a reaction of inelastic "acceleration" in the region up to 100 ev is $10^3 - 10^6$ times greater than the cross section of the inverse reaction — the excitation of the isomer state by a fast neutron.

Naturally, one must expect the cross section to obey the 1/v law at small energies, if there is no resonant level in the vicinity.

For a rough estimate of the ratio of the widths of the reactions (n, n′) and (n, γ) we assume[1] that for nuclei with A > 80 the neutron and radiation widths become equalized at ~1 kev. Since the escaping neutron has a large momentum l, then for nuclei with kR < 1 (see reference 1)

$$\Gamma_{n'} / \Gamma_\gamma \approx \sqrt{E'}\,(E' A^{2/3} \cdot 10^{-4})^l / [(2l-1)!!]^2, \qquad (1)$$

where E is the energy of the emitted neutron in kev.

Isomer	T	I_m	I	E', kev	$10^3 \frac{\Gamma_{n'}}{\Gamma_\gamma}$
$_{41}Nb^{91m}$	64 d	1/2−	9/2+	104	0.4
$_{43}Tc^{97m}$	91 d	1/2−	9/2+	99.2	0.4
$_{47}Ag^{110m}$	270 d	6−	2+	116	0.9
$_{48}Cd^{113m}$	5.11 yr	11/2−	1/2+	265	$2 \cdot 10^{-3}$
$_{52}Te^{125m}$	58 d	11/2−	3/2+	110	1

For certain long-lived isomers[2] the table lists the lifetimes T, the spins I and I_m and the parities of the final and initial states, the transition energy E′, and also the ratio $\Gamma_{n'}/\Gamma_\gamma$, estimated from formula (1). For all the isomers given, with the exception of Cd^{113m}, the (n, n′) reaction is accompanied by spin flip, since ΔI = 4, and the parities of the initial and final states are opposite. The table lists the values of $\Gamma_{n'}/\Gamma_\gamma$, for $l = 3$. For $l = 5$ these values are $10^4 - 10^5$ times smaller. Thus, given the intensity of the fast neutrons produced in the (n, n′) reaction, we can determine the spin of the compound nucleus for these isomers.

In conclusion, I express my gratitude to V. N. Gribov, A. D. Piliya, and M. I. Pevzner for discussion of our work.

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ON THE INCLUSION OF EXCHANGE IN THE THEORY OF COLLISIONS

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THE formulation of the problem of scattering of electrons by atoms, which has been given by Drukarev,[1] reduces in the case of the hydrogen atom to the solution of the system of integro-differential equations

$$(\Delta_1 + k_\alpha^2) F_\alpha^\pm (\mathbf{r}_1) = \underset{\beta}{S} F_\beta^\pm (\mathbf{r}_1)\, 2 \int \psi_\alpha^* (\mathbf{r}_2) \left(\frac{1}{r_{12}} - \frac{1}{r_1}\right) \psi_\beta (\mathbf{r}_2)\, d\mathbf{r}_2 \pm A_\alpha^\pm (\mathbf{r}_1), \qquad (1a)$$

with boundary conditions

$$F^\pm (\mathbf{r}) \underset{r\to\infty}{\sim} \delta_{\alpha 0} \exp (i k_\alpha z) + a_\alpha^\pm (\theta, \varphi)\, r^{-1} \exp (i k_\alpha r), \qquad (1b)$$

where α and β denote the sets of quantum numbers characterizing the hydrogen atomic states, for example (nlm) or (klm); S denotes summation over the discrete and integration over the continuous spectrum; $k_\alpha^2 = 2(E - \epsilon_\alpha)$, where ϵ_α are the energy levels of the hydrogen atom;

$$A_\alpha^\pm (\mathbf{r}_1) = \underset{\beta}{S} \psi_\beta (\mathbf{r}_1)\, 2 \int \psi_\alpha^* (\mathbf{r}_2) \left(\frac{1}{r_{12}} + \varepsilon_\beta - \frac{1}{2} k_\alpha^2\right) F_\beta^\pm (\mathbf{r}_2)\, d\mathbf{r}_2 = \int \psi_\alpha^* (\mathbf{r}_2) (H - E) \underset{\beta}{S} \psi_\beta (\mathbf{r}_1) F_\beta^\pm (\mathbf{r}_2)\, d\mathbf{r}_2, \qquad (2)$$

$$H = -\tfrac{1}{2}\Delta_1 - \tfrac{1}{2}\Delta_2 - 1/r_1 - 1/r_2 + 1/r_{12}. \qquad (3)$$

For a unique solution the function $\Phi^\pm(\mathbf{r}_1, \mathbf{r}_2) = \underset{\alpha}{S} [F_\alpha^\pm(\mathbf{r}_1)\psi_\alpha(\mathbf{r}_2) \pm F_\alpha^\pm(\mathbf{r}_2)\psi_\alpha(\mathbf{r}_1)]$ must be required to have the asymptotic form*

$$\Phi^\pm (\mathbf{r}_1, \mathbf{r}_2) \underset{r_1\to\infty}{\sim} \underset{\alpha}{S} \psi_\alpha (\mathbf{r}_2) [\delta_{\alpha 0} \exp (i k_\alpha z_1) + a_\alpha'^\pm (\theta_1, \varphi_1)\, r_1^{-1} \exp (i k_\alpha r_1)]. \qquad (1c)$$

For practical calculations, we solve instead of the infinite system (1a) a reduced system consisting, for example, of one or two equations. In this case one obtains appreciable differences between a_α^+ and a_α^-.

It is not always sufficiently clearly recognized that for an accurate solution of the infinite system the relation

$$a_\alpha^+ = \alpha_\alpha^- \qquad (4)$$

must be satisfied.

To prove this it is sufficient to show that there is a solution for which $A_\alpha^\pm = 0$. But this property describes the solution which is distinguished by the absence of $A_\alpha^\pm$ in (1a) [because the function $\Phi(\mathbf{r}_1, \mathbf{r}_2) = \underset{\alpha}{S}\psi_\alpha(\mathbf{r}_1) F_\alpha(\mathbf{r}_2)$ satisfies the Schrödinger equation $(H - E)\Phi(\mathbf{r}_1, \mathbf{r}_2) = 0$].

The equality (4) becomes nearly obvious if we note that the exact solution of the Schrödinger equation need not be sought in an explicitly symmetric form but may be found first as an asymmetric solution and then symmetrized.

Further, if we calculate the cross sections using only $a_\alpha^\pm$,[1] they will be the same for both signs. However one cannot agree with such a definition. Indeed, if the wave function of the two electrons has the form $\Phi^\pm(\mathbf{r}_1, \mathbf{r}_2) = \underset{\alpha}{S}[F_\alpha(\mathbf{r}_1) \times \psi_\alpha(\mathbf{r}_2) \pm F_\alpha(\mathbf{r}_2)\psi_\alpha(\mathbf{r}_1)]$, we obtain the following expression for the radial component of the scattered flux:

$$j^\pm(\mathbf{r}) = 2\,\mathrm{Im}\Big\{\underset{\alpha}{S} f_\alpha^*(\mathbf{r})\frac{\partial}{\partial r}f_\alpha(\mathbf{r}) + \underset{\alpha\beta}{S}\psi_\alpha^*(\mathbf{r})\frac{\partial}{\partial r}\psi_\beta(\mathbf{r})\int f_\alpha^*(\mathbf{r}') \times f_\beta(\mathbf{r}')\,d\mathbf{r}' \pm \underset{\alpha\beta}{S}\Big[\psi_\alpha^*(\mathbf{r})\frac{\partial}{\partial r}f_\beta(\mathbf{r})\int f_\alpha^*(\mathbf{r}')\psi_\beta(\mathbf{r}')\,d\mathbf{r}' + f_\alpha^*(\mathbf{r})\frac{\partial}{\partial r}\psi_\beta(\mathbf{r})\int\psi_\alpha^*(\mathbf{r}')f_\beta(\mathbf{r}')\,d\mathbf{r}'\Big]\Big\}, \quad (5)$$

where the f_α differ from the F_α by the absence of incident waves.

Hence we can see that the difference between the cross sections must appear not because of the inequality of a_α^+ and a_α^- but because of the exchange term in the flux, which Drukarev ignores, since he does not consider the flux produced by the functions ψ_α of the continuous spectra. Taking account of the exchange according to references 2 and 3 is simply taking account of the flux of atomic electrons excited to corresponding levels of the continuous spectrum.

*It should be remarked that Eqs. (1b) and (1c) are not wholly accurate in the case of ionization, in which case the asymptotic form of the wave function of one electron cannot be given independently of the other electron.

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ON THE PAPER OF V. A. BELOKON': "THE PERMANENT STRUCTURE OF SHOCK WAVES WITH JOULE DISSIPATION"

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In his discussion of the magnetohydrodynamic shock wave, V. A. Belokon'[1] writes down equations for the momenta and for the heat flow for the one-dimensional stationary motion of a non-viscous, non-heat-conducting, but electrically conducting gas. It is asserted that this system of equations leads, on the one hand, to the necessity of the existence of a maximum of the entropy inside the region of flow, and on the other hand, to the impossibility of a decrease in the entropy. On the basis of these facts, the author comes to the following conclusion: "In view of the absurdity of a continuous solution, we consider it unavoidable to postulate a Riemann isentropic discontinuity in the flow parameters within a compression wave of any amplitude, by analogy with the isothermal discontinuity for purely heat-conducting gases." The magnetic field at this discontinuity is considered continuous.

We cannot agree with this basic postulate. Indeed, if the magnetic field is continuous in the passage through the discontinuity and the gas is considered non-viscous and non-heat-conducting before and after the discontinuity, then this is a gas-dynamical discontinuity which always leads to an increase in the entropy.

The same kind of problem concerning the structure of the shock wave was considered earlier by Burgers.[2] He showed that two cases are possible: a) in strong magnetic fields all parameters inside the region of flow, including the entropy, change monotonically. The entropy reaches its maximum value at the point corresponding to $x = +\infty$; b) in weak magnetic fields the region of flow consists of two parts, the region of continuous variation of the parameters, at the end of which the entropy reaches some value $S^* \neq S_\infty$, and a compression discontinuity with a constant field, at which the entropy increases from S^* to S_∞.

The problem proposed by Belokon', therefore, has a complete solution without any additional postulates, and the postulate put forward in his paper is incorrect.

It is furthermore impossible to accept the following assertion of the author with respect to the