THE CONTRIBUTION OF THREE-PARTICLE FORCES TO THE BINDING ENERGIES OF HYPERNUCLEI

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The three-particle Λ-nucleon potential is calculated in the lowest order of meson theory. It is shown that the contribution of this potential to the potential energy of a Λ particle in a hypernucleus is positive and of insignificant amount. The estimates so obtained do not confirm Spitzer's[1] conclusion that three-particle forces play a large part in hypernuclei.

ASSUMPTIONS that many-particle forces can play a large part in the interactions of Λ particles with nucleons have been put forward repeatedly.[2-5] The absence of interaction between a Λ particle and a nucleon through the exchange of a single π meson has the result that the radius of action of the pair forces is of the order of $1/2m_\pi$. At the same time, on the assumption of charge independence of the forces the exchange of single π mesons is allowed in three-particle interactions (see figure). This leads to forces with a radius of action of the order $1/m_\pi$. Therefore it might be expected that three-particle forces are of more importance in Λ-nucleon than in nucleon-nucleon interactions.

Spitzer[1] has calculated the three-particle forces in the lowest order of perturbation theory and found that these forces make a very large negative contribution to the potential energy of the Λ particle. This result is somewhat unexpected, since such forces do not play any important part in nucleon-nucleon interactions.[6] The study of these forces is continued in the present paper.

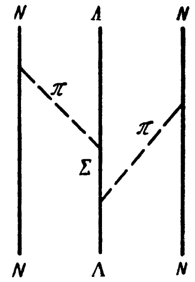
The eight diagrams of the type shown in the figure lead to the following expression for the potential-energy operator of the interaction between a Λ particle and a system of A nucleons:

$$V_3 = -\frac{f^4}{m_\pi^4}\frac{4M_N^2}{M_\Lambda^2}\sum_{i\neq j}^{A}\frac{(\tau_i\tau_j)}{(2\pi)^4}\iint\frac{(k_1k_2)(\sigma_i\mathbf{k}_1)(\sigma_j\mathbf{k}_2)}{\omega_1^3\omega_2^2}$$

$$\times\{\exp[i\mathbf{k}_1(\mathbf{x}-\mathbf{x}_i)+i\mathbf{k}_2(\mathbf{x}-\mathbf{x}_j)]\}\,v^2(k_1)v^2(k_2)\,d\mathbf{k}_1\,d\mathbf{k}_2; \quad (1)$$

$\mathbf{x}_i$ and $\mathbf{x}_j$ are the coordinates of the nucleons, $\mathbf{x}$ is the coordinate of the Λ particle, and $v(k)$ is the cutoff function.

In the calculation of the potential we have neglected the mass difference $\Delta = M_\Sigma - M_\Lambda$ and have assumed that the parities of the Λ and Σ particles



are the same as for nucleons. It must be noted that if the parities of Λ and Σ are different, then in the adiabatic approximation the forces of the type in question are zero.

The potential (1) does not contain the spin operator of the particle. In both the singlet and the triplet symmetric S state of a pair of nucleons the operator $(\tau_i\tau_j)(\sigma_i\mathbf{k}_1)(\sigma_j\mathbf{k}_2)$ has the value:

$$\langle(\tau_i\tau_j)(\sigma_i\mathbf{k}_1)(\sigma_j\mathbf{k}_2)\rangle = -(\mathbf{k}_1\mathbf{k}_2). \quad (2)$$

Therefore in the case of light hypernuclei the contribution to the potential energy of the Λ particle from the forces (1) is proportional to the number of pairs of nucleons in the core nucleus.

To obtain an estimate of the potential energy of the Λ particle in a light hypernucleus the distribution of nucleons in the core nucleus was taken to be a Gaussian distribution:

$$\rho(r) = a\exp\{-r^2/\gamma^2 r_\pi^2\}, \quad (3)$$

$r_\pi = 1/m_\pi$ is the Compton wavelength of the π meson, and a is a normalization constant (we take $\hbar = c = 1$). The parameter γ was chosen to make the density distribution in the core nucleus agree with the data on the scattering of electrons by light nuclei (taking account of the size of the proton).

The distribution of the Λ particle in the hypernucleus was also taken to be Gaussian:

$$\rho_\Lambda(r) = a_\Lambda \exp\{-r^2\eta^2/\gamma^2 r_\pi^2\}. \tag{4}$$

The variation parameter η characterizes the deviation of the distribution of the Λ particle from that of the nucleons. Using Eqs. (3), (4), and (2), we get the following expression for the potential energy of the Λ particle in a light hypernucleus, with the central part of the force fixed by Eq. (1):

$$U = f^4 \frac{8}{3\pi^2} \frac{M_N^2}{M_\Lambda^2 \gamma_2^6} m_\pi \frac{1}{2} A(A-1) F(\gamma^2, \gamma_1^2, \gamma_2^2), \tag{5}$$

$$F(\gamma^2, \gamma_1^2, \gamma_2^2) = \int_0^{x_m}\int_0^{x_m} \frac{(\gamma_2^4 x_1^3 x_2^3 - \gamma_2^2 x_1^2 x_2^2 + {}^1/_2 x_1 x_2)}{(x_1^2+1)(x_2^2+1)^{3/2}}$$

$$\times \exp[-\gamma_1^2(x_1-x_2)^2 - {}^1/_2\gamma^2 x_1 x_2]\, dx_1 dx_2,$$

$$\gamma_1^2 = {}^1/_4\gamma^2(1+\eta^{-2}), \qquad \gamma_2^2 = (4\eta^2)^{-1},$$

$$x = k/m_\pi, \qquad x_m = k_m/m_\pi, \tag{6}$$

k_m is the cutoff momentum (a rectangular cutoff is used). The function $F(\gamma^2, \gamma_1^2, \gamma_2^2)$ was calculated numerically for $x_m = 6$. The values of the variation parameter were found from the condition that the binding energy of the Λ particle be a minimum, as calculated by means of the two-particle potentials of the meson theory.[7] The constants in these potentials were taken to have the values given in reference 7. The table shows the values of γ^2 and η^2 that were used, and the potential energies calculated from Eq. (5) ($f^2 = 0.083$). The last line shows for comparison the potential energies for the pair forces caused by the exchange of two π mesons. The comparison shows that the contribution of the three-particle forces is insignificant.

Hypernucleus	H^3_Λ	H^4_Λ, He^4_Λ	He^5_Λ
γ^2	1.00	0.80	0.88
η^2	0.33	0.42	0.25
Three-particle forces, Mev	0.1	0.3	0.5
Pair forces, Mev	−8	−14	−18

From (5) it can be seen that as the number of particles in the core nucleus increases the contribution of the three-particle forces increases as A^2. In heavy hypernuclei, however, owing to the short-range nature of the forces, the contribution to the potential energy will be determined not by the number of particles in the core nucleus, but by the density of the nuclear matter. We can estimate the contribution of the potential (1) to the potential of a Λ particle in a heavy hypernucleus if we regard the nucleons as a degenerate Fermi gas and describe them by a wave function ψ that is a determinant made up of plane waves, taking account of the spin and isotopic spin of the nucleons. In this case the technique of calculation used earlier to find the contribution of the pair forces[8,9] leads to the following expression for the potential energy of a Λ particle in a heavy hypernucleus owing to the forces (1):

$$U = \langle\psi, V_3\psi\rangle = f^4 \frac{24}{\pi^2} m_\pi \frac{M_N^2}{M_\Lambda^2} \int_0^{2x_F} (x^2+1)^{-5/2}$$

$$\times \left(\frac{2}{3}x_F^3 x^6 - \frac{1}{2}x_F^2 x^7 + \frac{1}{24}x^9\right) dx, \tag{7}$$

$x_F = k_F/m_\pi$, and k_F is the maximum momentum of the nucleons in the nucleus. (In obtaining the expression (7) we have assumed that the number of protons is equal to the number of neutrons.) For nuclear radius $R = 1.2 A^{1/3}$ F, $x_F = 1.77$. With this value of x_F and $f^2 = 0.08$ we get from Eq. (7) U = 2.6 Mev. This result is to be compared with the contribution of the pair forces from interchange of two π mesons, which is $U^{(2\pi)} = -78$ Mev.[8] Thus the contribution from three-particle forces of the type under consideration is positive and of negligible size both for light and also for heavy hypernuclei. The results of these calculations do not support the conclusion of reference 1, that three-particle forces play a large part.

There can also be three-particle forces between a Λ particle and nucleons owing to the exchange of K and π mesons

$$N + \Lambda + N \to N + \widetilde{K} + N + \pi + N \to \Lambda + N + N,$$

$$N + \Lambda + N \to \Lambda + K + \Lambda(\Sigma) + \pi + N \to \Lambda + N + N.$$

Estimates show that forces of this type make a negative contribution to the potential energy, but here also it is negligibly small both in light and in heavy hypernuclei.

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