A POSSIBILITY FOR INVESTIGATING THE STRUCTURE OF NUCLEONS AND NUCLEI

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It is shown that in the interaction of high energy particles the sum of the quantities $(E - p\cos\vartheta)$ for all particles emitted after the interaction is equal to the mass of the target particle which effectively participates in the interaction. Thus the distribution of $\Sigma(E - p\cos\vartheta)$ is determined by the structure of the target particle. Application of this type of analysis to the experimental data shows that interactions with the part of the target which has a mass close to that of the π meson are dominant.

1. In reference 1 we derived a form of conservation laws for energy and momentum, proposed by S. N. Vernov, which is convenient for the kinematic analysis of interactions in which a high energy particle generates secondary particles.

If the target particle is at rest, we can write for sufficiently fast primary particles ($E_0 - p_0 = M_0^2/2p_0 \ll M_{tg}$)

$$\sum (E_i - p_i\cos\vartheta_i) = M_{\mathrm{tg}}. \qquad (1)$$

Here E, p, ϑ, and M are the total energy, the momentum, the angle of emission, and the mass of the particle. The index 0 denotes the primary particle, i the particle after the interaction, and tg the target particle. The summation in (1) goes over all particles emitted after the interaction.

If the particles emitted after the interaction are sufficiently fast, we can write (1) in the form

$$M_{\mathrm{tg}} = \sum \left(\frac{M_i^2}{2p_{\perp i}} \sin\vartheta_i + p_{\perp i} \tan\frac{\vartheta_i}{2} \right). \qquad (2)$$

In this form of writing the physical meaning of expression (1) becomes clear: it gives a connection between the mass of the target and the values of the angles and transverse momenta of the particles emitted after the interaction.

2. Let us generalize relation (1) to the case when the target particle moves.

With the notation $E_i - p_i\cos\vartheta_i = \Delta_i$, we obtain

$$\sum \Delta_i = M_{\mathrm{tg}} + T_{\mathrm{tg}} - p_{\mathrm{tg}}\cos\vartheta_{\mathrm{tg}}, \qquad (3)$$

where T_{tg} is the kinetic energy of the target particle, and $p_{tg}\cos\vartheta_{tg}$ is the projection of its momentum before the collision on the direction of motion of the incoming particle.

We assume further that the target particle is bound to some heavier particle. Let us consider, for example, the interaction with a nucleon inside the nucleus. The potential energy of the nucleon in the nucleus, u_n, is small in comparison with the mass of the nucleon. After the emission of the nucleon the residual nucleus $M_{nuc\,0}$ receives an impulse on the order of u_n, which causes it to recoil slightly. In studying the interaction with a definite moving nucleon, one must take account of the fact that the residual nucleus is not at rest before the interaction, but has a momentum which is equal and opposite to that of the nucleon in the nucleus. The quantity $\Delta_{nuc\,0} = E - p\cos\vartheta$ for the residual nucleus can be written in the form $\Delta_{nuc\,0} = M_{nuc\,0} - p_{nuc\,0}\cos\vartheta_{nuc\,0} + T_{nuc\,0}$. Neglecting $T_{nuc\,0}$ and using $\mathbf{p}_{nuc\,0} = -\mathbf{p}_n + \boldsymbol{\alpha}$, we obtain $\Delta_{nuc\,0} = M_{nuc\,0} + p_n\cos\vartheta_n - \alpha$, where α is a quantity of the same order as u_n.

We write relation (1) for the initial nucleus M_{nuc} in the form

$$\sum \Delta_i + \Delta_{\mathrm{nuc}\,0} = M_{\mathrm{nuc}},$$

where $M_{nuc} = M_{nuc\,0} + M_n + T_n - u_n$, and obtain

$$\sum \Delta_i = M_{\mathrm{n}} + T_{\mathrm{n}} - (u_{\mathrm{n}} - \alpha) - p_{\mathrm{n}}\cos\vartheta_{\mathrm{n}}. \qquad (4)$$

Relation (4) takes account of the recoil of the residual nucleus. If the recoil is neglected, we have

$$\sum \Delta_i = M_{\mathrm{n}} + T_{\mathrm{n}} - u_{\mathrm{n}} - p_{\mathrm{n}}\cos\vartheta_{\mathrm{n}}. \qquad (5)$$

The distribution of the quantities $\Sigma\Delta_i$, which we shall call M_t in the following, is given in this case by a line with a maximum at the value $M_n + \overline{(T_n - u_n)}$ (the average $\overline{p_n\cos\vartheta_n} = 0$, in any case for unpolarized nuclei). The width of the line is determined by the quantity p_n, the Fermi momentum of the nucleon in the nucleus.

3. Let us now consider the interaction of a sufficiently fast particle with a nucleus of mass M_{nuc}.

If there are interactions with one, two, or more nucleons, the spectrum of the quantities M_t will, according to the foregoing, have lines near the values of one, two, etc. nucleon masses.

In the region of target masses close to M_{nuc} the spectrum of the quantities M_t will be smeared out and appear to become continuous. Since the recoil of the residual nucleus cannot be neglected in this region, the position and the width of the line do not any more have the clear classical meaning expressed by relation (5). Even in this case, however, the form of the spectrum of M_t will reflect the structure of the nucleus.

Since in any case

$$\sum \Delta_i + \Delta_{\text{nuc } 0} = M_{\text{nuc}}, \tag{6}$$

where $\Delta_{\text{nuc } 0} = E - p \cos \vartheta$ for the recoiling residual nucleus as a whole or for its fission products, it should be noted that the distribution of the quantities $\Delta_{\text{nuc } 0}$ will simply reflect the distribution of the quantities $\Sigma\Delta_i$. A preference for either one of these quantities has to be based on experimental considerations.

4. This method can also be applied to the study of the structure of the nucleon by considering the interaction of the nucleon with particles whose wavelength is small compared with the dimensions of the nucleon.

In this case we extract from the sum of all Δ_i the term which corresponds to the recoil nucleon: $\Delta_\delta = E_\delta - p_\delta \cos \vartheta_\delta$. It follows from the analog of relation (4) that the value of $M_t = \Sigma\Delta_i$ will correspond to the value of the "effective masses" of those parts of the target nucleon which participate in the interaction.

The value of the quantities M_t obtained in this way will determine the relative probability of interaction with various regions of the nucleon.

If the nucleon has no structure, the spectrum of M_t should not have separate lines. If such lines are observed in experiment, it indicates that the nucleon is made up of separate structures. In the opinion of the authors, the question whether the experimentally observed structure is real or virtual is to be decided by the relative width of the lines in the distribution of M_t. From this point of view there is no physical borderline which separates a real particle from a virtual one; one can only speak of the degree of reality of a particle.

5. Since $\Sigma\Delta_i + \Delta_\delta = M_n$, we can obtain the distribution of M_t from $\Sigma\Delta_i$ or from the quantities Δ_δ for the recoil nucleon, whichever is more convenient from the experimental point of view.

In the region of small energies, where the identification of the recoil protons is possible, it is obviously more convenient experimentally to study the recoil nucleons.

In experiments with high energy particles, on the other hand, a number of circumstances are favorable for an exact determination of $\Sigma\Delta_i$ for the generated particles.

We emphasize, first of all, that $\Sigma\Delta_i$ and Δ_δ can be calculated without knowing the energy and the mass of the incoming particle, which appreciably simplifies the experimental procedure.

According to (2), the particles that give the greatest contribution to the sum are those emitted under relatively large angles with relatively small momenta. This circumstance allows us to relax the accuracy requirements with respect to the measurement of the momenta.

For sufficiently high energies of the secondary particles and sufficiently high number of generated π mesons, the error arising from the impossibility of identifying the fast nucleon is insignificant. If, in forming $\Sigma\Delta_i$, we treat the slow nucleon like a π meson with the same momentum, then Δ_δ will be automatically excluded from the sum of the Δ_i.

It can be shown that in the sum $\Sigma\Delta_i$ the errors in the measurement of the angles and momenta are averaged out in first approximation. An increase in the number of the π mesons generated in the shower leads, therefore, to an increase in the accuracy of the calculation of $\Sigma\Delta_i$.

Since the second term of the sum (2) gives the main contribution to the sum of the Δ_i, a small admixture of heavy mesons will not affect the value of $\Sigma\Delta_i$ appreciably.

6. We calculated $M_t = \Sigma\Delta_i$ from the available data on the following interactions of high energy particles with nucleons and nuclei: 1) proton-proton interaction with proton energy 3.7 Bev;[2] 2) interaction of cosmic ray protons with Be nuclei with an average proton energy of about 5 Bev;[1] 3) pion-proton interaction ($\pi^- + p$) with pion energies of about 5 Bev;[3] 4) generation of π mesons by μ mesons in lead nuclei, which was treated by Kessler and Maze[4] like the photoproduction of μ mesons by photons with energies of about 17 Bev; 5) interaction of 10^{11}-ev cosmic-ray particles* with LiH nuclei; 6) interaction of cosmic ray particles with energies $10^{12}-10^{13}$ ev with nuclei

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"Jets" in photoemulsions*

Type of star	E_0, eV	M_t/M_n	Remarks	Reference
2+16 p	$5\cdot10^{12}$	0.33	The $p\beta$ of the ionizing particles was determined	[5]
2+15 p	$2\cdot10^{13}$	0.16	" " "	[6]
1+39 p	$5\cdot10^{12}$	1.05	" " "	[8]
0+14 α	$\sim10^{14}$	0.08	The energy of the π^0 mesons was determined	[7]
0+7 p	10^{12}	0.10	The $\pi^\pm$ and π^0 mesons were measured	[9]

*In the table we list the high energy "jets" in which the angles and momenta of the secondary particles have been measured. The energy was estimated by the authors of the references on the basis of the angular distribution. In these cases the energy of the interaction given by the authors of references 5, 6, 7, and 9 is apparently much too high. The very poor sampling of the interactions does not allow us to make conclusions about the true contribution from the interactions with small M_t.

in photoemulsions, leading to the formation of so-called "jets," in those cases where data on the momenta of the particles exist.[5-9]

The calculated distributions of M_t are given in the table and in Figs. 1 to 5, the captions of which contain all the necessary notes.

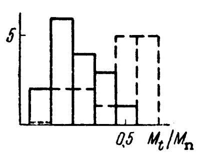

FIG. 1

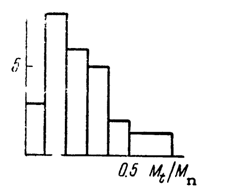

FIG. 2

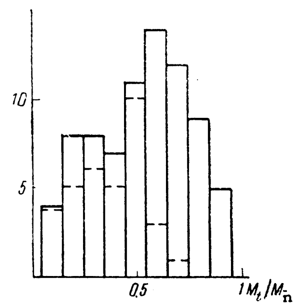

FIG. 3

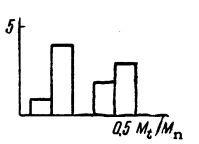

FIG. 4

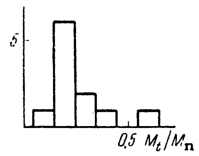

FIG. 5

FIG. 1. Interaction p + p, E_0 = 3.7 Bev. M_t was determined as the difference $M_n - \Delta_\delta$. The quantities Δ_δ were measured for the nucleons in the interactions $p + p \to p + N + \pi^+$ (solid line) and $p + p \to p + N + \pi^+ + \pi^0$ (dotted line), which could be reliably identified. The method of identification could lead to a distortion of the distribution in the region of large M_t. The comparison of the distributions of the two above-mentioned reactions shows, however, that the occurrence of a maximum in the region of small M_t for the reaction in which one pion is produced cannot be explained by the sampling. The absence of the maximum for the reaction leading to the production of two pions is connected with the small value of the center of mass energy (E = 500 Mev).

FIG. 2. Interaction p + Be, $\bar{E}_0$ = 5 Bev. M_t is, in most cases, determined as the difference $M_n - \Delta_\delta$. In the remaining cases it is measured directly as the sum of the Δ_i.

FIG. 3. Interaction $\pi^- + p$, E_0 = 5 Bev. M_t is determined as the difference $M_n - \Delta$. Dotted lines: reliably identified protons; solid lines: all events. The dotted curve shows the effect of the sampling in the region $M_t > 0.5$. The character of the sampling is analogous to that in the case of the interactions shown in Figure 1.

FIG. 4. Photoproduction of pions, $h\nu$ = 17 Bev. The figure shows the distribution of M_t for the penetrating showers generated by μ mesons of high energy incident on lead nuclei. M_t is computed by formula (2) with the value $\overline{p_{\perp l}} = 4.2\cdot10^8$ ev.

FIG. 5. Interaction of cosmic ray particles with LiH, $E_0 \approx 10^{11}$ ev. The angles and momenta of the particles are measured in the Wilson chamber. $M_t = 1.5\sum_{\text{charged}}\Delta_i$; the factor 1.5 corrects for the generation of π^0 mesons.

The experimental data used to obtain the distributions of M_t do not have sufficient statistical accuracy to lead to conclusive quantitative results. Nevertheless, it can be assumed that more abundant experimental material will lead to the following conclusions.

A. For nucleon-nucleon interactions in a large energy interval the distribution has a maximum in the region $M_t \approx 0.2\,M_n$. An analogous maximum is observed for interactions of photons with nucleons. The occurrence of this maximum indicates that the interaction with an "effective mass" close to the mass of the π meson plays the predominant role.

B. Let us assume that in the nucleon-nucleon interaction the number of cases in which the fast nucleon interacts with $M_t \approx M_\pi$ is equal to the number of cases in which the target is a nucleon and the incident nucleon interacts only through a part of itself, whose mass is close to M_π. Then the observed distributions of M_t can be interpreted by assuming that the cross section for the interaction of the "core" with π mesons is a few times larger than the cross section for the interaction of the nucleon "cores" with each other ($\sigma(N', \pi) \approx 4\sigma(N', N')$).

C. For the interaction of π mesons with nucleons the distribution of M_t does not have a maximum in the neighborhood of the π meson mass. This indicates that the cross section for the π-π interaction for energies of 5 Bev is relatively small ($\sigma(\pi, \pi) < \frac{1}{3}\sigma(\pi, N)$).

These results on the distribution of M_t for the interactions (N, N) and (π, N) permit us to estimate[1] the energy losses in the interaction of nucleons with energies $10^{10}-10^{11}$ ev with other nucleons and to explain the fact, established in cosmic-ray experiments, that the energy losses in the interaction of nucleons of about 10^{10} ev with light nuclei are small,[10] and the energy dependence of the energy losses is weak.[11]

7. From the standpoint of our method, the most fruitful experiments for the study of the structure of the nucleon appear to be the following:

1) Investigation of the photo production of π mesons by photons of high energy in nucleons and light nuclei. The distribution of M_t reflects, in this case, directly the relative frequency of the interaction with various parts of the target nucleon.

2) Comparison of the Δ spectra for the interactions $p+p$ and $\pi+p$ with similar effective energies. These experiments make it possible to separate the π - π interaction.

3) Experiments at such energies of the particles that there are no energetical limitations on interactions with small M_t. These experiments lead to a more accurate determination of the character of the tail of the Δ spectrum and allow us to solve the problem of the coupling and, in the last analysis, of the degree of reality of the π meson in the nucleon.

There is no doubt that this classical treatment of the Δ spectra is rather crude for the nucleon. One should, obviously, get a deeper insight by treating the interactions quantum-theoretically.

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ERRATA TO VOLUME 9

On page 868, column 1, item (e) should read:

(e). Ferromagnetic weak solid solutions. By way of an example, we consider the system Fe-Me with A2 lattice, where Me = Ti, V, Cr, Mn, Co, and Ni. For these the variation of the moment m with concentration c is

$$dm/dc = (N_d)_{Me} \mp 0.642 \; \{8\,(2.478 - R_{Me}) + 6\,|2.861 - R_{Me}| \mp [\,8\,(2.478 - R_{\mathrm{Fe}}) + 6\,(2.861 - R_{\mathrm{Fe}})]\},$$

where the signs – and + pertain respectively to ferromagnetic and paramagnetic Me when in front of the curly brackets, and to metals of class 1 and 2 when in front of the square brackets. The first term and the square brackets are considered only for ferromagnetic Me. We then have dm/dc = −3 (−3.3) for Ti, −2.6 (−2.2) for V, −2.2 (−2.2) for Cr, −2 (−2) for Mn, 0.7 (0.6) for Ni, and 1.2 (1.2) for Co; the parentheses contain the experimental values.

ERRATA TO VOLUME 10

Page	Reads	Should Read
224, Ordinate of figure	10^{23}	10^{29}
228, Column 1, line 9 from top	3.6×10^{-2} mm/min	0.36 mm/min
228, Column 1, line 16 from top	0.5 mm/sec	0.05 mm/min
329, Third line of Eq. (23a)	+(1/4 cosh r +...	+1/4(cosh r +...
413, Table II, line 2 from bottom	−0.0924±	−1.0924±
413, Table II, line 3 from bottom	+1.8730±	+0.8370±
479, Fig. 7, right, 1st line	92 hr	9.2 hr
499, Second line of Eq. (1.8)	$+\tilde{k}\sin^2\alpha/\omega_N^2 + \langle c^2\tilde{k}^2 \ldots$	$+\left(\tilde{k}/\omega_H\right)^2 \sin^2\alpha \;\langle c^2\tilde{k}^2 \ldots$
648, Column 1, line 18 from top	18 × 80 mm	180 × 80 mm
804, First line of Eq. (17)	$-{}^1/_3\,(\alpha_x^2\alpha_y^2 + \ldots$	$\ldots - 3\,(\alpha_x\alpha_y^2 + \ldots$
967, Column 1, line 11 from top	$\sigma(N', \pi) \approx 46\,(N', N')$	$\sigma(N', \pi) > \sigma(N', N')$
976, First line of Eq. (10)	$= \frac{e^2}{3r^2c^4}$	$= \frac{e^2}{3\hbar^2c^2}$
978, First line of Eq. (23)	$\left[\frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1)\sin^4(\theta/2)}\right.$	$\left[\frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1)^2\sin^4(\theta/2)}\right.$