INELASTIC COLLISIONS BETWEEN FAST POLARIZED PARTICLES AND ATOMS

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The differential cross section for scattering of electrons, positrons and μ mesons on atoms is derived in the Born approximation as a function of the polarization of the particles in the initial and final states. The change in the polarization vector from the scattering of the particles by free unpolarized electrons is also obtained.

1. INTRODUCTION

THE behavior of polarized electrons, positrons and μ mesons has recently been the subject of many experiments. In this connection it is of interest to calculate the cross sections for various processes involving the interactions of polarized particles with matter, in particular elastic and inelastic scattering of such particles by atoms in matter. Coulomb scattering of electrons including polarization effects was discussed in the review article by Tolhoek[1] and in a number of other papers.[2-5] Ivanter[6] obtained radiative corrections to the Coulomb scattering cross section of polarized electrons and μ mesons. Polarization effects in scattering of Dirac particles on free electrons were also considered by a number of authors.[7-12] However in reality the electrons in matter are not free. Effects due to the binding of electrons in matter are important for small angle scattering. This region of angles is particularly relevant when depolarization due to multiple scattering and bremsstrahlung of polarized particles is studied, since in these processes mainly small angle deviations are involved.

In this paper we study inelastic collisions between polarized particles and atoms. We obtain, in the Born approximation, the differential cross section $d\sigma_n(\theta, \boldsymbol{\zeta}_1, \boldsymbol{\zeta}_2)$ for scattering of the particle through an angle θ with excitation of the n-th level of the atom and change in polarization $\boldsymbol{\zeta}_1 \rightarrow \boldsymbol{\zeta}_2$, as well as the angular distribution of the inelastically scattered particles independently of the energy loss. We also study the change in polarization upon scattering by free unpolarized electrons. The calculations are performed for the general case of arbitrary polarization of one of the particles in the initial and final states.

2. SCATTERING ON ATOMS

The scattering of a relativistic unpolarized electron on an atom was first discussed by Bethe.[13] He found, in the Born approximation without taking into account exchange effects, the following expression for the cross section for a collision resulting in the atom making a transition to the n-th level:

$$d\sigma_n = 4\left(\frac{e}{\hbar c}\right)^4 \frac{\varepsilon\varepsilon'}{[q^2-(\Delta\varepsilon/\hbar c)^2]^2}\frac{p'}{p}|F_{n0}|^2 d\Omega, \tag{1}$$

$$F_{n0} = \int \psi_n^* \Big[ZA_0 + \sum_{j=1}^{z} e^{i\mathbf{q}\mathbf{r}_j}(-A_0 + \mathbf{A}\boldsymbol{\alpha}_j)\Big]\psi_0 d\tau. \tag{2}$$

Here ϵ, $\mathbf{p}$, ϵ', and $\mathbf{p}'$ are the total energy and momentum of the particle before and after scattering respectively, $\Delta\epsilon = \epsilon - \epsilon' = E_n - E_0$ is the energy transfer, $\hbar\mathbf{q} = \mathbf{p} - \mathbf{p}'$ is the momentum transfer, ψ_0 and ψ_n are the wave functions of the atom in the initial and final state, and α_j is a Dirac operator acting on the spin variables of the j-th atomic electron. The particle undergoing scattering is described by a plane wave with a spinor factor $u(\boldsymbol{\zeta})$ and $u(\boldsymbol{\zeta}')$ for the initial and final states respectively, where $\boldsymbol{\zeta}$ and $\boldsymbol{\zeta}'$ describe the particle polarization in its rest frame, u and u' are normalized to unity. At that

$$A_0 = (u'^*, u), \qquad \mathbf{A} = (u'^*, \boldsymbol{\alpha}u). \tag{3}$$

The expression for the transition matrix element

$$F_{n0} = \int \psi_n^* \Big\{ZA_0 + \sum_{j=1}^{z} e^{i\mathbf{q}\mathbf{r}_j}\Big[-A_0 + \frac{1}{c}\hat{\mathbf{v}}_j A + (\hbar/2mc)\mathbf{A}(\mathbf{q} + i[\boldsymbol{\sigma}_j\times\mathbf{q}])\Big]\Big\}\psi_0 d\tau. \tag{4}$$

on which the following discussion is based, is obtained by going over in Eq. (2), in a conventional manner, to the nonrelativistic approximation for the atomic electrons. Here $\mathbf{v}_j$ is the velocity operator for the j-th atomic electron and σ_j is a Pauli matrix.

Since Eq. (4) is rather complicated in the general case, we shall evaluate the matrix element F_{n0} in the dipole approximation, i.e., we assume that $qa \ll 1$, where a is the radius of the atom. This corresponds[14] to small scattering angles $\theta \ll p_0/p$ and not too large atomic excitation energies $\Delta\epsilon \ll p_0 v$, where p_0 is of the order of magnitude of the average momentum of the atomic electrons and v is the velocity of the particle. In this approximation, with $\exp(i\mathbf{q}\cdot\mathbf{r}_j)$ replaced by $1 + i\mathbf{q}\cdot\mathbf{r}_j$ and leaving terms proportional to q in Eq. (4), we find

$$F_{n0} = (i/e)\left[(\Delta\varepsilon/\hbar c)\,\mathbf{A}\mathbf{d}_{n0} - A_0\mathbf{q}\mathbf{d}_{n0}\right], \qquad \mathbf{d}_{n0} = e\int \psi_n^* \sum_j \mathbf{r}_j \psi_0 d\tau. \quad (5)$$

Here $\mathbf{d}_{n0}$ is the matrix element of the electric dipole moment. If the atoms in the initial and final states are unpolarized then $|F_{n0}|^2$ should be averaged over the initial and summed over the final states corresponding to different values of the projection of the angular momentum of the atom. Denoting this averaging and summing by a bar we obtain

$$\overline{|F_{n0}|^2} = \tfrac{1}{3}\overline{|\mathbf{d}_{n0}|^2}\,\{q^2|A_0|^2 - 2(\Delta\varepsilon/\hbar c) \times \mathrm{Re}(\mathbf{q}\mathbf{A})A_0 + (\Delta\varepsilon/\hbar c)^2|\mathbf{A}|^2\}. \quad (6)$$

For calculational purposes it is convenient to express the bispinor expressions appearing in Eq. (6) in terms of traces of certain 2×2 operators. This method was used by Olsen.[15] Consider, for example, the calculation of $|A_0|^2$. We express the bispinor amplitudes involved in A_0 in the form

$$u = \sqrt{\frac{\varepsilon+m}{2\varepsilon}}\begin{pmatrix}1\\ \sigma\mathbf{p}/(\varepsilon+m)\end{pmatrix}V, \quad u' = \sqrt{\frac{\varepsilon'+m}{2\varepsilon'}}\begin{pmatrix}1\\ \sigma\mathbf{p}'/(\varepsilon'+m)\end{pmatrix}V', \quad (7)$$

where V, V′ are spinors describing the initial and final polarization states. With the help of Eq. (3) we obtain

$$A_0 = (V'^{+}TV).$$

Here T is a certain 2×2 operator whose explicit form follows from Eq. (7). Introducing the projection operator onto a state with polarization $\boldsymbol{\zeta}$

$$P(\zeta) = \tfrac{1}{2}(1 + \zeta\sigma), \quad (8)$$

we find

$$|A_0|^2 = \tfrac{1}{4}\,\mathrm{Sp}\,\{T^{+}(1+\zeta'\sigma)\,T\,(1+\zeta\sigma)\}. \quad (9)$$

In the following we shall measure q in units of $mc/\hbar$ and will use the abbreviations $\gamma = \epsilon/mc^2$ and $\beta = v/c$. When obtaining an explicit expression for $d\sigma_n$ it is necessary to keep in mind that in the range of validity of the dipole approximation $\Delta\gamma$ and θ are small quantities. We therefore expand the right hand side of Eq. (1) in a series and keep only the leading terms in q^2 and $(\Delta\gamma)^2$. These terms may be of the same order in the extreme relativistic limit; in the nonrelativistic limit the terms with $(\Delta\gamma)^2$ are negligibly small. We obtain the following final expression for the differential scattering cross section:

$$\frac{d\sigma_n}{d\Omega} = \frac{e^2}{3r^2c^4}\overline{|\mathbf{d}_{n0}|^2}\frac{\gamma^2}{[q^2-(\Delta\gamma)^2]^2}\{[2q^2 - 2(\Delta\gamma/\gamma)^2(\gamma^2+1)] \times (1+\zeta_1\zeta_2) + [-q^2(\gamma-1)^2/\gamma^2 + (\Delta\gamma/\gamma)^2(\gamma-1) \times (\gamma-3)]\cdot[(\mathbf{n}_1\zeta_1)(\mathbf{n}_1\zeta_2) + (\mathbf{n}_2\zeta_1)(\mathbf{n}_2\zeta_2)] + [q^2(\gamma-1) \times (3\gamma-1)/\gamma^2 + (\Delta\gamma/\gamma)^2(\gamma-1)(-3\gamma+5)/\gamma^2] \times (\mathbf{n}_1\zeta_1) \times (\mathbf{n}_2\zeta_2) + [-q^2\beta^2 + (\Delta\gamma/\gamma)^2(\gamma^2-1)](\mathbf{n}_2\zeta_1)(\mathbf{n}_1\zeta_2)\}, \quad (10)$$

where $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors in the direction of the particle momentum before and after scattering. As can be seen from Eq. (10), an initially unpolarized beam remains unpolarized after scattering. However if transitions between states with definite projection of the angular momentum of the atom are registered, then terms linear in $\boldsymbol{\zeta}$, $\boldsymbol{\zeta}'$ will appear in Eq. (10). This means that the initially unpolarized beam becomes polarized after scattering.

Now let us determine the angular distribution of the polarized particles in inelastic scattering by the atom, independent of the energy loss:

$$d\sigma(\theta, \zeta, \zeta') = \sum_{n\neq 0} d\sigma_n. \quad (11)$$

It is shown in reference 14 that if the scattering angle satisfies the condition $\theta \gg p_0v_0/pv$, where v_0 is the average velocity of the atomic electrons, then q is independent of n and the summation in Eq. (11) may be carried out in a general form by using rules of matrix multiplication. The term $(\Delta\epsilon/\hbar c)^2$ in Eq. (1) may be neglected in comparison with q^2. The sum over n entering into Eq. (11) is transformed as follows:

$$\sum_{n\neq 0}|F_{0n}|^2 = (FF^{+})_{00} - |F_{00}|^2. \quad (12)$$

The operator F in Eq. (12) may be taken in the form

$$F = \left(Z - \sum_{j=1}^{Z}\exp\{i\mathbf{q}\mathbf{r}_j\}\right)A_0; \quad (13)$$

the remaining terms will contain, after averaging over the ground state of the atom, an additional factor of order $(v_0/c)^2$ and may be neglected. Substituting Eq. (13) into Eq. (12) we obtain

$$\sum_{n\neq 0}|F_{0n}|^2 = ZS(q)\,|A_0|^2, \quad (14)$$

where S(q) is the "incoherent scattering function" introduced by Heisenberg[16] and tabulated by Bewilogua.[17]

$$S(q) = 1 - \frac{1}{Z} F_{\theta}^2(q) + \frac{1}{Z} \sum_{j \neq k} \exp\{i\mathbf{q}(\mathbf{r}_j - \mathbf{r}_k)\}, \quad (15)$$

where F(q) is the atomic form factor. From Eqs. (14), (11), and (1) we obtain the following expression for the angular distribution of polarized particles in inelastic scattering through a small angle ($\theta \ll 1$):

$$d\sigma/d\Omega = 2Z(e^2/pv)^2 S(q)\theta^{-4}\{1 + \zeta_1\zeta_2 - \tfrac{1}{2}\beta^2(\mathbf{n}_2\zeta_1)(\mathbf{n}_1\zeta_2) + \left[\tfrac{1}{2}\beta^2 + (\gamma-1)^2/\gamma^2\right](\mathbf{n}_1\zeta_1)(\mathbf{n}_2\zeta_2) - [(\gamma-1)^2/2\gamma^2][(\mathbf{n}_1\zeta_1)(\mathbf{n}_1\zeta_2) + (\mathbf{n}_2\zeta_1)(\mathbf{n}_2\zeta_2)]\}. \quad (16)$$

The dependence of the cross section on polarization is precisely the same as that obtained in Coulomb scattering of electrons in the Born approximation. This is explained by the fact that the neglect of $\Delta\epsilon$ is equivalent to going over from inelastic to elastic scattering in a certain spherically symmetric field.

Equations (10) and (16) are valid for electrons, positrons, and μ mesons. Exchange effects, which exist in the scattering of electrons and positrons and which were not taken into account, are unimportant in the region of validity of these formulas ($\theta \ll 1$).

3. SCATTERING ON FREE ELECTRONS

If the scattering angle satisfies the condition $\theta \gg p_0/p$ (which means that the energy transfer is much larger than the binding energy) then the binding effects of the atomic electrons may be neglected. In this region inelastic scattering by an atom is equivalent to scattering by free electrons. The change in the polarization vector of electrons and positrons in scattering by free electrons was calculated for certain special cases in a number of papers. Ford and Mullin[8] obtained the change in the polarization of a longitudinally polarized electron when scattered by an electron. Mukhtarov and Perov[12] calculated the longitudinal component of positron polarization after scattering (the initial polarization being assumed to be also longitudinal). Lastly, Kresnin and Rozentsveĭg[7] determined the polarization of an initially unpolarized electron beam after scattering by polarized electrons. Since the results in reference 7 are given in the center of mass system one could derive from them an expression for the polarization vector of an electron beam deflected by an angle θ after scattering by unpolarized electrons, by replacing θ by $\pi - \theta$ in formulas (23) — (27) of that paper. However there apparently are mistakes in the indicated formulas since, for example, it follows from Eq. (26) that in the nonrelativistic limit in scattering through an angle $\theta = 0$ the polarization is reduced four fold which is obviously false. From Eq. (27) one obtains the information that in the scattering of a longitudinally polarized electron in the extreme relativistic limit its spin keeps its original direction independent of the scattering angle, which is also false. Therefore we give here expressions for the polarization of an electron beam after scattering by unpolarized electrons, obtained by the conventional method of projection operators:

$$\zeta_2(d\sigma/d\Omega) = A_1\zeta_1 + [A_2(\mathbf{n}_2\zeta_1) + A_4(\mathbf{n}_1\zeta_1)]\mathbf{n}_1 + [A_3(\mathbf{n}_1\zeta_1) + A_4(\mathbf{n}_2\zeta_1)]\mathbf{n}_2. \quad (17)$$

In this formula $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ refer, as before, to the rest system of the electron; however $\mathbf{n}_1$ and $\mathbf{n}_2$ now indicate the momentum direction before and after scattering in the center of mass system; $d\sigma/d\Omega$ is the Möller scattering cross section of unpolarized electrons

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{4}\frac{(2\gamma^2-1)^2}{\gamma^2(\gamma^2-1)^2}\left[\frac{4}{\sin^4\theta} - \frac{3}{\sin^2\theta} + \frac{(\gamma^2-1)^2}{(2\gamma^2-1)^2}\left(1 + \frac{4}{\sin^2\theta}\right)\right]. \quad (18)$$

Here $r_0 = e^2/mc^2$, and γ and θ are the energy and deflection angle of the electron in the center of mass system. The energy is measured in units of mc^2. The coefficients A_i are given by

$$A_i = [r_0^2/2\gamma^2(\gamma^2-1)^2]\,a_i,$$

$$a_1 = (2\gamma^2-1)^2(1+\cos\theta)/\sin^4\theta - (2\gamma^4-1)/\sin^2\theta,$$

$$a_2 = -(\gamma^2-1)[(2\gamma^2-1) + 2\gamma^2\cos\theta]/\sin^4\theta,$$

$$a_3 = (\gamma-1)[(2\gamma^2-1)(3\gamma+1) + 2(3\gamma^3+\gamma^2-2\gamma-1)\times\cos\theta]/\sin^4\theta - (\gamma-1)^2(2\gamma^2-1)(1+\cos\theta)/\sin^2\theta,$$

$$a_4 = (\gamma-1)[-2\gamma^3+2\gamma+1-\gamma(2\gamma^2-1)\cos\theta]/\sin^4\theta - (\gamma-1)(2\gamma^2-1)/\sin^2\theta. \quad (19)$$

In the nonrelativistic limit we obtain

$$\zeta_2 = \frac{2(1+\cos\theta)\cos\theta}{1+3\cos^2\theta}\zeta_1. \quad (20)$$

In this case the spin does not turn since there is no spin-orbit interaction, however the degree of polarization is reduced due to exchange effects. In the extreme relativistic case we obtain

$$\zeta_2 = 4(3+\cos^2\theta)^{-2}\{(1+\cos\theta)^2\zeta_1 - (1+\cos\theta)(\mathbf{n}_2\zeta_1)\mathbf{n}_1 + (1+\cos\theta)(3-\sin^2\theta)(\mathbf{n}_1\zeta_1)\mathbf{n}_2 - (1+\cos\theta+\sin^2\theta)\times[(\mathbf{n}_1\zeta_1)\mathbf{n}_1 + (\mathbf{n}_2\zeta_1)\mathbf{n}_2]\}. \quad (21)$$

If the initial polarization is longitudinal then after scattering through a small angle ($\theta \ll 1$) $\boldsymbol{\zeta}_2 = \zeta_1\mathbf{n}_1$, i.e., the polarization remains longitudinal.

The polarization of final state positrons in the scattering of a polarized positron beam by electrons may also be expressed in the form of Eq. (17). However $d\sigma/d\Omega$ now stands for the scattering cross section of unpolarized positrons by electrons

$$\frac{d\sigma}{d\Omega}=\frac{r_0^2}{16\gamma^2}\left[\frac{(2\gamma^2-1)^2}{(\gamma^2-1)^2\sin^4(\theta/2)}-\frac{8\gamma^4-1}{\gamma^2(\gamma^2-1)\sin^2(\theta/2)}+\frac{12\gamma^4+1}{\gamma^4}-\frac{4(\gamma^2-1)(2\gamma^2-1)}{\gamma^4}\sin^2\frac{\theta}{2}+4\frac{(\gamma^2-1)^2}{\gamma^4}\sin^4\frac{\theta}{2}\right]. \quad (22)$$

In this case the coefficients A_i are

$$A_1=\frac{r_0^2}{16\gamma^2}\left[\frac{(2\gamma^2-1)^2}{(\gamma^2-1)\sin^4(\theta/2)}-\frac{8\gamma^4-1}{\gamma^2(\gamma^2-1)\sin^2(\theta/2)}+\frac{4(\gamma^2+1)}{\gamma^2}\right],$$

$$A_2=\frac{r_0^2}{32\gamma^2}\left[-\frac{4\gamma^2-1}{(\gamma^2-1)\sin^4(\theta/2)}+\frac{4}{\sin^2(\theta/2)}\right],$$

$$A_3=\frac{r_0^2}{32\gamma^2}\left[\frac{12\gamma^3+4\gamma^2-7\gamma-3}{(\gamma+1)^2(\gamma-1)^2\sin^4(\theta/2)}-\frac{4(7\gamma^4+4\gamma^3-3\gamma^2+1)}{\gamma^2(\gamma+1)^2\sin^2(\theta/2)}+\frac{16(\gamma-1)(2\gamma^2-1)}{\gamma^2(\gamma+1)}-\frac{16(\gamma-1)^2}{\gamma^2}\sin^2\frac{\theta}{2}\right],$$

$$A_4=\frac{r_0^2}{32\gamma^2}\left[-\frac{(2\gamma+1)^2}{(\gamma+1)^2\sin^4(\theta/2)}+\frac{2(2\gamma^3-4\gamma^2-\gamma+1)}{\gamma^2(\gamma+1)\sin^2(\theta/2)}+\frac{8(\gamma-1)}{\gamma^2}\right]. \quad (23)$$

In the nonrelativistic limit the positron polarization is not changed by the scattering, i.e., $\zeta_1=\zeta_2$. This is explained by the fact that exchange effects are absent in positron-electron scattering in the nonrelativistic limit.

In the extreme relativistic limit we obtain

$$\zeta_2=\cos^2\frac{\theta}{2}\left\{\cos^2\frac{\theta}{2}\zeta_1-\frac{1}{2}[(\mathbf{n}_2\zeta_1)(\mathbf{n}_1+\mathbf{n}_2)+(\mathbf{n}_1\zeta_1)\mathbf{n}_1]+\left(1+\frac{1}{2}\cos^2\theta\right)(\mathbf{n}_1\zeta_1)\mathbf{n}_2\right\}\times\left[\sin^4\frac{\theta}{2}+\left(1+\sin^4\frac{\theta}{2}\right)\cos^4\frac{\theta}{2}\right]^{-1}. \quad (24)$$

In the scattering of a longitudinally polarized positron through a small angle we have, as in the electron case, $\zeta_2=\zeta_1\mathbf{n}_2$.

In the scattering of μ mesons by electrons the exchange effects are absent and the masses of the colliding particles are different. Meson polarization after scattering is given by Eq. (17) in which $d\sigma/d\Omega$ now stands for the scattering cross section of unpolarized mesons

$$\frac{d\sigma}{d\Omega}=\left(\frac{e^2}{c^2}\right)^2\frac{p^4\cos^2\theta+p^2(1+2\gamma\varepsilon+\Delta^2)\cos\theta+p^2(3\gamma^2+2\gamma\varepsilon-2)+\Delta^2(\gamma^2+1)}{8(\varepsilon+\gamma)^2p^4\sin^4(\theta/2)}, \quad (25)$$

where $\Delta=m/\mu$. The coefficients A_i are:

$$A_i=\left(\frac{e^2}{\mu c^2}\right)^2 a_i\left[8(\varepsilon+\gamma)^2p^4\sin^4\frac{\theta}{2}\right]^{-1},$$

$$a_1=p^2(2\gamma^2+2\gamma\varepsilon-1+\Delta^2)\cos\theta+p^2(2\gamma^2+2\gamma\varepsilon-1)+\Delta^2(\gamma^2+1),$$

$$a_2=-2p^4-p^2(2\gamma\varepsilon+\Delta^2),$$

$$a_3=p^2(\gamma-1)^2\cos^2\theta+2(\gamma-1)^2[(\gamma+1)\times(\gamma+2\varepsilon)+\Delta^2]\cos\theta+3p^4+2p^2\gamma\varepsilon+p^2\Delta^2,$$

$$a_4=p^2(\gamma-1)\cos\theta-p^2(\gamma-1)(2\gamma+1)-2p^2\varepsilon(\gamma-1)-(\gamma-1)^2\Delta^2. \quad (26)$$

In Eqs. (25) and (26) γ and ϵ denote the total energy of the meson and electron respectively measured in units of μc^2, and p is the momentum in units of μc where μ is the meson mass. If the meson is longitudinally polarized then the probability of the spin flipping with respect to its initial direction is given for low meson velocities by the formula*

$$Q=\left(\frac{m}{\mu}\right)^2\beta_L^2\sin^2\frac{\theta}{2}\left(1+\sin^4\frac{\theta}{2}\right), \quad (27)$$

where β_L is the meson velocity in the laboratory system in units of c. In the extreme relativistic case we find

$$\zeta_2=\{4(1+\cos\theta)\zeta_1-4(\mathbf{n}_2\zeta_1)\mathbf{n}_1+(\cos^2\theta+6\cos\theta+5)\times(\mathbf{n}_1\zeta_1)\mathbf{n}_2-4[(\mathbf{n}_1\zeta_1)\mathbf{n}_1+(\mathbf{n}_2\zeta_1)\mathbf{n}_2]\}\times(\cos^2\theta+2\cos\theta+5)^{-1}. \quad (28)$$

For a longitudinally polarized meson Eq. (28) becomes

$$\zeta_2=\zeta_1\mathbf{n}_2.$$

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Note added in proof (October 5, 1959). After this paper was submitted to the editor, G. V. Frolov has shown us his unpublished work which is a continuation of his previous work.[18] Our formulas (25) – (27) are special cases of Frolov's results.

*In the paper by Ford and Mullin[8] the analogous formula (20) contains an superfluous term $-\sin^4(\theta/2)$.

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Translated by A. M. Bincer
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ERRATA TO VOLUME 9

On page 868, column 1, item (e) should read:

(e). Ferromagnetic weak solid solutions. By way of an example, we consider the system Fe-Me with A2 lattice, where Me = Ti, V, Cr, Mn, Co, and Ni. For these the variation of the moment m with concentration c is

$$dm/dc = (N_d)_{Me} \mp 0.642 \ \{8 \ (2.478 - R_{Me}) + 6\,|2.861 - R_{Me}| \mp [\,8\,(2.478 - R_{\text{Fe}}) + 6\,(2.861 - R_{\text{Fe}})]\},$$

where the signs – and + pertain respectively to ferromagnetic and paramagnetic Me when in front of the curly brackets, and to metals of class 1 and 2 when in front of the square brackets. The first term and the square brackets are considered only for ferromagnetic Me. We then have dm/dc = –3 (–3.3) for Ti, –2.6 (–2.2) for V, –2.2 (–2.2) for Cr, –2 (–2) for Mn, 0.7 (0.6) for Ni, and 1.2 (1.2) for Co; the parentheses contain the experimental values.

ERRATA TO VOLUME 10

Page	Reads	Should Read
224, Ordinate of figure	10^{23}	10^{29}
228, Column 1, line 9 from top	3.6×10^{-2} mm/min	0.36 mm/min
228, Column 1, line 16 from top	0.5 mm/sec	0.05 mm/min
329, Third line of Eq. (23a)	+(1/4 cosh r +...	+1/4(cosh r +...
413, Table II, line 2 from bottom	–0.0924 ±	–1.0924 ±
413, Table II, line 3 from bottom	+1.8730 ±	+0.8370 ±
479, Fig. 7, right, 1st line	92 hr	9.2 hr
499, Second line of Eq. (1.8)	$+\tilde{k}\sin^2\alpha/\omega_N^2 + \langle c^2\tilde{k}^2 \ldots$	$+\left(\tilde{k}/\omega_H\right)^2 \sin^2\alpha \ \langle c^2\tilde{k}^2 \ldots$
648, Column 1, line 18 from top	18 × 80 mm	180 × 80 mm
804, First line of Eq. (17)	$-{}^1/_3\left(\alpha_x^2\alpha_y^2 + \ldots\right.$	$\ldots - 3\left(\alpha_x\alpha_y^2 + \ldots\right.$
967, Column 1, line 11 from top	$\sigma(N', \pi) \approx 46\,(N', N')$	$\sigma(N', \pi) > \sigma(N', N')$
976, First line of Eq. (10)	$= \frac{e^2}{3r^2c^4}$	$= \frac{e^2}{3\hbar^2c^2}$
978, First line of Eq. (23)	$\left[\frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1)\sin^4(\theta/2)}\right.$	$\left[\frac{(2\gamma^2 - 1)^2}{(\gamma^2 - 1)^2\sin^4(\theta/2)}\right.$