$$i\hbar^{-1}(E_n - E_0)\langle\Phi_n, \varphi\Phi_0\rangle = -I_0^{-1}\langle\Phi_n, N_Z\Phi_0\rangle. \tag{5}$$

From (5) and from the commutation relations one can easily derive the expressions

$$2\sum_{n\neq 0}\frac{\langle\Phi_0, M_Z\Phi_n\rangle\langle\Phi_n, N_Z\Phi_0\rangle}{E_n - E_0} = -I_0, \quad 2\sum_{n\neq 0}\frac{|\langle\Phi_n, N_Z\Phi_0\rangle|^2}{E_n - E_0} = I_0. \tag{6}$$

Exchanging L_Z by $M_Z + N_Z$ in (3) and applying (6) we immediately obtain (4).

Thus the expressions (3) and (4) are equivalent.

*Here and in the following we speak about a rotation of the nucleus around a fixed axis. Consideration of the rotation around a free axis will only introduce complications in the intermediate expressions and will not lead to any essential changes in the final results.

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EFFECT OF VACUUM FLUCTUATIONS ON THE POLARIZATION OF ELECTRONS MOVING IN A MAGNETIC FIELD

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OWING to the anomalous magnetic moment, the spin of an electron moving in a uniform magnetic field does not preserve its orientation along or opposite to the direction of motion, but precesses about the direction of the momentum. The quasi-classical interpretation of this effect was given by Mendlowitz and Case,[1] who obtained the equations of motion for the spin operator in the Heisenberg representation, from where they derived the precession. The corresponding experimental investigations have also been carried out.[2]

It seems useful to give a consistent quantum mechanical description of this effect which is valid for electrons with arbitrary energy. We start from the Dirac equation with radiative corrections taking into account the effects of the photon vacuum (see, for example, reference 3), which, in first approximation, has the form

$$(i\hat{\partial} + e\hat{A} - m)\psi(x) = -ie^2\int\gamma^\nu S^c(x, x')\gamma_\nu D^c(x - x')\psi(x')\,d^4x', \tag{1}$$

where D^c is the causal photon function, and S^c is the causal Green's function of the electron, expressed in terms of the exact solutions of the Dirac equation for an electron moving in a magnetic field. The time integration transforms (1) into

$$(E - \mathcal{H})\psi(\mathbf{r}) = \int K(\mathbf{r}, \mathbf{r}')\psi(\mathbf{r}')\,d\mathbf{r}', \tag{2}$$

where $\mathcal{H}$ is the Hamiltonian of the Dirac equation.

Each energy level is doubly degenerate with respect to the quantum number $s = \pm 1$ characterizing the projection of the spin on the direction of the momentum. The right hand side of (2) is a constant perturbation and causes periodic transitions between these states. Writing ψ as a superposition of ψ_1 and ψ_{-1}, multiplying (2) by ψ_s^+ and integrating over $\mathbf{r}$, we obtain a system of equations which determines the two energy values and the coefficients of the expansion. We introduce a time dependent wave function which satisfies the initial condition $\Psi(0) = \psi_1$, and find the following expression for the average value of the projection of the spin on the direction of the momentum:

$$\left\langle\frac{\sigma\mathbf{k}}{k}\right\rangle = \int\Psi^+(t)\frac{\sigma\mathbf{k}}{k}\Psi(t)\,d\mathbf{r} = \cos^2\delta t + \frac{1}{4}A^{-2}[(W_{1,1} - W_{-1,-1})^2 - 4W_{-1,1}^2]\sin^2\delta t, \tag{3}$$

where

$$A = {}^1/_2[(W_{1,1} - W_{-1,-1})^2 + 4W_{1,-1}W_{-1,1}]^{1/2}, \qquad \delta = A/\hbar,$$

$$W_{ss'} = \int\psi_s^+(\mathbf{r})K(\mathbf{r}, \mathbf{r}')\psi_s(\mathbf{r}')\,d\mathbf{r}\,d\mathbf{r}'.$$

Expression (3) has no divergencies connected with the mass of the field. Only for the calculation of the energy of the interaction with the vacuum it becomes necessary to introduce the corresponding compensating term in (1). If the electron moves in the direction of the field, we find $W_{s,-s} = 0$ and $\langle\sigma\mathbf{k}/k\rangle = 1$, i.e., the spin of the electron preserves its initial orientation. In the other limiting case — motion in the plane perpendicular to the direction of the field — we have $W_{1,1} = W_{-1,-1}$ and (3) takes the form

$$\langle\sigma\mathbf{k}/k\rangle = \cos 2\delta t, \qquad \delta = |W_{1,-1}|/\hbar. \tag{4}$$

Also $\langle\sigma_z\rangle = 0$. These relations can be interpreted as the precession of the spin in the plane of the orbit. However, in view of the fact that only the projections of the spin on the direction of the momentum are integrals of the motion (±1), it would be more consistent to avoid the term "precession" and to speak of the transition time between these states or of the transition probability per unit time.

With regard to the calculation of $W_{1,-1}$, the following should be noted. It is impossible to expand the Green's function in powers of the potential, since the potential of the uniform field is not a perturbation. Indeed, the vector potential depends on a coordinate which can become very large in the relativistic case (for example, in the relativistic case, $\langle e^2\hat{A}^2\rangle \sim e^2 \times H^2 \langle r^2\rangle \sim E^2$). Similarly, with any other method of expansion, one must guard against the appearance in the neglected terms of expressions which depend on coordinates which after integration could lead to large values. In our case the expansion in terms of H/H_0 ($H_0 = m^2c^3/e\hbar \sim 10^{13}$ oe) was introduced in the last phase of the calculations, after the integration over space and the summation over the virtual states. As a result we obtained in first approximation in H/H_0 the following value for $W_{1,-1}$, which is valid both in the relativistic and nonrelativistic regions:

$$W_{1,-1} = -(\alpha/2\pi)\mu H. \tag{5}$$

This result could have been derived from the operator $(\alpha/2\pi)(\sigma H)\mu$, but the use of this operator in the relativistic region would, according to the considerations above, require a special justification.

The time for the spin-flip caused by the interaction of the electron with the photon vacuum is, therefore, equal to $\pi/2\delta = 2\pi^2 mc/\alpha eH$. The ratio of this over the period of rotation of the electron is equal to $(\pi/\alpha)mc^2/E \sim 450\ mc^2/E$. The last quantity decreases as the energy becomes larger, and reaches the value 1 at energies of ~200 Mev.

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MINIMAL ERROR IN THE EXPERIMENTAL OBSERVATION OF ASYMMETRY

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MANY experiments with elementary particles, aimed at proving or disproving the conservation of spatial parity or proving the existence of spin in a particle, reduce to the observation of a definite asymmetry in the distribution of the particles, produced in a certain reaction. It becomes useful to estimate the probability of the error committed when conclusions concerning the presence or absence of asymmetry are drawn from such an experiment.

In the observation of asymmetry, all particles are separated (during the course of the experiment or during the data reduction) into two groups, such that in the absence of asymmetry of the observed process a particle can belong to either group with equal probability. Usually the probability of registration of each particle in one of the groups is independent of the number of particles already accumulated in these groups. Therefore, if the data are corrected for possible systematic errors, the number of particles in the two groups, n_+ and n_-, have Poisson distributions with mean values $\frac{1}{2}n(1 \pm F)$, where $n = n_+ + n_-$, and F is a constant that characterizes the force of the interaction that leads to violation of the symmetry.

It can be shown that for $n_+ > n_- \gg 1$ the relation

$$t = (n_+ - n_- - 1)/\sqrt{n} \tag{1}$$

has a Student's t-distribution with f degrees of freedom, where

$$\frac{1}{f} = \left(\frac{n_+}{n}\right)^2 \frac{1}{n_+ - 1} + \left(\frac{n_-}{n}\right)^2 \frac{1}{n_- - 1} \approx \frac{1 + 2(n_+ - n_-)^2 n^{-3}}{n-2}. \tag{2}$$

When $n \gg 1$ the value of t tends to $F\sqrt{n}$.

When $F = 0$, relation (1) satisfies, with probability $1 - \alpha$, the inequality

$$t < t_{1-\alpha/2}(f), \tag{3}$$

where $t_P(f)$ is the number that has the probability P of satisfying the inequality $t < t_P$. If the value obtained for t does not satisfy inequality