DISPERSION RELATIONS FOR INELASTIC K-MESON PROCESSES

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Submitted to JETP editor June 7, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 1379-1384 (November, 1959)

Dispersion relations for a process of the type $\widetilde{K} + N \to Y + \pi$ are written down. The structure of the amplitude for scalar and pseudoscalar K mesons is described.

1. Amati and Vitale[1] considered nonrelativistic dispersion relations for inelastic K-meson processes; relativistic dispersion relations for $\pi + N \to Y + K$ (Y means Λ or Σ) and $\gamma + N \to Y + K$ were studied by Polivanov and Okubo.[2] In the present work the processes $\widetilde{K} + N \to Y + \pi$ are considered, in particular $K^- + p \to \Sigma^0 + \pi^0$. Because of the difference in the M_Λ and M_Σ masses, which particles also occur in intermediate states, the energy spectra are different for each process.

2. We make use here of a complex field for $K^\pm$ and a real field for π^0 which directly represent the physical K and π particles.

Using the notation of Bogolyubov and Shirkov[3] for a complex scalar field and the definitions

$$j_K(x) = i\frac{\delta S}{\delta \overset{*}{\varphi}_K(x)} S^+,$$

$$j_\pi(x) = i\frac{\delta S}{\delta \varphi_\pi(x)} S^+,$$

we get for $K^- + p \to \Sigma^0 + \pi^0$, dropping the factor

$$\delta(q + p - p' - q') \,/\, \sqrt{2q_0 q_0'},$$

$$\langle p' \,|\, S + S^+ \,|\, p\rangle = i\,[T^{ret}(k) - T^{adv}(k)] = iT(k),$$

where $|p\rangle$ is a state of momentum p, and p′ and p are the momenta of the hyperon and nucleon, while q′ and q are the pion and K-meson momentum, where

$$(p')^2 = M_Y^2, \quad (p)^2 = M^2, \quad (q')^2 = m_\pi^2, \quad (q)^2 = m_K^2.$$

Here, using the Bogolyubov causality condition[4]

$$T^{ret}(k) = \int d\eta e^{i(k\eta)} \left\langle p' \left| \frac{\delta j_K(-\eta/2)}{\delta\varphi_\pi(\eta/2)} \right| p \right\rangle = \int d\eta e^{i(k\eta)} F^{ret}(\eta),$$

$$T^{adv}(k) = \int d\eta e^{i(k\eta)} \left\langle p' \left| \frac{\delta j_\pi(\eta/2)}{\delta\overset{*}{\varphi}_K(-\eta/2)} \right| p \right\rangle = \int d\eta e^{i(k\eta)} F^{adv}(\eta),$$

where $k = \frac{1}{2}(q' - q)$ and $(k\eta) = k_0\eta_0 - \mathbf{k}\boldsymbol{\eta}$. In getting the second term we use the condition $SS^+ = 1$ and the invariance of the vacuum and one-particle states.

Later we shall work with the Fourier transforms

$$T(k) = 2iA(k) = \int d\eta e^{i(k\eta)} F(\eta),$$

$$D(k) = \int d\eta e^{i(k\eta)} \bar{F}(\eta),$$

where

$$F(\eta) = i\langle p' \,|\, [j_K(-\eta/2),\ j_\pi(\eta/2)] \,|\, p\rangle$$

and

$$\bar{F}(\eta) = {}^1\!/_2\,[F^{ret}(\eta) + F^{adv}(\eta)],$$

which represent the antihermitian part A(k) and the Hermitian part D(k) of the retarded matrix $T^{ret}(k)$.

3. We introduce the coordinate system in which $p_0 = p_0'$ and $q_0 = q_0'$. We put $\mathbf{p}' = a\mathbf{p}$; the condition $p_0 = p_0'$, $\sqrt{\mathbf{p}^2 + M^2} = \sqrt{a^2\mathbf{p}^2 - M_Y^2}$ gives

$$a = \pm\sqrt{1 - \Delta/\mathbf{p}^2} = \pm\alpha,$$

where $\Delta = (M_Y^2 - M^2)$. The sign of α is determined below from the condition that there is a break in the energy spectrum, and it turns out to be positive. Therefore

$$\mathbf{p}' = \alpha\mathbf{p}.$$

Using momentum conservation and the definition $\mathbf{k} = \frac{1}{2}(\mathbf{q}' + \mathbf{q}) = \lambda\mathbf{e} - \gamma\mathbf{p}$, we get from the conditions $\mathbf{ep} = 0$ and $\mathbf{e}^2 = 1$

$$\gamma = -\delta/2(1-\alpha)\mathbf{p}^2, \quad \delta = (m_K^2 - m_\pi^2), \quad \lambda^2 = q_0^2 - E_\tau,$$

where

$$E_\tau^2 = {}^1\!/_4(1-\alpha)^2\mathbf{p}^2 + \gamma^2\mathbf{p}^2 + m,$$

and $m = \frac{1}{2}(m_K^2 + m_\pi^2)$.

Therefore, for $k = \frac{1}{2}(q' + q)$, where $k_0 = \frac{1}{2}(q_0' + q_0)$, we get

$$k = (E;\ \mathbf{k}) = (E;\ \lambda\mathbf{e} - \gamma\mathbf{p}),$$

$$\lambda = \pm\sqrt{E^2 - E_\tau^2}.$$

Here E_τ is the threshold energy in our coordinate system.

4. We thus have

$$T(E) = \int d\eta \exp\{iE\eta_0 - i\lambda\eta\mathbf{e} + i\gamma\eta\mathbf{p}\} F(\eta). \quad (4.1)$$

For the process $\pi^0 + p \rightarrow \Sigma^0 + K^+$ we get

$$T_c(E) = -\int d\eta \exp\{iE\eta_0 - i\lambda\eta\mathbf{e} - i\gamma\eta\mathbf{p}\} F(-\eta). \quad (4.2)$$

Expanding the members of the commutator in the complete system of intermediate states we find that the first member in T(E) contains the state

$$|\nu\rangle = p,\ p + \pi^0, \ldots$$

and the second

$$|\mu\rangle = \Lambda,\ \Sigma^0 + \pi, \ldots,$$

i.e., from baryon and strangeness conservation we have a nucleon branch $|\nu\rangle$ and a hyperon branch $|\mu\rangle$.

Integrating over η, we get

$$T(E) = i(2\pi)^4 \Big\{\sum_\nu \langle p'|j_K(0)|p_\nu\rangle\langle p_\nu|j_\pi(0)|p\rangle\,\delta(E - p_0 + p_0^\nu) - \sum_\mu \langle p'|j_\pi(0)|p_\mu\rangle\langle p_\mu|j_K(0)|p\rangle\,\delta(E + p_0 - p_0)\Big\} \quad (4.3)$$

and the momenta of the intermediate states, determined by the delta-functions

$$\mathbf{p}_\nu = {}^1/_2(1+\alpha)\mathbf{p} - \mathbf{k} = [(1+\alpha)/2 + \gamma]\mathbf{p} - \lambda\mathbf{e},$$

$$\mathbf{p}_\mu = {}^1/_2(1+\alpha)\mathbf{p} + \mathbf{k} = [(1+\alpha)/2 - \gamma]\mathbf{p} - \lambda\mathbf{e}. \quad (4.4)$$

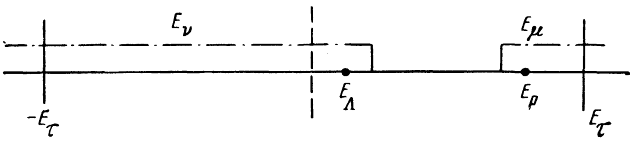
The delta-functions determine the spectrum of the nucleon branch

$$-2p_0E_\nu = (M_\nu^2 - M^2 - m) - \mathbf{p}^2(1-\alpha) - \frac{\delta}{2}\left(\frac{1+\alpha}{1-\alpha}\right). \quad (4.5')$$

and the hyperon branch

$$2p_0E_\mu = (M_\mu^2 - M^2 - m) - \mathbf{p}^2(1-\alpha) + \frac{\delta}{2}\left(\frac{1+\alpha}{1-\alpha}\right). \quad (4.5'')$$

Here M_ν and M_μ mean the mass of the intermediate states. The spectrum has the form given in the figure.



Form of the spectrum for the process $K^- + p \rightarrow \Sigma^0 + \pi^0$

5. Examination shows that the two regions of the continuous spectrum are not joined for all values of $\mathbf{p}^2$ within the limits

$$1.04\,\Delta < \mathbf{p}^2 < \infty$$

(Δ is the minimum $\mathbf{p}^2$ value).

We see that here the continuous spectrum in the unobservable region takes up an even greater area than it does in the K-meson scattering case, where we also have a continuous spectrum even for forward scattering. In this case the reaction $K^- + p \rightarrow Y + \pi$ contributes in the unphysical region, since $M + m_K > M_Y + m_\pi$ always. For dispersion relations in the K-scattering case, Galzenati and Vitale[5] got reasonable results, in the sense of comparisons with experiment, by neglecting the integrals over the unphysical region; lowest order perturbation theory for $K^- + N \rightarrow Y + \pi$ shows[6] that the introduction of an amplitude below the threshold is smooth for the case of a pseudoscalar K meson.

In all reactions of the $K^- + p \rightarrow Y + \pi$ type, there is a contribution in the unphysical region from the reactions $\pi_1 + Y_1 \rightarrow Y_2 + \pi_2$ and $\pi_1 + N_1 \rightarrow N_2 + \pi_2$; estimating these contributions is made more difficult by the fact that both branches still contain states of the type $(p + 2\pi)$ or $(Y + 2\pi)$ and so on, corresponding to different processes of the strong pion interaction.

6. The spectrum of the function $T_c(E)$ for the reaction $\pi^0 + p \rightarrow \Sigma^0 + K^+$ is reflection symmetric relative to the spectrum of the function T(E).

From (4.1) and (4.2), we have

$$S_eT(-E) = -S_eT_c(E),$$
$$\mathfrak{A}_eT(-E) = +\mathfrak{A}_eT_c(E), \quad (6.1)$$

where S_e, $\mathfrak{A}_e$ mean symmetrization with respect to $\mathbf{e}$, which excludes the doublevaluedness of λ (see reference 7). The expression (6.1) corresponds to the "crossing symmetry" and affords the possibility of excluding the negative-energy region in dispersion relations.

Dispersion relations can be obtained by substituting the Cauchy integral formula in the function T(E); the contour of integration is evident from the figure. Under the same conditions as in reference 7, we get the relation between D(E) and A(E):

$$S_eD(E) = \frac{1}{\pi}\mathrm{P}\int_0^\infty d\varepsilon\left[\frac{S_eA(\varepsilon)}{\varepsilon - E} + \frac{S_eA_c(\varepsilon)}{\varepsilon + E}\right],$$

$$\mathfrak{A}_eD(E) = \frac{1}{\pi}\mathrm{P}\int_0^\infty d\varepsilon\left[\frac{\mathfrak{A}_eA(\varepsilon)}{\varepsilon - E} - \frac{\mathfrak{A}_eA_c(\varepsilon)}{\varepsilon + E}\right]. \quad (6.2)$$

Combining equal relations for $T_c(E)$ we can write the symmetric expressions

$$S_e\{D(E) - D_c(E)\} = \frac{2E}{\pi}\mathrm{P}\int_0^\infty \frac{d\varepsilon}{\varepsilon^2 - E^2}\{S_eA(\varepsilon) - S_eA_c(\varepsilon)\},$$

$$\mathfrak{A}_e\{D(E) - D_c(E)\} = \frac{2}{\pi}\mathrm{P}\int_0^\infty \frac{\varepsilon\,d\varepsilon}{\varepsilon^2 - E^2}\{\mathfrak{A}_eA(\varepsilon) - \mathfrak{A}_eA_c(\varepsilon)\}, \quad (6.3)$$

where the subtraction of the amplitudes plays a "cutoff" role at high energies.

7. For pseudoscalar K mesons the amplitude of the process is scalar.

Then

$$T^{ret} = \delta_{s's} T^0 + i\lambda [\mathbf{e} \times \mathbf{p}] \langle \sigma \rangle_{s's} T^1, \quad (7.1)$$

where s′ and s are the spin states of the hyperon and the nucleon. The other possible invariant **ep** is equal to zero in our coordinate system, so that T^0, T^1 do not depend on the sign of **e** or **p**. Separating T^0 and T^1 into hermitian and antihermitian parts, corresponding to T_c^0 and T_c^1 we get from (6.3), applying the symmetrizations S_e and $\mathfrak{A}_e$

$$D^1(E) - D_c^1(E) = \frac{2}{\pi} \mathrm{P} \int_0^\infty \frac{\varepsilon \, d\varepsilon}{\varepsilon^2 - E^2} \{A^1(\varepsilon) - A_c^1(\varepsilon)\},$$

$$D^0(E) - D_c^0(E) = \frac{2E}{\pi} \mathrm{P} \int_0^\infty \frac{d\varepsilon}{\varepsilon^2 - E^2} \{A^0(\varepsilon) - A_c^0(\varepsilon)\} \quad (7.2)$$

For scalar K mesons the amplitude is pseudoscalar. The only non-vanishing pseudoscalars are $\boldsymbol{\sigma} \cdot \mathbf{p}$ and $\boldsymbol{\sigma} \cdot \mathbf{e}$, which expresses the fact that we have here only the spin-flip amplitudes; the spin flip compensates for the change in internal parity.

Therefore

$$T^{ret} = \{\langle \sigma \rangle_{s's} \mathbf{p}\} T^{(p)} + \{\langle \sigma \rangle_{s's} \mathbf{e}\} T^{(e)}. \quad (7.3)$$

This gives, together with (6.3), the relationship between the hermitian and antihermitian part of the amplitudes $T^{(p)}$ and $T^{(e)}$

$$D^{(p)}(E) - D_c^{(p)}(E) = \frac{2}{\pi} \mathrm{P} \int_0^\infty \frac{d\varepsilon}{\varepsilon^2 - E^2} \{A^{(p)}(\varepsilon) - A_c^{(p)}(\varepsilon)\},$$

$$D^{(e)}(E) - D_c^{(e)}(E) = \frac{2}{\pi} \mathrm{P} \int_0^\infty \frac{\varepsilon \, d\varepsilon}{\varepsilon^2 - E^2} \{A^{(e)}(\varepsilon) - A_c^{(e)}(\varepsilon)\}. \quad (7.4)$$

8. Let us examine the contribution of the poles. The first terms in A(E) for one nucleon or one hyperon in the intermediate state give the amplitudes

$$a_p = |1 - E_p / p^0| \sum_{s''} \langle p' | j_K(0) | p_\nu, s'' \rangle \times \langle p_\nu, s'' | j_\pi(0) | p \rangle \delta(E - E_p),$$

$$a_\Lambda = |1 - E_\Lambda / p^0| \sum_{s''} \langle p' | j_\pi(0) | p_\mu, s'' \rangle \times \langle p_\mu, s'' | j_K(0) | p \rangle \delta(E + E_\Lambda).$$

They contain the the coupling constants $(gg_{\Sigma Kp})$ and $(g_{\Sigma\Lambda\pi} g_{\lambda pK})$ where g is the pion-nucleon coupling constant. We note, however, that the energy E_p corresponding to a pole is not the same as the pole contribution for pion-nucleon scattering.

In both the scalar and pseudoscaler K-meson cases, the contributions from the poles are

$$(gg_{\Sigma Kp}) \frac{E}{(2\pi)^2 (E_p^2 - E^2)} \{+ E_p F(\mathbf{p}^2) + f(\mathbf{p}^2)\} - (g_{\Sigma\Lambda\pi} g_{\Lambda pK}) \frac{E}{(2\pi)^2 (E_\Lambda^2 - E^2)} \{E_\Lambda \Phi(\mathbf{p}^2) + \varphi(\mathbf{p}^2)\}, \quad (8.2')$$

and

$$(gg_{\Sigma Kp}) \frac{E_p}{(2\pi)^2 (E_p^2 - E^2)} \Psi(\mathbf{p}^2) + (g_{\Sigma\Lambda\pi} g_{\Lambda pK}) \frac{E}{(2\pi)^2 (E_\Lambda^2 - E^2)} \Psi(\mathbf{p}^2), \quad (8.2'')$$

corresponding to the first and second equations in (7.2) and (7.4).

The functions $F(\mathbf{p}^2)$, $\Phi(\mathbf{p}^2)$, etc depend on our coordinate system and we do not give their general form.

9. It has already been remarked that these relations contain integrals over the unphysical regions, even more than the K-scattering case. According to perturbation theory results[6] or to the dispersion relations for K scattering, this difficulty may not be so serious for scattering; but this question is still open.

In our case the possibility of estimating the coupling constants is reduced by the existence of the nucleon branch of the pion-nucleon coupling.

As an approximation, as for example a one-meson approximation, we can express the amplitudes $\langle \Sigma, \pi | j_\pi | Y \rangle$, for which there are no experimental data, in terms of other amplitudes by making some assumptions about strong interaction symmetries.

I express my thanks to A. A. Logunov for suggesting the problem and to M. K. Polivanov for valuable discussions.

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Translated by W. Ramsay
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