starting from second order equations.

The assumption b = g is, however, as we have already seen, totally unjustified.

[1] K. Popov, JETP **28**, 257 (1955), Soviet Phys. JETP **1**, 336 (1955).

[2] Kh. Karanikolov, JETP **28**, 283 (1955), Soviet Phys. JETP **1**, 265 (1955).

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TWO-PARTICLE EXCITATIONS OF SUPERFLUID FERMI-SYSTEMS

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IT was shown in a paper by Ch'en Ch'un-Hsien[1] that the superfluidity of a gas of weakly interacting Fermi-particles can easily be explained if one "uncouples" the infinite set of coupled Schwinger equations for the Green functions, reducing it to a second order system. The characteristic spectrum of elementary excitations is determined in this way. Up to now there are, however, no prescriptions for the "uncoupling" when one studies more complicated problems, for instance, to find the two-particle excitation spectrum. In the following we state a method of obtaining a complete set of equations to determine the two-particle Green function using the formalism, proposed by Bogolyubov,[2] the u, v-transformation. Although the u, v-transformation does not contain the total number of particles, there are grounds for believing that the results obtained with it are the same as the results of a different consideration, but in a higher approximation.[3]

We shall consider a system of nonrelativistic Fermi particles, the Lagrangian density function of which has the form

$$L = \sum_s \psi_s^+(x)\left[-i\partial/\partial t + \nabla^2/2M + E_F\right]\psi_s(x) + L_{\text{int}}.$$

$$L_{\text{int}} = (g^2/2)\sum_{ss'} \psi_s^+(x)\,\psi_{s'}^+(x)\,\psi_{s'}(x)\,\psi_s(x). \qquad (1)$$

(the interaction is, as usual, localized in a spherical shell $E_F - \omega < E < E_F + \omega$). We shall determine the time dependent functions ψ and ψ^+ in the interaction representation

$$\psi(\mathbf{x}, t) = e^{iH_0 t}\,\psi(\mathbf{x}, 0)\,e^{-iH_0 t};\quad \psi^+(\mathbf{x}, t) = e^{-iH_0 t}\psi^+(\mathbf{x}, 0)\,e^{iH_0 t},$$

$$H_0 = \int \psi^+(\mathbf{x}, 0)\left[-\nabla^2/2M - E_F\right]\psi(\mathbf{x}, 0)\,dx. \qquad (2)$$

We shall consider two functions

$$F = \langle T(\psi\psi\psi^+\psi^+ S)\rangle / \langle TS\rangle, \quad \Phi = \langle T(\psi\psi^+\psi^+\psi^+ S)\rangle / \langle TS\rangle,$$

where $S = \exp\{i\int L_{\text{int}}\,dx\}$; the averaging is over the state determined by the vector C. If C is the wave-function of the ground state of the system C_V, $\Phi = 0$, and F is the exact two-particle Green function. Our approximate method consists in approximating the exact ground state function C_V by the function of the "vacuum without interaction," introduced by N. N. Bogolyubov.[2] Expanding

$$\psi_s = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}} a_{\mathbf{k}s}\exp\left\{i\mathbf{k}\mathbf{x} - i\left(\frac{k^2}{2M} - E_F\right)\right\},$$

we determine the new Fermi amplitudes $\alpha_{\mathbf{k}s}$ which are connected with the old $a_{\mathbf{k}s}$ through the u, v-transformation. The normalized function C satisfies the relation $\alpha_{\mathbf{k}s}C = 0$, and u and v must be found from the condition that the average energy value of the system be a minimum.

We shall construct the equations for F and Φ in the weak coupling approximation using the generalized Wick theorem[4] and the following rules: a) the system of equations must be complete, b) the spin dependence of F and Φ and also the additional time dependence of Φ must be the same as for $g^2 = 0$, c) the integral kernels in the equations must be of the kind $\langle T(\psi\psi^+ S)\rangle/\langle TS\rangle$ (up to terms of order g^2) which corresponds to taking into account a number of terms of second order in g^2. Determining these integral kernels, going over to Fourier components for all functions and throwing away terms described by unconnected diagrams we obtain after a number of transformations a system of equations for the Fourier components of F and Φ which are integrated over the relative four-momentum (K is the total four-momentum)

$$A(K)F(K) + B(K)\Phi(K) = F_0(K);$$

$$C(K)F(K) + D(K)\Phi(K) = \Phi_0(K); \qquad (3)$$

A, B, C, D, F_0, and Φ_0 are some complicated functions.

The energies E_2 of the two-particle excitations are defined as the zeroes of the determinant $\begin{vmatrix} A & B \\ C & D \end{vmatrix}$ relative to the fourth component of the vector K. The coefficients A, B, C, D must be determined

for small $C = \omega e^{-1/\rho}$, $|K|/C$, E_2/C ($\rho = g^2 \times (dn/dE)_{E=E_F}$). The secular equation becomes in that case of the form

$$(E_2^2 - s^2|K|^2/3) + \rho(s|K|/C)^2 f(E_2/s|K|) = 0. \quad (4)$$

In zeroth approximation

$$E_2 = s|K|/\sqrt{3}; \quad s = k_F/M, \quad (5)$$

which agrees with the result of Bogolyubov[2] and Galitskiĭ.[5] It is necessary to note that these authors found the energy E_2 by studying a model but not the true Hamiltonian. Such a procedure leads as a matter of principle to difficulties when one tries to determine corrections to E_2. Among other things, the approximate method stated in the foregoing enables us in principle to increase the accuracy of determining E_2 by improving the approximation in the wave function (and apart from this, of course, by calculating terms of higher order in g^2). In the framework of this method one can completely analogously study also more complicated than two-body excitations of Fermi-systems.

In conclusion I express my gratitude to Academician N. N. Bogolyubov for suggesting this work and to D. V. Shirkov for useful discussions.

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[2] Bogolyubov, Tolmachev, and Shirkov, Новый метод в теории сверхпроводимости (A New Method in the Theory of Superconductivity), M., Acad. Sci. U.S.S.R., 1958, Fortschr. Physik **6**, 605 (1958).

[3] N. N. Bogolyubov, Usp. Fiz. Nauk **67**, 549 (1959), Soviet Phys. Uspekhi **2**, 236 (1959).

[4] N. N. Bogolyubov and D. V. Shirkov, Introduction into the Theory of Quantum Fields, Interscience, 1959.

[5] V. M. Galitskiĭ, JETP **34**, 1011 (1958), Soviet Phys. JETP **7**, 698 (1958).

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ON THE INFLUENCE OF THE PAULI PRINCIPLE AND OF SHORT-RANGE NUCLEAR FORCES ON THE ABSORPTION OF PHOTONS BY NUCLEI IN THE OSCILLATOR MODEL

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As has been shown by Brink,[1] the collective[2] and the independent-particle[3] descriptions are identical in the case of the oscillator potential. This is due to the circumstance that the Schrödinger equation is separable in this case both in the single-particle and the Jacobi coordinates. In particular one can take for one of the Jacobi coordinates the difference of the coordinates of one proton and one neutron. Thus in this case the two-nucleon (quasi-deuteron) mechanism[4] will also be identical with the previous two.

This equivalence is violated on going over to a real nucleus, mainly because of the short range of the nuclear forces and because of the Pauli principle.

The influence of the short-range forces can be estimated taking for the zeroth approximation the oscillator Hamiltonian H_{osc} and considering $H - H_{osc}$ as a perturbation. In the zeroth order the nuclear wave function is a product wave function. In first order this multiplicative character will be violated. The separability of a particular coordinate (i.e., the degree of applicability of the corresponding mechanism) can reasonably be indicated by the integral $N_{\sigma_0\tau_0}$ of the square of the modulus of the nonfactorizable part of the wave function:

$$N_{\sigma_0\tau_0} = \sum_{\sigma,\tau}{}' \left\{ \frac{\langle f_\sigma \varphi_\tau | H - H_{osc} | f_{\sigma_0}\varphi_{\tau_0}\rangle}{E_{\sigma\tau} - E_{\sigma_0\tau_0}} \right\}^2. \quad (1)$$

Here f_σ and φ_τ are the zeroth-approximation oscillator functions corresponding to the factorized coordinate and the remaining variables respectively; the indices zero indicate the ground state. The function $N_{\sigma_0\tau_0}$ equals zero for a function factorizable in the coordinate singled out and equals unity for a function containing no factorizable part. The evaluation of (1) requires the application of the Talmi transformation[5] and is in general very involved.