer take place.

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