lation and obtained T_1 for the general case of an arbitrary field orientation both for the hexagonal and for the tetragonal lattice. We do not give here the final formula, which is complicated. It has been established that the anisotropy of T_1 is not more than 5%, so that in measurements of T_1 on single crystals or polycrystalline material one would expect practically identical results.

4. The crystal lattice of deuterium was first determined by Kogan, Lazarev, and Bulatova[8] (tetragonal, a = 3.35, c = 5.79) and it seemed useful to explore the possibility of confirming these results by data on the anisotropy of nuclear resonance. We have limited ourselves to examining orthodeuterium, as the intensity of the resonance is almost an order of magnitude greater than for paradeuterium. The rotational state of orthodeuterium has spherical symmetry, so that intramolecular broadening should be absent. According to Hatton and Rollin[3] one can also neglect the line broadening caused by the quadrupole moment of the deuterium nucleus. The summation (2) leads to considerable anisotropy:

$$\sum_k r_{ik}^{-6}(3\cos^2\theta_{ik}-1)^2 = a^{-6}(8.32-3.2\cos^2\theta). \qquad (5)$$

In conclusion I would like to express my sincere thanks to B. G. Lazarev for his interest in the work.

*The present work was started in connection with an experimental search for anisotropy in nuclear resonance in hydrogen, carried out by A. A. Galkin and I. V. Matyash, to whom we are grateful for suggesting the problem.

†The measurements of the specific heat anomaly[1] and Nakamura's theory[2] indicate that in this temperature range an appreciable anistropy of molecular orientation is already starting. The intramolecular resonance line broadening connected with this can, in principle, be calculated from Moriya and Motizuki's theory,[6] but such a calculation is extremely unwieldy. We should point out that the experimental data given by Sugawara et al.,[10] with values of the second moment determined on polycrystalline material, indicates the relatively small intramolecular broadening, at least for small ortho-hydrogen concentrations ($\rho \sim$ 10–20%).

[1] R. W. Hill and B. W. A. Ricketson, Phil Mag. **45**, 277 (1954).

[2] T. Nakamura, Prog. Theor. Phys. **14**, 135 (1955).

[3] J. Hatton and B. V. Rollin, Proc. Roy. Soc. **A199**, 222 (1949).

[4] F. Reif and E. M. Purcell, Phys. Rev. **91**, 631 (1953).

[5] M. Bloom, Physica **23**, 767 (1957).

[6] T. Moriya and K. Motizuki, Prog. Theor. Phys. **18**, 183 (1957).

[7] Keesom, de Smedt, and Mooy, Proc. Konink. Akad. Wetenschap, Amsterdam **33**, 814 (1930).

[8] Kogan, Lazarev, and Bulatova, JETP **37**, 678 (1959), Soviet Phys. JETP **10**, 485 (1960).

[9] J. H. Van Vleck, Phys. Rev. **74**, 1168 (1948).

[10] Sugawara, Masuda, Kanda, and Kanda, Phys. Rev. **95**, 1355 (1954).

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DENSITY OF CHARGED PARTICLES IN THE CHANNEL OF A SPARK DISCHARGE

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In order to determine the density of charged particles in the channel of a spark discharge we have investigated the shape of the He II line λ = 4686 A produced in a discharge in helium. Under the experimental conditions in the present work (p = 1.5 – 12 atmos, C = 0.05 μf, U = 2 – 12 kv, L = 0.18 – 3.6 μh) up to 0.3 μsec after the initiation of the discharge only the spark lines of helium (4686 and 3203 A) are radiated; however, the shapes of these lines could not be examined quantitatively because of smearing. The line shapes were recorded by means of a photoelectric system in which traces are made at two different instants of time after the initiation of the discharge.[1,2] It has been established that at the beginning of the discharge the 4686 A line is highly broadened and shifted toward the red, although there is no noticeable asymmetry. The displacement was measured with respect to the position of the same line at later instants of time, when the line exhibits essentially no displacement (t ≈ 1 μsec). It is reasonable to assume that the red shift of the line is due to the quadratic Stark effect.[3] The absolute values of the displacement (up to 8 A) and the half width (up to 50 A) indicate that in the initial stages of the discharge the density of charged particles is quite appreciable.

This density can be estimated by three methods:

1. Using the shape of the 4686 A line computed by Unsöld on the basis of the Holtsmark theory for the linear Stark effect it is possible to find the density of charged particles N by matching (at the wings of the line) the experimental and theoretical shapes (cf. for example, references 5 and 6).

It should be noted that for line shapes which

Discharge conditions			t, μsec	$N \cdot 10^{-18}$, cm^{-3}		
p, atmos	L, μh	U, kv		at the wings of the line	according to ref. 1	according to ref. 3
1.5	0.18	4	0.08	0,62	0,52	0,77
2.5	0.18	2.5	0,15	0,42	0,45	0,75
			0.3	0.47	0.36	0.49
5	0.18	6	0.05	—	0.69	0,96
			0.2	0,62	0.56	0,69
8	0.18	6	0.05	—	0,94	1.7
			0.25	—	—	1,2
12	0.18	10.5	0.05	—	1.1	2
			0.25	—	—	1,28
2.5	3.6	3,5	0.15	0,18	0.15	0.23
			0.45	—	0.19	0.29

are distorted by reabsorption and for cases of high density (> 10^{18} cm^{-3}) the curves cannot be matched satisfactorily.

2. By measuring the half width of the line it is possible to estimate the density of charged particles from the well-known relation given by Holtsmark.[7] Using the Stark constant for the outermost components of the splitting (n = 6) we obtain the following relation between the half width of the line γ (in A) and the density:

$$\gamma = 4 \cdot 10^{-11} N^{2/3}. \qquad (1)$$

Both of these methods are based on the Holtsmark statistical theory for the linear Stark effect. Thus it is assumed that the broadening of the lines is due to the linear effect and that the second-order effect can be neglected.*

3. Using the experimentally determined value of the line shift, it is possible to find the density from the theory of the quadratic Stark effect in collision broadening:[10]

$$\Delta\omega = 33.4 C^{2/3} v^{1/3} N. \qquad (2)$$

A very rough estimate indicates that the existence of a linear effect does not lead to a significant change in Eq. (2) for the case of the 4686 A line (cf. also reference 11).

The mean value of the Stark constant for several Stark components (n = 0, 1, 2, 3) is 1.2×10^{-15} cm^4 sec^{-1}. Assuming that the temperature of the channel is 10 ev, we obtain the following relation between the line shift Δ (in A) and the density of charged particles N (per cm^3):

$$\Delta = 2.6 \cdot 10^{-18} N. \qquad (3)$$

In investigations of line shape it is important to take account of distortions due to reabsorption. Starting from the ratio of the intensity of the background, spark and arc lines, using the Saha formula for $N = 10^{18}$ cm^{-3} it is possible to place a lower limit on the temperature of the discharge channel. Carrying out this estimate for $t \approx 0.1\,\mu$sec, we find $T \geq 6-7$ ev. The maximum measured radiant temperature of the channel (for $\lambda = 4686$ A) is 2 ev. Whence it follows that for $t < 0.1\,\mu$sec there is no noticeable reabsorption in the channel. At later moments of time and pressures $p \geq 8$ atmos reabsorption does become significant; this is indicated by the depression of the line peaks.

The results of calculations of the charged-particle density for various experimental conditions are given in the table. In view of the rough simplifications which have been used in the calculations, the results obtained by different methods seem to be in satisfactory agreement.

It should be noted that the question of whether or not it is permissible to use existing theories of line broadening for the case of such a dense plasma ($N \approx 10^{18}$ cm^{-3}) has not yet been resolved;[8,10] in particular, under these conditions the conditions for an ideal plasma are not satisfied.[8]

In conclusion the author wishes to thank M. P. Vanyukov for his interest in the work and discussion of the results.

*It should be noted that at the present time it is difficult to evaluate the role of the electrons quantitatively by the use of the statistical theory (cf. for example, refs. 8 and 9); for this reason there is some uncertainty in the determination of the density.

[1] B. A. Ermakov and A. A. Mak, Приборы и техника эксперимента (Instrum. and Meas. Engg.) No. 3 (1959).

[2] Vanyukov, Ermakov, Mak, and Muratov, Вестник ЛГУ, серия физ. хим. (Herald, Leningrad State University, Phys. and Chem. Series.) No. 16, 25, (1959).

[3] A. Sommerfeld, Atomic Structure and Atomic Spectra, I. Methuen, London 1934 (Russ. Transl. Gostekhizdat, 1956).

[4] A. Unsöld, Z. Astrophys. **23**, 75 (1944).

[5] J. D. Craggs and W. Hopwood, Proc. Phys. Soc. (London) **59**, 755 (1947).

[6] H. Wulff, Z. Physik, **150**, 614 (1958).

[7] J. Holtsmark, Ann. Phys., **58**, 577 (1919).

[8] V. I. Kogan, Физика плазмы и проблема управляемых термоядерных реакций (Plasma Physics and the Problem of a Controlled Thermonuclear Reaction), IV, Academy of Sciences Press (U.S.S.R.) (1958) p. 258.

[9] W. Lochte-Holtgreven, Probleme des Plasmas in Physik und Astronomie, Akademie-Verlag, Berlin 1958, p. 156.

[10] I. I. Sobel'man, Usp. Fiz. Nauk **54**, 551 (1954).

[11] H. Griem, Z. Physik **137**, 280 (1954).

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THE CROSS SECTION FOR ELECTRON-ELECTRON SCATTERING AT HIGH ENERGIES

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SINCE it is intended very soon to make experiments on the scattering of electrons by electrons at high energies,[1] it is desirable to derive the general formula for the scattering of an electron by an electron in the case in which the charges of the electrons are smeared out in an invariant way.

The calculation has been made for the case of the exchange of one photon. As is well known, in this case the vertex operator for the interaction of an invariantly smeared out electron and a photon can be written in the form

$$e\Gamma_\mu(q) = e[\gamma_\mu f_1(q^2) + (1/2m)(\hat{q}\gamma_\mu - \gamma_\mu\hat{q}) f_2(q^2)]. \quad (1)$$

Here q is the momentum transferred, and $f_1(q^2)$ and $f_2(q^2)$ are functions describing the distributions of charge and current in the electron.

If a particle of spin ½ has a point charge e and a point anomalous magnetic moment μ, then $f_1 = 1$, $f_2 = -i\mu/2$; for small values of the magnitude of the momentum transfer q the functions f_1 and f_2 are respectively the distributions of charge and anomalous magnetic moment in the electron. If, however, the magnitude of the momentum transfer becomes larger than or of the order of the reciprocal of the length characterizing the distributions of charge and anomalous magnetic moment in the electron, this simple interpretation of the functions $f_1(q^2)$ and $f_2(q^2)$ becomes incorrect, and both functions describe both the charge distribution and the anomalous magnetic-moment distribution. Just for this reason, although the anomalous magnetic moment of the electron is of radiative origin and is very small ($\mu \sim \alpha/2\pi$), the function $f_2(q^2)$ can be very important for the description of the charge and current distributions in the electron at small distances.

After averaging and summing over the spins of the electrons in the initial and final states we get the following formula for the scattering cross section in the center-of-mass system:

$$d\sigma / d\Omega = r_0^2 X / 4\gamma^2, \quad (2)$$

where

$$X = \alpha / 4(1-\lambda)^2 + \beta / 4(1-\mu)^2 - (\delta_1 + \varepsilon_1) / 4(1-\mu) \times (1-\lambda);$$

$$\alpha = 2[2 - 2\lambda + \varkappa^2 + \mu^2]|f_1|^4 + 8(1-\lambda) \times [4 - 3\lambda + \lambda^2 - \mu^2 - \varkappa^2]|f_1|^2|f_2|^2 + 4(1-\lambda)^2 \times [7 - 2\lambda - \lambda^2 + 2(\varkappa^2 + \mu^2)]|f_2|^4 - 16(1-\lambda)\,\mathrm{Im}(f_1 f_2^*) \times [|f_1|^2(\lambda - 2) + |f_2|^2(1-\lambda)(\lambda - 5)] + 48(1-\lambda)^2[\mathrm{Im}(f_1 f_2^*)]^2;$$

$$\delta_1 + \varepsilon_1 = 2\{2\,\mathrm{Re}(f_1'^2 f_1^{*2})[\lambda + \mu + \varkappa - \varkappa^2 - 1] - 2\,\mathrm{Im}(f_1 f_2 f_1'^{*2}) \times (1-\lambda) \times [5\varkappa + 3\mu + \lambda - 3] - 2\,\mathrm{Re}(f_1'^2 f_2^{*2})(1-\lambda) \times [5\varkappa + \mu + 2\lambda - 1 - \lambda^2 - 2\lambda\mu] - 2\,\mathrm{Im}(f_1' f_2' f_1^{*2})(1-\mu) \times [5\varkappa + 3\lambda + \mu - 3] + 8\,\mathrm{Re}(f_1' f_2' f_1^* f_2^*)(1-\mu)(1-\lambda) \times (2\varkappa - 3) - 4\,\mathrm{Im}(f_1' f_2' f_2^{*2})(1-\lambda)(1-\mu)[6 - 4\lambda - \mu - \varkappa] - 2\,\mathrm{Re}(f_2'^2 f_1^{*2})(1-\mu) \times [5\varkappa + \lambda + 2\mu - 1 - \mu^2 - 2\lambda\mu] - 4\,\mathrm{Im}(f_1 f_2 f_2'^{*2})(1-\lambda)(1-\mu) \times [6 - \lambda - \varkappa - 4\mu] + \mathrm{Re}(f_2'^2 f_2^{*2})(1-\lambda)(1-\mu)[\lambda^2 + \mu^2 - 5\varkappa^2 - 13];$$

the coefficient β can be obtained from the formula for α if we make the following replacements:

$$f_{1,2} \to f'_{1,2}; \quad \lambda \to \mu, \quad \mu \to \lambda.$$

Besides this, we have used the notations:

$$f_{1,2} \equiv f_{1,2}(q^2), \quad f'_{1,2} \equiv f_{1,2}(q'^2);$$

$$\varkappa = -m^{-2}(p_1 p_2) = -m^{-2}(p'_1 p'_2) = 2\gamma^2 - 1,$$

$$\mu = -m^{-2}(p_1 p'_2) = -m^{-2}(p_2 p'_1) = \gamma^2 + (\gamma^2 - 1)\cos\vartheta,$$

$$\lambda = -m^{-2}(p_1 p'_1) = -m^{-2}(p_2 p'_2) = \gamma^2 - (\gamma^2 - 1)\cos\vartheta,$$

$$\varepsilon = \gamma m.$$

ϑ is the scattering angle in the center-of-mass system; p_1, p_2 are the initial and p'_1, p'_2 the final momenta; $q = p_1 - p'_1 = p'_2 - p_2$, $q' = p_1 - p'_2 = p'_1 - p_2$.