ON POPOV'S RELATION BETWEEN THE SYMMETRY OF TRANSPORT COEFFICIENTS AND THERMODYNAMICS

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K. POPOV[1] tried to show that the Onsager relations

$$dx_i/dt = \sum_{k=1}^{n} L_{ik} X_k \qquad (i = 1, 2, \ldots n) \tag{1}$$

together with the symmetry properties

$$L_{ik} = L_{ki} \tag{2}$$

are connected with the existence of first integrals of a more general system of differential equations

$$d^2x_i/dt^2 = \ddot{x}_i = \sum_{j=1}^{n} g_{ij} x_j \tag{3}$$

if one assumes that the solution stays finite when $t \rightarrow +\infty$. According to Popov the g_{ij} are essentially thermodynamic quantities determined by the relation

$$\Delta S = -\tfrac{1}{2} \sum_{i,j=1}^{n} g_{ij} x_i x_j, \tag{4}$$

although they occur in the transport equation (3). We shall show in the following that this assumption (taken over and further developed by Karanikolov[2]) is not compatible with the theory of irreversible processes.

If we define the g-matrix using (4) and start with an equation of the form (3) one can, indeed, obtain relations which are similar in form to (1) and (2). One obtains, namely, ultimately a set of equations

$$\dot{x}_i + \sum_{j=1}^{n} \mathscr{L}_{ij} X_j = 0, \qquad \mathscr{L}_{ij} = \mathscr{L}_{ji}, \tag{5}$$

where the $\mathscr{L}$-matrix is completely determined by the g-matrix. Although Eq. (5) is formally similar to (1) and (2) the $\mathscr{L}$-matrix is not the matrix of the Onsager coefficients which we shall denote by L. Indeed, if $g^{1/2}$ is the positive definite square root of g one can easily show that $\mathscr{L} = g^{1/2}$. It is at the same time essential for the theory of irreversible processes that the Onsager coefficients are independent of the g-matrix and can therefore not determine the $\mathscr{L}$-matrix.

Moreover, starting from (3) Popov shows that the inverse relaxation time matrix τ^{-1} given by its values $-r_1, -r_2, \ldots, -r_n$ is given by the equation

$$\tau^{-1} = g^{1/2}. \tag{6}$$

In actual fact, however, it is well known from the theory of irreversible processes that the τ^{-1} matrix satisfies the relation

$$\tau^{-1} = Lg. \tag{7}$$

Equations (6) and (7) are only compatible if $L = g^{-1/2} = \tau$, which is clearly incorrect.

The source of the contradictions mentioned here lies in Popov's initial assumption (3). The assumption which establishes a connection between the transport properties of the system and the purely thermodynamic quantities is totally inadmissable and contradicts all we know about irreversible processes.

We shall briefly give an analysis of (3) starting from the usual equations of the thermodynamics of irreversible processes

$$X_i = \sum_{j=1}^{n} g_{ij} x_j. \tag{8}$$

The g-matrix is here defined according to (4) and in contradistinction to (3) it is assumed that

$$d^2x_i/dt^2 = \sum_{j=1}^{n} b_{ij} x_j, \tag{9}$$

where $b \neq g$. Putting afterwards $b = g$ we can see immediately that (5) follows from that.

Starting from (9) one shows easily that

$$\dot{x}_i + \sum_{k=1}^{n} K_{ik} x_k = 0, \tag{10}$$

where the K-matrix is symmetric; it is namely the positive definite square root of b, i.e.,

$$K_{ik} = K_{ki}, \tag{11}$$

$$K = b^{1/2}. \tag{12}$$

If we now substitute x from (8) into (10), we get

$$\dot{x}_i + \sum_{j=1}^{n} \mathscr{L}_{ij} X_j = 0, \text{ where } \mathscr{L}_{ij} = \sum_{k=1}^{n} K_{ik} g_{kj}^{-1}. \tag{13}$$

The $\mathscr{L}$-matrix is thus in the general case not at all symmetric, but if we take Popov's assumption that $b = g$ it follows from (12) and (13) that $\mathscr{L}_{ij} = g_{ij}^{-1/2} = K_{ij}^{-1}$. This matrix is symmetric according to (11). This is just the basis on which Popov constructed the derivation of the symmetry relations, starting from second order equations.

The assumption $b = g$ is, however, as we have already seen, totally unjustified.

[1] K. Popov, JETP **28**, 257 (1955), Soviet Phys. JETP **1**, 336 (1955).

[2] Kh. Karanikolov, JETP **28**, 283 (1955), Soviet Phys. JETP **1**, 265 (1955).

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TWO-PARTICLE EXCITATIONS OF SUPERFLUID FERMI-SYSTEMS

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It was shown in a paper by Ch'en Ch'un-Hsien[1] that the superfluidity of a gas of weakly interacting Fermi-particles can easily be explained if one "uncouples" the infinite set of coupled Schwinger equations for the Green functions, reducing it to a second order system. The characteristic spectrum of elementary excitations is determined in this way. Up to now there are, however, no prescriptions for the "uncoupling" when one studies more complicated problems, for instance, to find the two-particle excitation spectrum. In the following we state a method of obtaining a complete set of equations to determine the two-particle Green function using the formalism, proposed by Bogolyubov,[2] the u, v-transformation. Although the u, v-transformation does not contain the total number of particles, there are grounds for believing that the results obtained with it are the same as the results of a different consideration, but in a higher approximation.[3]

We shall consider a system of nonrelativistic Fermi particles, the Lagrangian density function of which has the form

$$L = \sum_s \psi_s^+(x)\,[-i\partial/\partial t + \nabla^2/2M + E_F]\,\psi_s(x) + L_{\text{int}},$$

$$L_{\text{int}} = (g^2/2)\sum_{ss'} \psi_s^+(x)\,\psi_{s'}^+(x)\,\psi_{s'}(x)\,\psi_s(x). \qquad (1)$$

(the interaction is, as usual, localized in a spherical shell $E_F - \omega < E < E_F + \omega$). We shall determine the time dependent functions ψ and ψ^+ in the interaction representation

$$\psi(\mathbf{x}, t) = e^{iH_0t}\,\psi(\mathbf{x}, 0)\,e^{-iH_0t};\quad \psi^+(\mathbf{x}, t) = e^{-iH_0t}\psi^+(\mathbf{x}, 0)\,e^{iH_0t},$$

$$H_0 = \int \psi^+(\mathbf{x}, 0)\,[-\nabla^2/2M - E_F]\,\psi(\mathbf{x}, 0)\,dx. \qquad (2)$$

We shall consider two functions

$$F = \langle T(\psi\psi\psi^+\psi^+S)\rangle / \langle TS\rangle, \quad \Phi = \langle T(\psi\psi^+\psi^+\psi^+S)\rangle / \langle TS\rangle,$$

where $S = \exp\{i\int L_{\text{int}}\,dx\}$; the averaging is over the state determined by the vector C. If C is the wave function of the ground state of the system C_V, $\Phi = 0$, and F is the exact two-particle Green function. Our approximate method consists in approximating the exact ground state function C_V by the function of the "vacuum without interaction," introduced by N. N. Bogolyubov.[2] Expanding

$$\psi_s = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} a_{\mathbf{k}s}\exp\left\{i\mathbf{kx} - i\left(\frac{k^2}{2M} - E_F\right)\right\},$$

we determine the new Fermi amplitudes $\alpha_{\mathbf{k}s}$ which are connected with the old $a_{\mathbf{k}s}$ through the u, v-transformation. The normalized function C satisfies the relation $\alpha_{\mathbf{k}s}C = 0$, and u and v must be found from the condition that the average energy value of the system be a minimum.

We shall construct the equations for F and Φ in the weak coupling approximation using the generalized Wick theorem[4] and the following rules: a) the system of equations must be complete, b) the spin dependence of F and Φ and also the additional time dependence of Φ must be the same as for $g^2 = 0$, c) the integral kernels in the equations must be of the kind $\langle T(\psi\psi^+S)\rangle/\langle TS\rangle$ (up to terms of order g^2) which corresponds to taking into account a number of terms of second order in g^2. Determining these integral kernels, going over to Fourier components for all functions and throwing away terms described by unconnected diagrams we obtain after a number of transformations a system of equations for the Fourier components of F and Φ which are integrated over the relative four-momentum (K is the total four-momentum)

$$A(K)\,F(K) + B(K)\,\Phi(K) = F_0(K);$$

$$C(K)\,F(K) + D(K)\,\Phi(K) = \Phi_0(K); \qquad (3)$$

A, B, C, D, F_0, and Φ_0 are some complicated functions.

The energies E_2 of the two-particle excitations are defined as the zeroes of the determinant $\begin{vmatrix} A & B \\ C & D \end{vmatrix}$ relative to the fourth component of the vector K. The coefficients A, B, C, D must be determined