INVESTIGATION OF THE HIGH ENERGY μ-MESON FLUX IN EXTENSIVE ATMOSPHERIC SHOWERS

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Experimental data on the possible existence of narrow beams of μ mesons (diameter in observation plane < 0.5 m) have been obtained with the aid of apparatus which permits one to study simultaneously extensive atmospheric showers on the surface of the earth and underground. Data on extensive atmospheric showers obtained at the surface of the earth can be used to construct a picture of generation of narrow μ meson beams.

INTRODUCTION

THE experimental study of μ meson flux of high energy in the composition of an extensive atmospheric shower is of great interest from the viewpoint of the study of nuclear interactions at super high energies. The μ mesons are themselves the "tracks" of nuclear interactions which reflect both the angular distribution of secondary particles and their energy spectrum. The connection of the quantitative characteristics of the μ-meson flux of high energy with the quantitative characteristics of the event of interaction of nuclear-active particles of an extensive atmospheric shower is simplified, since the decay of the high energy μ mesons plays almost no role, while the ionization losses in the atmosphere are small in comparison with the energies of the μ mesons.

This permits us to assume that the flux of high energy μ mesons belonging to the extensive atmospheric shower reflects at sea level the whole set of nuclear interactions which have taken place in the atmosphere in the development of the shower.

It can be expected that the separate acts of nuclear interaction can also leave a track in the lateral distribution of μ mesons of high energy. Actually, against the background of the equilibrium density fluctuations of μ mesons, irregularities can appear as the result of groups of μ mesons which maintain approximately the direction of the μ mesons given by the last nuclear interaction in the event. Thus the study of the lateral distribution of the μ-meson flux of high energy can give experimental information bearing on the events of nuclear interaction at super high energies.

We have carried out an investigation of the high-energy μ-meson flux underground on apparatus, which makes it possible to obtain simultaneously the identical data on the extensive atmospheric shower on the surface of the earth. The arrangement described below forms part of the installation at Moscow State University for the exhaustive investigation of extensive atmospheric showers. The work was carried out during 1957-1958.

The present work is devoted to a study of the peculiarities in the lateral distribution of high-energy μ mesons. Other problems will be taken up in subsequent papers.

DESCRIPTION OF THE APPARATUS

One of the most important difficulties in the simultaneous study of μ mesons of high energy underground and extensive atmospheric showers on the surface of the earth is the necessity of recording the extensive atmospheric shower over a large area. Actually, a large region of possible positions of the axis of the shower on the surface of the earth, the dimensions of which are determined by the angular distribution of the axes of the extensive atmospheric shower in the depths of the earth, corresponds to a fixed position of the axis of the shower under the earth. Therefore, apparatus which studies the showers individually and which possesses a sufficient aperture ratio should permit us to determine the axis and the number of particles in the shower over an area comparable with the area of the region indicated.

The arrangement we used on the surface of the earth made it possible to record the showers individually within a circle of radius 25 m. At a maximum depth in the earth of 40 m and with the location of the meson detector directly under the center of this circle one could effectively study the central region of showers with a number of particles from 10^4 to 10^6.

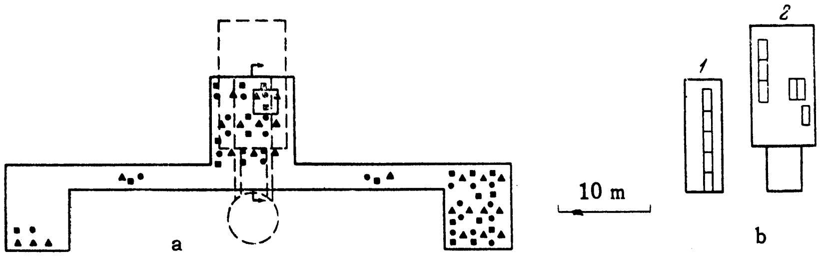

FIG. 1. Geometrical arrangement of the counters: a) on the surface of the earth, b) in the underground chamber. ▲ – group of 24 counters with counter area 330 cm², ■ – group of 24 counters with counter area of 100 cm², ● – group of 24 counters with counter area 24 cm². The dashed lines indicate the contours of the underground chambers. The open rectangles are banks of detectors of μ mesons of the type Fig. 3a.

For counting the extensive atmospheric shower and the meson flux in it we used the well-known method of correlated hodoscopes (see reference 1). The total number of Geiger-Müller counters contained in the hodoscope on the surface of the earth amounted to 1680. The geometry of the location of the counters on the earth's surface is shown in Fig. 1a. The underground chambers were along the vertical under the central chamber of Fig. 1a. The geometry of the position of the counters in the underground chamber is shown in Fig. 1b. The cross section of the arrangement along the vertical to the line ↑—↑ marked in Fig. 1a is shown in Fig. 2. The amount of material on the underground chamber was determined by a vertical profile of the ground and the building (the density of the ground amounted to 1.7 g/cm³). Moreover, for the same purpose, controlled measurements of the vertical intensity of cosmic rays were carried out in the underground chambers with the aid of a telescope of counters separated by 10 cm of lead. The measured intensity amounted to $(1.5 \pm 0.05) \times 10^{-3}\, l/\mathrm{cm}^2$ sec. sterad in chamber 1 and $(0.8 \pm 0.1) \times 10^{-3}\, l/\mathrm{cm}^2$ sec. sterad in chamber 2.

The resultant intensities were compared with the data given in reference 2 on the dependence of the vertical intensity of cosmic rays on depth underground. The depth of the ground determined in this way amounted to: for chamber 1 — 20 m of water equivalent, for chamber 2 — 40 m of water equivalent along the vertical. The minimum energy of μ mesons necessary for penetration to this depth along the vertical amounts to 5 and 10 Bev.

Recording of mesons in the underground chambers was carried out by means of Geiger-Müller counters included in the hodoscope and screened by lead and iron. Figure 3a shows a group of counters which, together with the screen, forms a meson recording unit. The dimensions of the counters beneath the screen were 60 × 550 mm. The total number of counters was 120 in chamber 1 and 144 in chamber 2. In other setups the number of counters in chamber 2 was increased to 480.

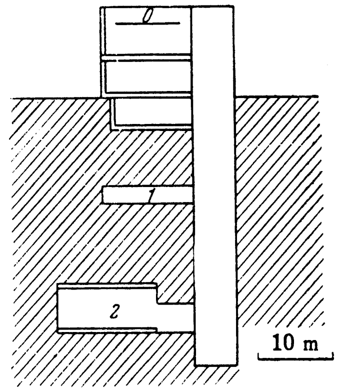

FIG. 2. Vertical profile of the chamber along the line ↑–↑ of Fig. 1a. 0 – level of position of the counters on the surface of the earth, 1 – underground chamber 1, 2 – underground chamber 2.

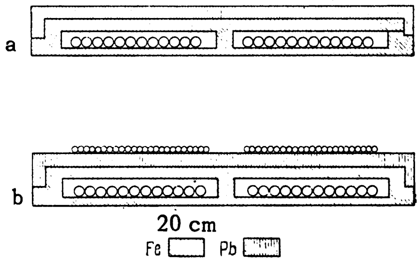

FIG. 3. Blocks of detectors of μ mesons.

An iron covering on the ceiling and walls of the chamber, the thickness of which amounted on average to 7.5 cm of iron or 4.7 t-units, served as an additional screen in chamber 2.

Hodoscopes with cold-cathode thyratrons of the Korablev type[3] were used in the research. The resolving time of the units in the underground chambers amounted to 15 — 20 microseconds; the number of random coincidences became comparable with the number of true ones at a meson density of 0.01 particles/m². This density was the lower limit of the recorded range of densities and made possible the study of the central region of the shower with a number of particles up to 5000. The upper limit was determined by the density at which the number of fired counters drew close to the total number of counters. For the total number of counters in the underground chamber this limit amounted to 100 particles/m², and for the chamber as a whole was not achieved experimentally throughout the whole time of operation of the apparatus. However, cases were observed in which the density at isolated points in the chamber exceeded this limit. During the entire period of operation of the apparatus, a control was maintained of the resolving time of the individual hodoscopic components and of the operation of the counters. We also note that the possibility of parasitic coupling (induction) between the individual components of the applied hodoscope was eliminated.

Operation of the apparatus was carried out by various means. In the present paper experimental material is used which was obtained in the operation of the apparatus with coincidences of six groups of counters each of area 0.132 m^2. These groups of counters were located in the central chamber at ground level practically on the same vertical with the underground chambers. In the particular variant that is described the apparatus was operated over a period of 1740 hours.

METHOD OF RECORDING THE μ-MESON FLUX

The method of recording μ-meson flux with the use of Geiger-Müller counters that we have described possesses the advantage that it permits the use of a large area for the recording. However, the hodoscopic pictures thus obtained need special analysis for the separation of cases actually connected with the μ-meson flux. Unscreened counters can be triggered by the electron-photon component in equilibrium with the μ-meson. The former arises as a result of formation of δ electrons by μ mesons, and also due to the radiation retardation of μ mesons and to the direct creation by them of electron-positron pairs. The μ meson can therefore be accompanied (with a definite probability) by a current of electrons which is also recorded by the unscreened counters. The iron jacket of the ceiling in chamber 2 leads to an increase of particles in such a flow, although it decreases the mean energy of particles in it. Therefore, in the investigation of rare events (for example, the passage of the axis of the extensive atmospheric shower through the underground chamber) the frequency of which is comparable with the frequency appearance of δ showers, screening of the counters appears necessary.

Let us estimate the probability that a μ meson with energy W will have an electron-photon accompaniment, which passes through the filter we have used (12 t-units of lead and 4 t-units of iron). We obtain an upper estimate of this probability if we assume that the filter contains 16 t-units of lead. The probabilities of exciting an electron-photon shower capable of giving three particles under a filter with energies > 0 to one meson with energy W is equal to

$$C(W) = \int_0^W \frac{N^3(E)}{3!} \exp[-N(E)]\, U(W, E)\, dE,$$

where $N(E) = 3 \times 10^{-3} (E/\beta_{Pb})^{1.3}$ is the result of approximation for cascade showers in lead developed in reference 4; $N^3(E) \exp[-N(E)]/3!$ is the Poisson distribution; $U(W, E)$ is the equilibrium spectrum of electrons and photons accompanying a μ meson with energy W in iron; β is the critical energy:

$$U(W, E)\, dE \approx \int_E^W \{C_\delta(W, E')\, dE' + C_{h\nu}(W, E')\, dE' + C_e(W, E')\, dE'\} \frac{E'}{2.3E^2}\, dE.$$

Here $C_\delta(W, E')$, $C_{h\nu}(W, E')$ and $C_e(W, E')$ are the probabilities of transfer through one t-unit of energy E' to a δ electron, Bremsstrahlung photon and an electron-positron pair, respectively. For a μ meson with energy $W = 10^{10}$ ev, we obtain $C \approx 3 \times 10^{-4}$ and for $W = 10^{13}$ ev, we obtain $C \approx 1$.

Thus, introduction of a screen above the counters permits a significant decrease in the probability of recording electrons and photons arising from the ground. But, on the other hand, μ mesons create δ electrons and δ showers in the screen itself, which can mask the groups of mesons. It is easy to differentiate such showers from groups of mesons if the mesons are recorded simultaneously above the screen by means of an additional row of counters. For μ mesons with $W \ll 10^{13}$ ev, the requirement of correspondence of the firing of the counters of the upper and lower rows practically eliminates the ambiguity in interpretation in the triggering of counters by mesons.

With the aim of investigating the pattern of the distribution of μ mesons in an individual shower, four units of type a of Fig. 3 were changed in chamber 2 to a two-row unit of type b of Fig. 3. In the upper row, counters were used with a smaller diameter (30 × 550). The frequency of appearance of δ electrons and δ showers under the screen classified according to the number of fired counters, was determined from the results of the action of units of type b and is plotted in Table I.

The frequency of appearance of δ showers was shown to be less than for particles which arise for detectors at the surface of the earth,[1] owing to the low altitude of the cavity of the block.

RESULTS

In the present work results are given of the work with the apparatus described, pertaining to the possible existence of irregular spatial distribution of μ-meson flux of high energy in an extensive atmospheric shower.

In eight hundred hours of operation of the two-

TABLE I

a	2	3	4	5	6
b	3800	3800	3800	3800	3800
c	105	23	8	4	3
d	4.0±0.4	0.6±0.1	0.2±0.08	0.1±0.05	0.08±0.05

a is the number of fired counters in the δ shower; b is the number of μ mesons which penetrate the unit, c is the number of δ showers, d is the percent of δ showers per meson.

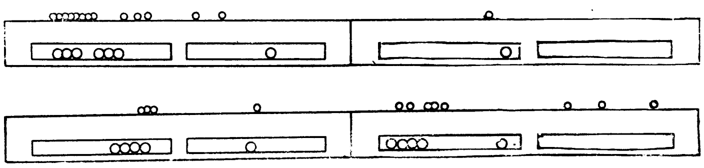

FIG. 4. Hodoscopic pictures for recording groups of μ mesons. Fired counters in units of the type of Fig. 3b are denoted by the circles.

row units of total area 3.1 m^2, we recorded 17 events of simultaneous firing of three and more counters in a line in an upper or lower row (see, for example, Fig. 4).

The readings of the upper row of counters establish the fact that the events recorded were not connected with electron-photon showers from the ground which are found close to the maximum of their development (the mean density of mesons obtained on the upper row of counters ρ_u and the density of mesons determined by the lower row ρ_l are the same: $\rho_u = 2.7 \pm 0.3$ particles/m^2 and $\rho_l = 2.2 \pm 0.3$ particles/m^2). It is possible that such events can be connected with very "young" electron-photon showers from the ground, perhaps with the simultaneous generation of a young shower in the ground and a second shower in the screen by a single meson. A precise estimate of the probability of appearance of such events is made difficult by the fact that the number of particles in the assumed δ showers is unknown experimentally. Moreover, data are lacking on the spectrum of μ mesons in the region $W > 10^{12}$ ev in the composition of extensive atmospheric showers. We shall consider below the set of experimental data which takes into account events that follow a group of μ mesons.

The following data can be compared for each group: 1) the minimum number of mesons in the group, equal to the number of counters of the upper row fired in the group; 2) the total amount of mesons recorded in a given shower in the underground chambers; 3) location of the axis of the shower on the surface of the earth and the number of particles in the shower. A summary of these data with respect to all recorded events is given in Table II.

The location of the axis and the number of particles in the shower was determined by the method suggested in reference 5, and with the aid of the arrangement of reference 6, which employs the knowledge of the spatial distribution function of the electrons in the shower. The accuracy of the determination of the axis and the number of particles in the shower were, respectively, 1 m and 20% close to and inside the surface chamber, and 4–5 m and 40% at distances of the order of 20 m from the surface chamber.

We shall show that the chosen groups of μ mesons cannot be the random accumulation of mesons connected with the Poisson statistical fluctuations of the particle flux. Bearing in mind that the characteristic phenomenon in the passage of a group of mesons is the successive firing of the counters in a row of unit b of Fig. 3, we shall consider each picture as consisting of group firings of the following classes: first class, o●o; second class,

TABLE II

№	a	b	c	d	e	f
1	$2\cdot10^5$	8	1	—	3	5
2	$5\cdot10^4$	13	1	—	3	2
3	$2\cdot10^5$	15	1	—	6	5
4	$2.5\cdot10^5$	12	1	—	5	3
5	$3\cdot10^5$	12	1	—	10	11
6	$2\cdot10^5$	25	1	—	9	2
7	$1\cdot10^5$	10	1	—	5	6
8	$2\cdot10^6$	11	2	1	5;4	18
9	$8\cdot10^5$	20	1	—	6	8
10	$5\cdot10^5$	20	1	—	6	11
11	$8\cdot10^5$	7	1	—	4	7
12	$1\cdot10^6$	25	1	—	5	11
13	$7\cdot10^5$	17	2	2	4;6	8
14	$3.5\cdot10^6$	15	2	1.5	4;5	14

a – number of particles in the shower; b – distance in meters from the axis of the shower on the earth's surface to the vertical passing through the point of observation of the group of mesons; c – number of groups of mesons in the shower; d – distance in meters between groups; e – number of fired counters in the vertical row of unit b of Fig. 3 at the point of passage of the group of mesons; f – number of mesons recorded in a given shower apart from the group of mesons (on an area of 4.35 m^2).

o●●o; third class, o●●●o, etc. We assume that of the total number n of counters, m were fired and the distribution of mesons in the plane of observation is uniform on the average. We shall calculate with what probability a given picture of fired counters can be observed if the trajectories of the mesons are independent.

A formula was derived in reference 7 for the probability which is of interest to us; it has the form

$$W(n, m, \nu, p) = \frac{C^p_{n-m+1}\, p!}{\prod_i \nu_i!\, C^m_n},$$

Here p is the number of groups of all classes in the given shower, ν_i is the number of groups of the i-th class. However, practical use of this formula is difficult because of the roughness of the calculations for large n. Moreover, we have it in mind to extend the same approach to the treatment of the data of single row units, making use of a knowledge of the frequency of appearance of δ showers. Therefore, we apply another approach to the determination of the probability of the given picture of fired counters by classes, based on a comparison of the theoretically expected number of groups of a given class with the experimental observations.

Under the observed conditions, the probability is m/n that a chosen counter fires, and $1 - m/n$ that it does not fire. Consequently, the probability of the event of first class is $(m/n)(1-m/n)^2$; of the second class, $(m^2/n^2)(1-m/n)^2$; of the third class, $(m^3/n^3)(1-m/n)^2$; etc. The appearance of an event of an arbitrary class, generally speaking, changes the probability of the remaining events, but if $m \ll n$, then the change is probably small. For $m \ll n$, the number of events of different classes is given by

$$M_1 = n\frac{m}{n}\left(1 - \frac{m}{n}\right)^2;$$

$$M_2 = n\frac{m^2}{n^2}\left(1 - \frac{m}{n}\right)^2 \text{ etc.}$$

For each shower, we have theoretical and experimental distributions of the events over the classes. Calculating the consistency criterion of Pearson

$$\chi^2 = \sum_i (M_{i\,\mathrm{exp}} - M_i)^2 / M_i\,.$$

we find the probability of appearance (connected with the statistical fluctuations) of a given picture of the distribution of mesons in the plane of observation.

Let us compare the frequency of appearance of pictures with groups of mesons (as a group of mesons we take events of third and higher classes) with the frequency of events expected because of statistical fluctuations. For this purpose, we analyze all pictures of fired counters in chamber 2 according to the total number of counters fired. The number of groups of mesons expected because of statistical fluctuations and the observed number of groups is shown in Table III.

TABLE III

a	5–8	9–14	15–26	>26
b	264	69	29	7
c	1	0,5	0.3	0.1
d	3	5	7	2

a – total number of discharge counters (m) under the screen in chamber 2; b – number of showers with given m; c – expected number of groups of mesons because of statistical fluctuations; d – observed number of groups of mesons.

Comparison shows that the observed groups of μ mesons cannot be accounted for by statistical fluctuations in the meson distribution. Consequently, the observed groups of mesons are peculiarities of the lateral distribution of μ mesons of high energy.

We now attempt to determine the role of such groups in the general lateral distribution of mesons of high energy. It is seen from Table II that in showers with a number of particles $N \sim 10^5$, one group of mesons is observed, while in showers with $N \sim 10^6$ cases are encountered with two groups on an area of $3.1\,\mathrm{m}^2$. It is possible that in showers with different N, the groups of mesons play different roles in the lateral distribution of the mesons. Therefore, let us consider separately showers with $N \sim 10^5$ and showers with $N \sim 10^6$. From Table II we can obtain the mean density of the meson accompaniment of a group of mesons for showers with $N < 5 \times 10^5$ and $N > 5 \times 10^5$:

$$\rho(\bar{N} = 2\cdot 10^5) = (1.0 \pm 0.2) \text{ particles/m}^2$$

$$\rho(\bar{N} = 1.3\cdot 10^6) = (2.3 \pm 0.3) \text{ particles/m}^2$$

It follows from the geometrical arrangement in chamber 2 (Fig. 1b) that these densities relate to a distance of $3-4\,\mathrm{m}$ from the group of mesons. The densities of mesons measured by the same apparatus at a distance of 14 m from the axis of the extensive atmospheric shower (description of this experiment will be given in subsequent papers) amount to (0.22 ± 0.03) particle/m^2 and (1.6 ± 0.6) particles/m^2, respectively, for showers with $\bar{N} = 2 \times 10^5$ and with $\bar{N} = 1.3 \times 10^6$. It is thus seen that in showers with $\bar{N} \sim 10^5$ a sharp increase in the density of μ mesons is observed upon approach to a group of mesons, while in showers with $N \sim 10^6$ this increase is less marked.

In this connection, it is of interest to compare the frequency of appearance of groups of μ mesons with the frequency of passage of the axes of the extensive atmospheric showers through the detector of μ mesons.

Let us now calculate the number of axes of extensive atmospheric showers with a different number of particles passing through the area of the double-row units in 800 hours of operation for the experimental setup that has been described. The intensity of the axes of extensive atmospheric showers with a number of particles N and with angles θ and φ in the underground chamber is equal to

$$(A / N^{\varkappa+1})\, dN \cdot \cos^{\nu}\theta \sin\theta d\theta d\varphi.$$

The probability of recording such a shower at the surface of the earth is

$$[1 - \exp\{-kNf(r)\sigma\}]^6,$$

where $kNf(r)$ is the density of electrons at the surface of the earth in the extensive atmospheric shower, $\sigma = 0.132\ \mathrm{m}^2$. The connection between the angle θ and r is the following: $r = H \tan\theta$, where H is the distance between the surface and underground chambers measured along the vertical. Then the number of axes passing through the area of the double row units is

$$I = \frac{A \cdot 3{,}1 \cdot 800}{N^{\varkappa+1}} dN \int\limits_0^{2\pi}\int\limits_0^{\pi/2} \cos^{\nu}\theta \sin\theta d\varphi d\theta$$
$$\times [1 - \exp\{-kNf(H\tan\theta)\sigma\}]^6.$$

Taking $\nu = 8$ in accord with reference 8, and integrating numerically, we obtain the results shown in Table IV. It is seen from the table that the frequency of the groups in showers with $N \sim 10^5$ corresponds to the frequency of the axes of the showers while the frequency of the groups of mesons in showers with $N \sim 10^6$ is much greater than the frequency of axes of such showers.

It is quite probable that in showers with a number of particles $N < 10^5$, there are no groups of μ mesons of three or more mesons, for the mesons in these groups have a smaller lateral divergence.

Up to now we have been discussing data obtained by double-row units. If we analyze the data from single-row units, it is possible to broaden the statistical material for chamber 2 and obtain data on chamber 1 at a depth of 20 m under the screen. This permits us to trace the dependence of the number of groups of mesons on the depth. The simulation of groups of mesons by the δ showers from the screen serves as an obstacle to this course. However, the frequency of appearance of the δ showers is known from the experiment with double row units. We include the probability of appearance of the δ showers, determined experimentally, in the consideration of pictures of the discharge of counters in single-row units, similarly to what was done for double-row units. Then the probability of appearance of an event of the first class does not change, of the second class becomes $(m^2/n^2)(1 - m/n)^2 + W_{\delta_2}$, where W_{δ_2} is the probability of appearance of the δ electron; for the third class, $(m^3/n^3)(1 - m/n)^2 + W_{\delta_3}$, where W_{δ_3} is the probability of appearance of the δ shower with three discharge counters, etc. The values of W_{δ_2}, W_{δ_3}, W_{δ_4} are shown in Table I. Further, we obtain new theoretical values of M_i and find the probability of pictures of discharges for individual showers, just as for double-row units. Table V lists the results obtained from an analysis of the pictures in single-row units for chambers 1 and 2. As is seen from the table, the expected value of δ showers for pictures with $m \geq 9$ is small in comparison with the number of observed groups of discharges of counters, and at the same time the frequency of the groups of mesons determined according to double-row units corresponds to the frequency observed on single-row units. Consequently, in both cases we observe the groups of mesons (for $m \geq 9$). One can draw the conclusion from Table V that the ratio of the frequencies of appearance of groups of mesons at depths of 20 and 40 meters under the screen amounts to 1 ± 0.5. For groups of mesons observed in chamber 1, a mean density of mesons accompanying the groups for showers with $10^5 < N < 5 \times 10^5$ was obtained.

This density amounted to $\rho(\bar{N} = 2 \times 10^5)$

TABLE IV

a	$1 \cdot 10^4$—$1 \cdot 10^5$	$1 \cdot 10^5$—$5 \cdot 10^5$	$5 \cdot 10^5$—10^6	$>10^6$
b	4;5	3	0.5	0.3
c	1	7	5	4

a – number of particles in the shower; b – expected number of axes of showers; c – number of observed groups of μ mesons.

TABLE V

	Chamber 1*				Chamber 2**			
Number of discharging counters in the chamber (m)	5—8	9—14	15—26	>26	5—8	9—14	15—26	>26
Number of showers with given m	400	120	30	20	450	170	37	22
Expected number of δ showers and groups of mesons connected with statistical fluctuations	3.4	1	0.3	0.2	4;5	1;2	0.4	0.2
Observed number of groups of discharged counters (class 4 and higher, see text)	4	7	5	5	4	9	10	6

*For chamber 1 the data are given on showers with m < 26 after 700 hours of operation, for showers with m > 26, after 1740 hours.

**For chamber 2, data are given for showers with m < 26 after 1070 hours of operation, and for showers with m > 26 after 1740 hours.

= (0.9 ± 0.1) particles/m^2 and is equal to the density of accompaniment of the groups in the same showers in chamber 2.

DISCUSSION OF RESULTS

Thus the study of the flux of high-energy μ mesons in extensive atmospheric showers points to the possible existence close to the axis of the showers of beams of μ mesons, the number of which increases with increase in the intensity of the shower.

Let us return to the problem of the possible nature of the observed phenomenon. First of all, we consider the possibility of generation of groups of μ mesons in the ground above the apparatus.

As a result of the nuclear-active components of high energy of the extensive atmospheric shower, formation of beams of π mesons in the ground is possible; these decay with finite probability with the formation of μ mesons.

The number of μ mesons is determined by the expression

$$n_\mu(\geqslant E) = \int\limits_E^{E_{\mathrm{na}}} \int\limits_0^\infty n_\pi(x, E')\, dE' \, \frac{dx \lambda m_\pi c^2}{\rho \tau_0 c E'}, \qquad (1)$$

where $n_\pi(x, E')\,dE'$ is the number of π mesons in the shower at a depth x, expressed in path lengths for the interaction λ possessing energies from E′ to E′ + dE′, $\rho = 1.7$ g/cm^3 is the density of the earth, $\tau_0 = 2 \times 10^{-8}$ sec is the half-life of the π meson, and m_π is the mass of the π meson. The limit of integration of E for chamber 1 amounts to 5×10^9 ev, for chamber 2, 10^{10} ev. Consideration of the cascade process by the method of successive generations leads to the equation

$$n_\pi(E', x)\, dE' = \sum_{i=0}^{\infty} \frac{e^{-x}}{i!} x^i \Pi_i(E')\, dE',$$

where i is the number of the generation, Π_i is the spectrum of π mesons of the i-th generation. Substituting this expression in (1), we obtain

$$n_\mu(\geqslant E) = \int\limits_E^{E_{\mathrm{na}}} \sum_{i=0}^{\infty} \Pi_i(E')\, dE' \, \frac{\lambda m_\pi c^2}{\rho \tau_0 c E'}. \qquad (2)$$

In Appendix 1 it is shown that in the case of not too high an energy loss of the nuclear-active particles of the avalanche (which is evolved upon the formation of the electron-photon component), Eq. (2) has the form:

$$n_\mu = K E_{\mathrm{na}} / E^2, \qquad (3)$$

where K is connected with the character of the acts of nuclear interaction in the avalanche. According to the estimate given in the Appendix, the energy of the nuclear-active particle responsible for a group of μ mesons of three particles must amount to 4×10^{13} ev. The experimental data on nuclear-active particles of high energy in extensive atmospheric showers at sea level show[9] that the appearance of such particles in showers with $N \sim 10^5$ is very unlikely. A comparison of experimental data at depths of 20 and 40 m below the screen also contradicts the picture we have given of the generation of μ-meson groups. Actually, if the energy $E_{1\mathrm{na}}$ which exists in showers with a number of particles N_1 suffices for the formation of a group of μ mesons with a given number of particles n_μ recorded at a depth of 40 m below the screen, then for the formation of a group observed at a depth of 20 m below the screen, the energy $E_{2\mathrm{na}} = E_{1\mathrm{na}}/4$ is sufficient [see Eq. (3)]; this exists in showers with $N_2 = N_1/4$. Then the frequency of appearance of groups at a depth of 20 m below the screen will exceed the frequency of appearance of groups at a depth of 40 m below the screen by a factor q, and this ratio will amount to (taking into account the spectrum of the showers in terms of the number of particles and the effectiveness of the separation of the showers)

$$q = \frac{\int_{N_0}^{\infty}\int_{0}^{\pi/2} \frac{A}{N^{\varkappa+1}} \cos^8\theta \sin\theta \,[1-\exp\{-kNf(H_1\tan\theta)\,\sigma\}]^6 \,dN\,d\theta}{\int_{N_1}^{\infty}\int_{0}^{\pi/2} \frac{A}{N^{\varkappa+1}} \cos^8\theta \sin\theta \,[1-\exp\{-kNf(H_2\tan\theta)\,\sigma\}]^6 \,dN\,d\theta} \approx 4.$$

The ratio 1 ± 0.5 is observed experimentally (Table V), which contradicts the given picture of the generation of groups of μ mesons.

The observed group of μ mesons can also arise in the atmosphere because of decay of π and K mesons generated in acts of nuclear interaction. In this picture of the generation of groups of μ mesons, the character of the distribution of transverse momenta of secondary particles in the nuclear interaction is extremely important. Recent research[10,11] on the study of nuclear interaction in photoemulsions shows that the distribution of transverse momenta of secondary particles in nuclear interactions with energies $10^{12} - 10^{13}$ ev can be represented in the form

$$\varphi(p_\perp)\,dp_\perp = \frac{dp_\perp}{9\cdot 10^8} \quad \text{and } 10^8\text{ ev/c}; \leqslant p_\perp \leqslant 10^9\text{ ev}/c;$$

$$\varphi(p_\perp) = 0 \quad \text{and } p_\perp > 10^9\text{ ev/c}. \quad p_\perp < 10^8\text{ ev}/c. \tag{4}$$

It can also be shown (see Appendix 2) that with such a distribution of transverse momenta, the generation of a group of μ mesons of five particles, which cover a region with linear dimension d = 0.5 m at sea level in a single act of nuclear interaction, is highly improbable according to present-day representations. Let us therefore consider the possibility of the excitation of a group of μ mesons because of the decay of π mesons in the passage through the atmosphere of a complete nuclear avalanche of an extensive atmospheric shower. Such a group of μ mesons can be formed in the core of an extensive atmospheric shower. We shall find the number of μ mesons at sea level in an avalanche with primary energy E_0 which is diverging at a distance d from the axis of the shower:

$$n_\mu = \int_{x_0}^{1033}\int_{E_{min}}^{E_0}\int_{p_{\perp min}}^{p_{\perp max}} n_\pi(E_0, E, x)\,dE\,\frac{B\,dx}{xE}\,\varphi(p_\perp)\,dp_\perp, \tag{5'}$$

where x is the depth of the atmosphere in g/cm^2, $n_\pi(E_0, E, x)$ is the differential spectrum of π mesons at a depth x, $B = 1.39 \times 10^{11}$ ev is the decay constant of the π mesons. Here it is assumed that the μ meson acquires a transverse momentum $p_\perp$ in the decay of the π meson, which up to this point preserved the direction of the primary particle. Such an approximation gives an upper bound to the number of μ mesons in the circle d. In the case of a distribution of transverse momenta according to Eq. (4), we have

$$n_\mu = \int_{x_0}^{1033}\int_{H(x)\cdot 10^8/d}^{E_0} n_\pi(E, x)\,dE\,\frac{dx}{x}\,B\,\frac{1}{9}\left(\frac{d}{H(x)\cdot 10^8} - \frac{1}{E}\right), \tag{5''}$$

where H(x) is the distance above sea level corresponding to a depth x in the atmosphere.

For the calculation of n_μ according to Eq. (5″), it is necessary to give the energy spectra of the π mesons which have not yet been experimentally investigated with sufficient completeness. We shall consider only the theoretical models of the development of the nuclear cascade in the atmosphere which are considered in references 12 and 13. Numerical calculation carried out with the spectra of π mesons from the research of reference 12 according to Eq. (5″) gives the number of μ mesons in a circle with d = 0.5 m, $n_\mu = 0.2$ for $E_0 = 2 \times 10^{15}$ ev. However, we note that this model of the development of a nuclear cascade in the atmosphere does not give the number of nuclear-active particles of high energy at sea level corresponding to experiment. According to this model, $N_{na}(E > 10^{12}\text{ ev})/N = 2 \times 10^{-6}$, while experiment[9] gives $N_{na}(E > 10^{12}\text{ ev})/N = 2 - 5 \times 10^{-5}$. The calculation carried out in reference 13 gives better agreement with experiment on the number of nuclear-active particles of high energy at sea level in an extensive atmospheric shower [here $N_{na}(E > 10^{12}\text{ ev})/N = 5 \times 10^{-5}$]. In this calculation the following assumptions were made: 1) In the collision of the nucleon with the nucleus of air, one-half of its energy goes into the formation of $n = 2(0.5\,E_0)^{1/4}$ secondary particles; 2) secondary particles obtain equal energy; 3) the π mesons interact according to the same model; 4) charged μ mesons consist of 70% of all π mesons; 5) the path for the interaction $\lambda = 90$ g/cm^2.

Using $n_\pi(E, x)$ calculated under these assumptions by Eq. (5″) for $E_0 = 2 \times 10^{15}$ ev, we obtain $n_\mu = 5$, which corresponds to observation of a group of μ mesons in the core of the shower. In this case 3 or 4 μ mesons in a group possess energies $E_\mu \sim 10^{11}$ ev and 1 or 2 μ mesons possess energies of $E_\mu \sim 10^{12}$ ev.

From the point of view of such a picture of the generation of groups of μ mesons, the appearance of several groups of μ mesons in showers with high primary energies corresponds to the appearance of additional π-meson cores in the extensive atmospheric shower. The energy contained in the additional core can be many times smaller than in the original. Actually, in the additional group of μ mesons, for a shower with $E_0 \sim 10^{16}$ ev, as many mesons are observed as were in the fundamental group for showers with $E_0 \sim 10^{15}$ ev; therefore such an additional core can be connected with the emission of individual nuclear-active particles of high energy ($E_{na} \sim 10^{15}$ ev) from the original core. However, the observed distances between groups of μ mesons in showers with $E_0 \sim 10^{16}$ ev (~ 2 m) require very large transverse momenta for the nuclear-active particles generating the additional core:

$$p_\perp \approx \frac{2 \text{ m}}{2\cdot 10^4 \text{ m}} \cdot 10^{15} \text{ ev/c} = 10^{11} \text{ ev/c}.$$

Therefore, the existence of several groups of μ mesons in showers with $N \sim 10^6$ particles is improbable within the framework of the picture of the generation of μ mesons given here. The appearance of several groups of μ mesons in a single shower can be explained if it reduces the energy of the nuclear-active particle responsible for the group of μ mesons. It can be assumed that the total number of particles created in the act of nuclear interaction and also their energy and angle distributions undergo very large fluctuations. Then such acts of nuclear interactions are found which give narrow beams of π mesons, which yield in turn beams of μ mesons.

It can also be assumed that a process exists of much more rapid multiple production of μ mesons than the decay of π mesons (see, for example, reference 14). A substantially smaller energy of the nuclear-active particle ($E_{na} \ll 10^{15}$ ev) then suffices for the formation of the group of μ mesons. The rapid increase in the number of groups of μ mesons in a single shower in this case that is observed experimentally is explained by the increase in the flux of nuclear-active particles of the necessary energy ($N_{na} \sim E_0$). The fact that only a single group of μ mesons is observed in showers with $E_0 \sim 10^{15}$ ev means, in this interpretation, that the cross section of interaction with multiple production of μ mesons is significantly smaller than the total cross section of nuclear interaction.

The foregoing consideration shows that the final explanation of the nature of the particles which caused the events observed by us — the group discharge of counters screened by a filter under large thicknesses of earth in the passage of an atmospheric shower — is very important.

APPENDIX 1

For the calculation of the number of μ mesons observed underground and originating from π mesons reaching the ground, we consider a nuclear cascade process, in which the π mesons determine its development. We assume that in the collision of a nuclear-active particle with a nucleus of the earth, $n\pi$ mesons are produced while αn of them are nuclear active; in the collision of π mesons with the nuclei of the earth, $n\pi$ mesons are again produced, etc. Then

$$n_\pi(E, x) = \sum_{i=0}^{\infty} \frac{e^{-x}}{i!} x^i \Pi_i(E); \quad \Pi_i(E) = (\alpha n)^i \delta\left(E - \frac{E_{na}}{n^i}\right).$$

We are interested in the number of μ mesons from the decay of π mesons in the ground with energies $> E$. It is equal to

$$n_\mu(>E) = \int_E^{E_{na}} \int_0^\infty \sum_{i=0}^\infty (\alpha n)^i \delta\left(E' - \frac{E_{na}}{n^i}\right) \frac{dE' dx \lambda m_\pi c^2}{p\tau_0 c E'} \frac{e^{-x} x^i}{i!}$$

$$= \int_E^{E_{na}} \sum_{i=0}^\infty (\alpha n)^i \delta\left(E' - \frac{E_{na}}{n^i}\right) \frac{dE'}{E'} \frac{\lambda m_\pi c^2}{p\tau_0 c} = \frac{\lambda m_\pi c}{p\tau_0}$$

$$\times \sum_{i=0}^k (\alpha n)^i \frac{n^i}{E_{na}},$$

where k is determined from the relation $E = E_{na}/n^k$; $k = \log(E_{na}/E)/\log n$. Thus,

$$n_\mu(>E) = \frac{\lambda m_\pi c}{p\tau_0} \frac{[(\alpha n^2)^k - 1]}{E_{na}\,(\alpha n^2 - 1)} = \frac{\lambda m_\pi c}{p\tau_0} \left[\left(\frac{E_{na}}{E}\right)^{2+\log\alpha/\log n}\right.$$

$$\left. - 1\right] \frac{1}{E_{na}(\alpha n^2 - 1)} \approx \frac{\lambda m_\pi c}{p\tau_0 E_{na}} \left(\frac{E_{na}}{E}\right)^\gamma \frac{1}{\alpha n^2},$$

where

$$\gamma = 2 + (\log\alpha/\log n).$$

It follows from the form of the dependence of n_μ on E that $n_\mu \sim KE_{na}/E^2$ for sufficiently small leakage of energy from the nuclear cascade process. If we take $n = 5$ (a value which was determined experimentally for energies $E_{na} \geq 10^{11}$ ev, see reference 10) and $\alpha = 0.7$, then a nuclear-active particle on the surface of the earth must possess energies of 4×10^{13} ev for the production of a group of μ mesons of three particles.

APPENDIX 2

For the act of nuclear interaction with energy of the nuclear-active particle E_0 which takes place at a depth H above the level of observation, we have

$$n_\mu = \int_{E_{min}}^{E_0} \int_{p_\perp min}^{p_\perp max} n_\pi(E)\, dE \varphi(p_\perp)\, dp_\perp \frac{L_{int}}{L_{int} + E\tau_0/m_\pi c}$$

$$\times\left[1 - \exp\left(-\frac{H}{L_{eff}}\right)\right],$$

where $n_\pi(E)dE$ is the energy spectrum of π

mesons, $\varphi(p_\perp)\,dp_\perp$ is the distribution of transverse momenta of π mesons, L_{int} is the path length for the interaction of π mesons in centimeters,

$$1/L_{\text{eff}} = 1/L_{\text{int}} + m_\pi c^2/E\tau_0 c.$$

We assume that the distribution of transverse momenta of π mesons has the form (4). The energy spectrum of the π mesons produced in the act of nuclear interaction is given in two variants:

1. Interaction proceeds according to the Fermi-Landau theory; then the spectrum of π mesons has the form (see reference 19)

$$n_\pi(E)\,dE = \frac{n_0}{\sqrt{2\pi\mathscr{L}}}\exp\left\{-\frac{[\ln E/\bar{p}_\perp c]^2}{2\mathscr{L}}\right\}\frac{dE}{E},$$

$$n_0 = k(\bar{n}+1)(E_0/10^9)^{1/4}, \quad k\sim 1, \quad \bar{n}\sim 4,$$

$$\mathscr{L} = 0.56\ln(E_0/10^9) + 1.6\ln[2/(\bar{n}+1)] + 1{,}6,$$

$$\bar{p}_\perp c = 3\cdot 10^8 \text{ ev}.$$

2. The interaction proceeds according to the Heisenberg theory; then the spectrum of π mesons has the form

$$n_\pi(E)\,dE = E_0\left\{E^2 \ln\frac{M}{m_\pi}\sqrt{\frac{E_0}{2Mc^2}}\right\}^{-1} dE \quad \text{for } m_\pi c^2 \times\sqrt{\frac{E_0}{2Mc^2}} < E < E_0.$$

Results of the calculation of n_μ are given in Table VI for three values of H and the maximum energy of nuclear-active particles appearing at these levels in showers with the number of particles at sea level $N \sim 10^5$.

TABLE VI

H	$E_0=10^{13}$ ev		$E_0=10^{15}$ ev
	100 m	500 m	10 km
First variant	0.15	0.08	0.1
Second variant	0.3	0.07	0.08

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255