effect of viscosity on the structure of the shock wave:

"If dissipation occurs by way of viscosity in addition to Joule heating, then the isentropic discontinuity mentioned above will be smoothed out for all amplitudes, since for vanishing viscosity the curves for continuous evolution of the flow parameters pass arbitrarily close to the isentropic line S_{max}, coinciding with it only in a single point, at $+\infty$."

The problem of the family of integral curves for the one-dimensional stationary flow of a viscous and electrically conducting gas, corresponding to the problem of the structure of the magnetohydrodynamic shock wave, was discussed in detail by Ludford.[3] He showed that in the case of vanishing viscosity the corresponding curve approaches the curve obtained by Burgers.

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[1] V. A. Belokon', JETP **36**, 1316 (1959), Soviet Phys. JETP **9**, 932 (1959).

[2] J. M. Burgers, Penetration of a Shock Wave into a Magnetic Field. Magnetohydrodynamics, Ed. R. K. M. Landshoff, Stanford (1957) (Russ. Transl., Atomizdat, 1958).

[3] C. S. S. Ludford, J. Fluid Mechanics **5**, 67 (1959).

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ON THE PHASE OF THE SCATTERED WAVE (A REPLY TO V. V. MALYAROV)

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THE possibility of determining the phase of the scattered wave by numerical methods with the help of the differential equation for the phase

$$d\delta / d\rho = -[l(l+1)/\rho^2 + U(\rho)]\sin^2(\rho + \delta) \qquad (1)$$

was discussed in reference 1, and it was shown that it is necessary in this case to find the solution which satisfies the condition

$$\sin(\rho + \delta) = \rho/(l+1) \text{ for } \rho \to 0. \qquad (2)$$

V. V. Malyarov[2] correctly observed that one and the same symbol is used in reference 1 to denote two proportional constants, and also that the condition $\int^{\infty} U(\rho)\,d\rho < C$ should be replaced by $\int^{\infty} |U(\rho)|\,d\rho < C$.

Then V. V. Malyarov proposes, incorrectly, to obtain the second term in the expansion for $\delta(\rho)$ from the expression for $\delta(\rho)$ of reference 1, overlooking the fact that this expression was formally obtained from (1) and (2). Actually this term can be obtained by substituting the series

$$\delta(\rho) = -l\rho/(l+1) - a_2\rho^2 - a_3\rho^3 - \ldots$$

in (1). We then obtain for $\delta(\rho)$ the expansion

$$\delta(\rho) = -l\rho/(l+1) - \gamma_0\rho^2/2(l+1)^3 - \ldots, \qquad (3)$$

while the method of V. V. Malyarov would lead to the incorrect expression

$$\delta(\rho) = -l\rho/(l+1) - \gamma_0\rho^2/2(l+1)^2.$$

Finally, V. V. Malyarov alleges that the note[1] "... is of no interest for scattering theory and can lead the reader into error," and he bases this assertion on the following argument: "The use of the additional condition (5) is justified for $\rho \to \infty$, when $A(\rho) \to$ const and $\delta(\rho) \to$ const. For $\rho \to 0$, however, any other supplementary condition could be used instead of (5). Different additional conditions correspond to different phases $\delta(\rho)$. The problem is indeterminate. It follows from the definition of the phase of the scattered wave that the phase $\delta(\rho)$ obtained with the help of such a supplementary condition is not the phase of the scattered wave."

This remark of V. V. Malyarov cannot be applied to the contents of our note, because there we obtained (6) and (7) by using the classical method of variation of the arbitrary Lagrange constants, and we were concerned with the determination of the phase of the scattered wave at infinity [i.e., finding the limit of $\delta(\rho)$ for $\rho \to \infty$].

Applying (1) and (7) of reference 1 to the known cases where the scattering problem can be solved exactly, one easily sees that this phase is indeed the phase of the scattered wave, according to the interpretation of scattering theory.

The remarks of V. V. Malyarov concerning the note[1] are therefore without substance.

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[1] F. S. Los', On the Phase of the Scattered Wave, JETP **33**, 273 (1957); Soviet Phys. JETP **6**, 211 (1958).

[2] V. V. Malyarov, JETP **34**, 1039 (1958); Soviet Phys. JETP **7**, 719 (1958).

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