REDUCED WIDTHS FOR NUCLEON ASSOCIATIONS IN THE SHELL MODEL OF THE NUCLEUS

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A method is considered which can be employed to calculate the reduced widths of compound-nucleus levels decaying with emission of deuterons, tritons, and α particles. The analysis is based on the shell model of the nucleus.

It has been pointed out several times that the shell model and the α particle model are not mutually exclusive. For example, it can be easily shown[1,2] that, using oscillator functions, the wave functions for the lowest states 0^+, 2^+, and 4^+ of the Be^8 nucleus are the same in the shell model with LS coupling and in the α particle model. This is an illustration of the fact that the α association of the nucleons is compatible with the Young scheme of the orbital part of the wave function if this scheme contains the maximal number of four-groups.[3] An essential part of the features of the α particle model is, therefore, already contained in the symmetry properties of the wave functions of the lowest states of nuclei with shell structure, as has been noted earlier.[4]

It should be mentioned that by basing the analysis of the α association in the nucleus on the shell model, we can not only make a formal comparison between the models as mentioned above, but also obtain for the first time the possibility to calculate all those effects in which the α association (or the d, t, and other associations) plays the principal role. This means we can use the experimental data immediately to test the correctness of the approach to this problem. One of these effects is the decay of the compound nucleus under emission of d, t, He^3, or α. The probability of the decay through one of these channels is determined by the corresponding reduced level width γ^2, which, in the instance of α particles, is given by[5,6]

$$\gamma = \binom{A}{4}^{1/2} \frac{\hbar}{\sqrt{2\mu}} \sum_{M,\,M_1} (J_1 M_1 \Lambda M \mid J_0 M_0) \times \int X_\lambda^* \psi_1 \psi_\alpha Y_{\Lambda M}(\mathbf{R}_1 - \mathbf{R}_\alpha)\, d\sigma \tag{1}$$

where $|\mathbf{R}_1 - \mathbf{R}_\alpha| = a$. X_λ is the wave function for the level λ of the compound nucleus; ψ_1 and ψ_α are the functions of the final state of the nucleus and of the α particle, respectively; $Y_{\Lambda M}(\mathbf{R}_1 - \mathbf{R}_\alpha)$ is the angular part of the wave function describing the relative motion of the recoiling particles with the orbital angular momentum Λ and its projection M; μ is the reduced mass of the recoiling particles;

$$\binom{A}{m} = A!\,/\,m!\,(A-m)!;$$

a is the channel radius; J_0, M_0 and J_1, M_1 are the angular momenta and their projections of the initial and final states of the nucleus, respectively; $(J_1 M_1 \Lambda M \mid J_0 M_0)$ are Clebsch-Gordan coefficients; the integration is to be taken over the internal variables of the recoiling particles and over the angular coordinates of their relative motion.

Using oscillator functions to describe the relative motion of the nucleons in the states X_λ, ψ_1, and ψ_α, we can connect the internal functions of X_λ, ψ_1, and ψ_α with the shell model functions X_λ^{sh}, ψ_1^{sh}, and ψ_α^{sh}, corresponding to the lowest energy state for the center of mass of these nuclei. According to the theorem of Elliot and Skyrme,[4] we have for states with one unfilled state,

$$X_\lambda^{sh} = \psi_{00}(\mathbf{R})\, X_\lambda = |\,\beta,\, l^n [f]\, TSL\rangle, \tag{2a}$$

$$\psi_1^{sh} = \psi_{00}(\mathbf{R}_1)\, \psi_1 = |\,\beta,\, l^{n-4} [f_1]\, T_1 S_1 L_1\rangle, \tag{2b}$$

$$\psi_\alpha^{sh} = \psi_{00}(\mathbf{R}_\alpha)\, \psi_\alpha = |\, s^4 [4]\, 000\rangle, \tag{2c}$$

where $\psi_{00}(\mathbf{R})$ is an oscillator function describing the motion of the center of mass of the nucleus with the quantum numbers N = 0 and L = 0 [N is the principal quantum number: $E_{NL} = \hbar\omega(N + 3/2)$] β is a symbol denoting all closed shells, and n is the number of nucleons in the unfilled shell.

With the help of the technique of fractional parentage coefficients a group of four nucleons can be separated out of the function X_λ^{sh}. We shall be interested only in the fractional parentage coefficient

$$\langle \beta,\, l^n [f]\, TSL \,|\, \beta,\, l^{n-4} [f_1]\, TSL_1;\; l^4 [4]\, 00\Lambda \rangle,$$

corresponding to the separation of four nucleons in the state with T = 0, S = 0, and the same Young scheme $[f_2] = [4]$ as for ψ_α^{sh}. For these coefficients we have the following relation:

$$\langle \beta,\, l^n [f]\, TSL \,|\, \beta,\, l^{n-4} [f_1]\, T_1S_1L_1;\; l^4 [f_2]\, T_2S_2L_2 \rangle = \binom{n}{4}^{1/2} \times \binom{A}{4}^{-1/2} \langle l^n [f]\, TSL \,|\, l^{n-4} [f_1]\, T_1S_1L_1;\; l^4 [f_2]\, T_2S_2L_2 \rangle. \quad (3)$$

The function $|\, l^4 [4]\, 00\Lambda \rangle$ contains the state $\psi_3 = \psi_{N\Lambda}(\mathbf{R}_\alpha)\, \psi_\alpha$ with the coefficient

$$K(N\Lambda) = \langle l^4 [4]\, 00\Lambda \,|\, \psi_{N\Lambda}(\mathbf{R}_\alpha)\, \psi_\alpha \rangle,$$

for which (2c) gives the expression

$$K(N\Lambda) = \left\langle l^4 [4]\, 00\Lambda \left| \frac{\psi_{N\Lambda}(\mathbf{R}_\alpha)}{\psi_{00}(\mathbf{R}_\alpha)} \right| s^4 [4]\, 000 \right\rangle. \quad (4)$$

If we integrate in the right-hand side of (1) not only over the angular variables of the vector $\mathbf{R}_1 - \mathbf{R}_\alpha$, but also over the radial variable, we can rewrite the expression for γ in the following form:

$$\gamma = R_{N\Lambda}(|\mathbf{R}_1 - \mathbf{R}_\alpha| = a) \binom{A}{4}^{1/2} \frac{\hbar}{\sqrt{2\mu}} \times \int X_\lambda^* \psi_1 \psi_\alpha \psi_{N\Lambda}(\mathbf{R}_1 - \mathbf{R}_\alpha)\, d\sigma\, d(|\mathbf{R}_1 - \mathbf{R}_\alpha|). \quad (5)$$

(ψ_1 and $\psi_{N\Lambda}$ are coupled in the state J_0, M_0).

Let us consider the integral

$$I = \int X_\lambda^{sh*} \psi_1^{sh} \psi_3\, d\sigma\, d(|\mathbf{R}_1 - \mathbf{R}_\alpha|)\, d\mathbf{R} = \binom{n}{4}^{1/2} \binom{A}{4}^{-1/2} \times \langle l^n [f]\, TSL \,|\, l^{n-4} [f_1]\, TSL_1;\; l^4 [4]\, 00\Lambda \rangle\, K(N\Lambda). \quad (6)$$

Separating the relative motion of the recoiling particles and their center of mass motion with the help of the Talmi transformation[8] and using expression (5), we can write this integral in the form

$$I = \gamma \left\langle \begin{matrix} 0\,0 \\ N\Lambda \end{matrix} \right| A-4.4 \left| \begin{matrix} N\Lambda \\ 0\,0 \end{matrix} \right\rangle [R_{N\Lambda}(|\mathbf{R}_1 - \mathbf{R}_\alpha| = a)]^{-1} \binom{A}{4}^{-1/2}$$

$$\left\langle \begin{matrix} 0\,0 \\ N\Lambda \end{matrix} \right| A-4.4 \left| \begin{matrix} N\Lambda \\ 0\,0 \end{matrix} \right\rangle = \int \psi_{00}^*(\mathbf{R}_1)\, \psi_{N\Lambda}^*(\mathbf{R}_\alpha)\, \psi_{00}(\mathbf{R})\, \psi_{N\Lambda} \times (\mathbf{R}_1 - \mathbf{R}_\alpha)\, d\mathbf{R}_1 d\mathbf{R}_\alpha = \left(\frac{A-4}{A}\right)^{N/2}. \quad (7)$$

Hence

$$\gamma = \frac{\hbar}{\sqrt{2\mu}} \left(\frac{A}{A-4}\right)^{N/2} \binom{n}{4}^{1/2} \langle l^n [f]\, TSL \,|\, l^{n-4} [f_1]\, TSL_1;\; l^4 [4]\, 00\Lambda \rangle \times K(N\Lambda)\, R_{N\Lambda}(|\mathbf{R}_1 - \mathbf{R}_\alpha| = a). \quad (8)$$

Assuming that at the nuclear boundary $\frac{1}{2} a^3 R_{N\Lambda}^2(a) = 1$ (reference 5), we obtain for the reduced width for the decay of the compound nucleus under emission of an α particle

$$\gamma^2 = \frac{\hbar^2}{\mu a} \left(\frac{A}{A-4}\right)^N \binom{n}{4} \langle l^n [f]\, TSL \,|\, l^{n-4} [f_1]\, TSL_1;\; l^4 [4]\, 00\Lambda \rangle^2 K^2(N\Lambda). \quad (9)$$

This formula can be generalized to the case where a cluster with an arbitrary number of nucleons m and with spin $S_2 \neq 0$ is emitted:

$$\gamma^2_{m\Lambda J_0 J_1} = \frac{\hbar^2}{\mu a} \left(\frac{A}{A-m}\right)^N \binom{n}{m} \sum_{L_2 J_2} \langle l^n [f]\, TSL \,|\, l^{n-m} [f_1]\, T_1S_1L_1;\; l^m [f_2]\, T_2S_2L_2 \rangle^2 K^2(N\Lambda, L_2)\, U^2 \times (SL_1J_0L_2 : J_2L)\, U^2 (L_1S_1J_2S_2 : J_1S), \quad (10)$$

where J_0 and J_1 are the total angular momenta of the initial and final states, respectively, J_2 is the channel spin, U (ABCD : EF) is a Racah coefficient,[9] and

$$K(N\Lambda, L_2) = \left\langle l^m [f_2]\, T_2S_2L_2 \left| \frac{\psi_{N\Lambda}(\mathbf{R}_2)}{\psi_{00}(\mathbf{R}_2)} \right| \psi_2^{sh} \right\rangle.$$

For the deuteron we take $\psi_2^{sh} = |\, s^2 [2]\, 010 \rangle$, and for the triton, $\psi_2^{sh} = |\, s^3 [3]\, \frac{1}{2} \frac{1}{2}\, 0 \rangle$. Here we assume that the internal wave function of the recoiling particles is not changed appreciably during the transition in the external region. Rough estimates support this assumption. According to the shell model we describe the states of the nuclei H^3, He^3, and He^4 by the configurations s^3 and s^4 with oscillator wave functions. To obtain the correct binding energies for these nuclei, we must set $\hbar\omega = 16$ Mev (the "width of the well" is $r_0 = \sqrt{\hbar/m\omega} = 1.6 \times 10^{-13}$ cm). This value is very close to that used in the description of the p shell nuclei in the framework of the intermediate coupling model. Thus the internal wave functions of the three-nucleon and four-nucleon clusters, corresponding to the states $|\, s^n [f]\, TS_0 \rangle$ and $|\, p^n [f]\, TSL \rangle$, are close to one another. If we recall that in general the same form of potential for the nucleon interaction is used to calculate the binding energy of t, He^3, and α as well as the binding energies and level energies of p shell nuclei, we find that in these nuclei the binding energies of the t and α clusters (and of He^3 as well) agree with good accuracy (5 – 10%) with the experimental binding energies for the corresponding free particles. This gives rise to the hope that our "algebraic" separation of the

Reduced widths for the d, t, and α decays of levels of light nuclei

Nucleus	E* Mev	J, T	$^{2T+1,2S+1}L[f]$	Type of decay	Λ	$2\mu a\gamma^2/3\hbar^2$ Theory	$2\mu a\gamma^2/3\hbar^2$ Experiment	$a \cdot 10^{13}$cm
He^5	2.0	3/2,1/2	^{22}P [1]	$He^4 + n$	1	5/6	1[11]	
Li^6	2.189 4.52	3,0 2,0	^{13}D [2]	$He^4 + d$	2	3/4	0.5[12] 0.6	4.0
Be^7	4.65	7/2,1/2	^{22}F [3]	$He^4 + He^3$	3	$\frac{343}{432}$	0.3[13]	4.4
Be^8	2.9	2,0	^{11}D [4]	$He^4 + \alpha$	2	1	1[12]	5.0

three- and four-nucleon complexes in the framework of the shell model reproduces the most essential features of this association. In the case of deuterons the situation is somewhat worse. It should further be noted that the smallness of the binding energy of the deuteron and the triton may lead to an additional factor smaller than unity in the corresponding formulas for the reduced widths, as a result of the action of the nuclear boundary on the internal function of the emitted particle. At the same time one might think that our method of calculation will satisfactorily reproduce the relative values of the reduced widths for the emission of d and t for the different levels of the same nucleus. The results of this work are given in the table (E* is the excitation energy of the level).

Mang,[10] in his calculation of the α decay of heavy nuclei, also used oscillator functions for the separation of the internal motion of the nuclei and the α particles.

In conclusion it is of interest to anticipate the appearance of those effects of the peculiarities of the α association which are due to the correlations between the nucleons which are not taken account of in the shell model.

1. One might think that, if we use nucleon interaction parameters that reproduce the energies of nuclear states which do not admit of α association (for example, the levels with T = 1 in Be^8) to compute the levels which do admit of α association, i.e., which are characterized by a Young scheme with maximal number of 4-groups, the latter will lie above the corresponding experimental levels as a consequence of the additional energy connected with forming the four-nucleon complex.

2. The α associations should involve LS coupling, i.e., the states of the nuclei admitting α associations should be more adequately described by the LS coupling scheme than is the case in the intermediate coupling model. (Thus, for example, the Be^8 core in the nuclei of the type B^{10} is apparently better described by LS coupling, while the outer nucleons should conform to jj or intermediate coupling.) Correlations in the α associations which are not considered in the shell model can also show up in the β decay. Thus, these additional correlations can lead to a higher value of ft for the decay B^{12}-C^{12} than is obtained in the usual shell model theory with intermediate coupling

3. As another consequence of these additional correlations, the fractional parentage expansion of the true wave function of, for example, the Be^9 nucleus should contain the α associated states Be^8 with a greater statistical weight than that given by the fractional parentage coefficients of the shell model wave function.

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