THE CAPACITANCE OF p-n JUNCTIONS AT LOW TEMPERATURES

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The capacitances of p-n junctions in germanium and silicon were studied down to helium temperatures. It was established that at very low temperatures the capacitance of the p-n junction itself is not observed because of the small series capacitance of the base between the p-n junction and the electrodes. It was shown that all the observed phenomena could be explained using a simple equivalent circuit. The additional difference of potential affecting the value of the p-n junction capacitance was determined, and its connection with the screening effect of the inversion layer and the contact difference of potential was established.

An electron-hole or p-n junction occurs in a semiconductor in one portion of which there is a surplus of donor impurities and in another portion a surplus of acceptor impurities.

For the elements of the third and fifth groups, which are commonly used as impurities, the ionization energy W_i is ~ 0.01 ev in germanium and ~ 0.04 ev in silicon.

At sufficiently high temperatures, when $kT \gg W_i$ (k is the Boltzmann constant, T is the absolute temperature), the impurity atoms are almost completely ionized. The surplus ionized impurities not compensated by electrons in the one part and by holes in the other part of the semiconductor, form the space charge of the p-n junction.

At sufficiently low temperatures, when $kT \ll W_i$, the concentration of ionized surplus impurities becomes very small in the homogeneous parts of the semiconductor far from the p-n junction; but the junction itself remains, and the contact or diffusion potential difference U_k in it tends to the value corresponding to the difference in position of the donor and acceptor impurity energy levels:[1]

$$U_k = \frac{E_F - E_F'}{q} = \frac{E_d - E_a}{q} + \frac{kT}{q}\ln\left[\frac{N_d - N_a}{N_a}\right]\left[\frac{N_a' - N_d'}{N_d'}\right] \quad (1)$$

where E_F, E_F' are the Fermi levels in the n-type and p-type germanium, respectively; E_d is the energy level of the donor impurity; E_a the energy level of the acceptor impurity; N_d, N_a are the concentrations of donor and acceptor impurities in the n-type germanium; N_d', N_a' are the concentrations of donor and acceptor impurities in the p-type germanium; q is the electronic charge.

It is apparent that the large decrease of the electrical conductivity and the increase of the contact potential difference occurring at low temperatures should significantly alter all the properties of a p-n junction, including the differential capacity which typifies the space charge distribution in it. It is, therefore, of interest to study the properties of p-n junctions at low temperatures, particularly their capacitance, especially as recently intensive work has started on the use of p-n junctions as non-linear capacitors, as proposed by one of us several years ago.*

1. EXPERIMENTAL PROCEDURE

The measurements of p-n junction capacitances were made using an MLE-1 bridge at audio-frequencies, a Q-meter in the range of frequencies ν from 50 kcs to 1 Mcs and a special bridge at a fixed frequency of 100 kcs. The amplitude of the measuring signal during measurements with the bridges was about 1 — 5 mv, but for measurements with the Q meter at high Q factors it went as high as 2 v. The biasing voltage was supplied by a battery and measured directly.

The measurements were carried out in liquid helium, hydrogen and nitrogen at atmospheric or reduced pressure of the vapors. Intermediate temperatures from 4.2 to 14°K and from 20 to 65°K were obtained by heating the assembly in the vapors of the corresponding liquid.

The assembly consisted of a copper case inside which were the specimen to be studied and the thermometers (carbon and copper). The

*B. M. Vul. Inventor's Certificate No. 02407/460177, dated June 29, 1954.

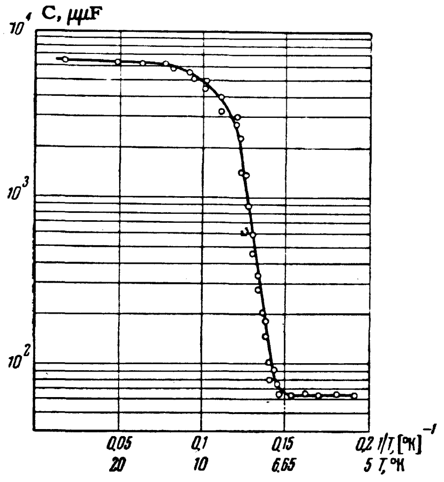


FIG. 1. The capacitance of a germanium diode (specimen no. 1) as a function of temperature. Bias voltage U = 0, frequency of measuring signal $\nu = 10^5$ cps, diode area $S = 1.88$ cm^2, thickness d = 0.4 mm.

lead to the specimen consisted of a coaxial cable, the capacity of which ($12\,\mu\mu$F) remained constant when the device was moved in the cryostat. The heating rate could be varied from 0.5 to 5° K per hour.

The measurements were made on alloyed germanium and silicon diodes of various dimensions fabricated in the All-Union Electro-Technical Institute[2] and in the P. N. Lebedev Physics Institute of the U.S.S.R. Academy of Sciences. For equivalent types of specimens, very similar results were obtained.

2. CAPACITANCE AT VERY LOW TEMPERATURES

The results of capacitance measurements on germanium and silicon diodes as a function of temperature are given in Figs. 1 and 2. As is seen from these, the capacitance of the diodes is practically constant over a wide temperature range, but diminishes sharply for germanium diodes around helium temperatures and for silicon diodes around hydrogen temperatures. The temperature interval in which the sharp change of capacitance occurs will be referred to below as the "transitional."

On further decreasing the temperature, the capacitance does not decrease further. Having attained definite small values, the capacitances become again almost constant quantities, determined only by the geometrical dimensions and dielectric permittivity of the specimens — a fact which was confirmed by additional measurements on specimens without p-n junctions.

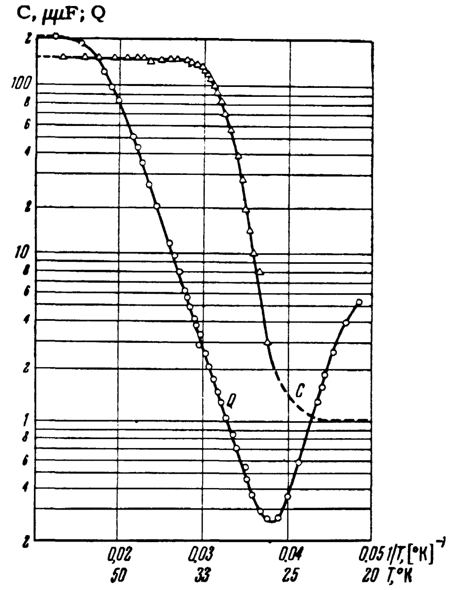


FIG. 2. The capacitance C and Q-factor of a silicon diode (specimen no. 6) as a function of temperature. U = 0, $\nu = 7.3 \times 10^4$ cps, S = 0.04 cm^2, d = 0.3 mm.

The apparent disappearance of the p-n junction at very low temperatures would seem to be connected with the fact that under these conditions the capacity of the p-n junction itself does not affect the measured capacity. This fact, and also the fact that at very low temperatures the differential real part of the conductivity of the germanium is very small in comparison with the imaginary part, allows the simple equivalent circuit shown in Fig. 3 to be used to calculate the capacitance in the intermediate range.

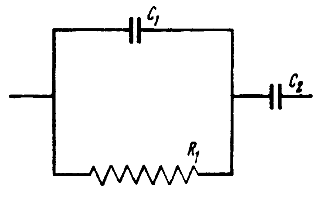


FIG. 3. Equivalent circuit; C_1, R_1, are the capacitance and resistance of the n-type part of the specimen; C_2 is the capacitance of the p–n junction.

For this circuit the measured capacitance is

$$C = C_2(1 + \omega^2 R_1^2 C_1^2) / (1 + \omega^2 R_1^2 C_1 C_2 + \omega^2 R_1^2 C_1^2) \qquad (2)$$

where C_2 is the capacitance of the p-n junction; C_1 and R_1 are the capacitance and resistance of the n-type part of the semiconductor. The effect of the p-type part can be neglected, because its thickness is small in comparison with the n-type. Since $C_2/C_1 \gg 1$, then at very low temperatures, when $\omega^2 R_1^2 C_1 C_2 \gg 1$, it follows from (2) that

$$C/C_2 = (1 + \omega^2 R_1^2 C_1^2) / \omega^2 R_1^2 C_1 C_2$$

or

$$C - C_1 = 1 / \omega^2 R_1^2 C_1. \qquad (3)$$

The change of electron mobility with temperature can be neglected in comparison with the concentration change and, therefore, it can be considered approximately that

$$1/R_1 \sim n, \tag{4}$$

and therefore from (3) we obtain

$$C - C_1 \sim n^2 \text{ for } \omega = \text{const}.$$

This is confirmed by calculating the ionization energy from the results given in Fig. 4.

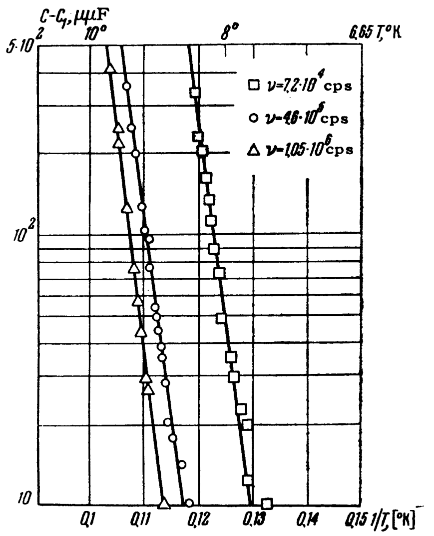


FIG. 4. The temperature dependence of the capacitance $C - C_1$ of a germanium diode (specimen no. 4) in the "transitional" temperature interval for various frequencies of the measuring signal ν. $C_1 = 67\ \mu\mu F$, $S = 2\ cm^2$, $d = 0.4$ mm.

From Eq. (2) it follows that if C = const, then ωR = const, or $\omega \sim 1/R \sim n$. Since it can be taken in this region of temperature that $n \approx \exp(-W_i/kT)$, then, using (4) we find

$$\ln \omega = \text{const} - W_i/kT.$$

In Fig. 5 a graph is presented of the dependence of $\ln \omega$ on $1/T$ for fixed capacitance of the germanium diode $C = 2C_1$. The ionization energy of the donor impurity atoms in germanium determined from the slope of the line in Fig. 5 is 1.1×10^{-2} ev, which agrees well with known values.[3] From analogous measurements on silicon diodes the value $W_i \approx 0.046$ ev was obtained.

To verify the equivalent circuit in the "transitional" region in addition to the capacitance measurements, the Q-factor was also measured. For the circuit given in Fig. 3 the Q-factor is

$$Q = (1 + \omega^2 R_1^2 C_1 C_2 + \omega^2 R_1^2 C_1^2) / \omega C_2 R_1. \tag{5}$$

It follows from (5) that, due to the change of R with temperature, the Q-factor has a minimum. The minimum Q-factor is

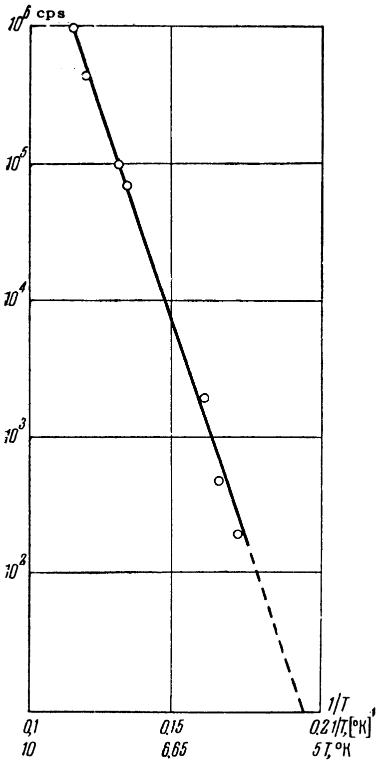


FIG. 5. The relationship between the temperature and the frequency of the measuring signal in the "transitional" temperature interval for constant capacitance $C = 2C_1$ – germanium diode (specimen no. 4)

$$Q_{\min} = 2\left[(C_1C_2 + C_1^2)/C_2\right]^{1/2} \approx 2\,(C_1/C_2)^{1/2},$$

since $C_1 \ll C_2$. The results of Q-factor measurements on germanium and silicon diodes are given in Figs. 2 and 6. The measurements were made with zero bias voltage. For the germanium power rectifier the values $C_1 = 67\ \mu\mu F$, $C_2 = 6750\ \mu\mu F$ were obtained, and correspondingly by calculation $Q_{\min} = 0.2$. The character of the change of Q with temperature, the presence of the minimum, and its value, show that the equivalent circuit used is a sufficiently close approximation under our conditions.

Since in actual devices the thickness of the p-n junction (δ) is much smaller than the base thickness (h), then $C_2 \gg C_1$ always and

$$Q = (1 + \omega^2 R_1^2 C_1 C_2) / \omega C_2 R_1.$$

If $\omega^2 R_1^2 C_1 C_2 \ll 1$, which is true at high temperatures, then

$$Q \approx 1/\omega C_2 R_1 \sim n.$$

The slope of the curve $Q(1/T)$ (Figs. 2 and 6) to the left of the minimum corresponds to the temperature dependence $Q \sim n$. If, on the other hand, $\omega^2 R_1^2 C_1 C_2 \gg 1$, which is true at reduced temperatures, then

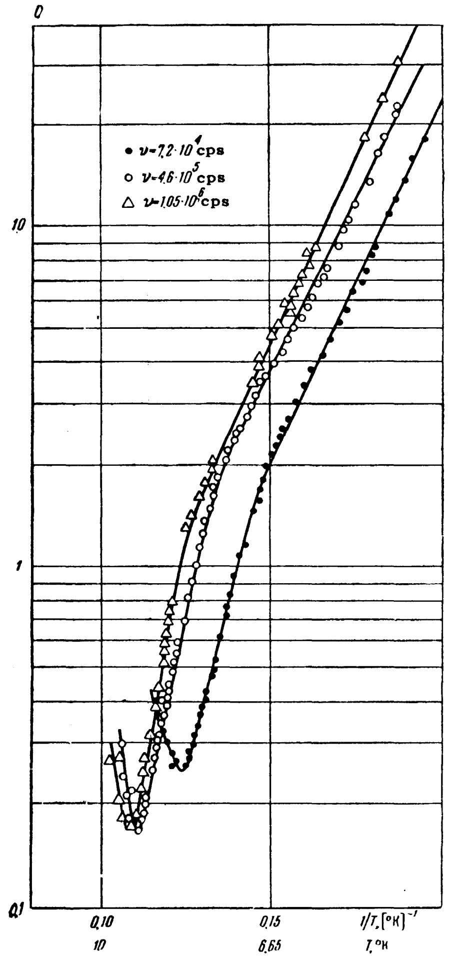


FIG. 6. Q-factor of a germanium diode (specimen no. 4) at low temperatures for various frequencies of the measuring signal ν. Bias voltage U = 0.

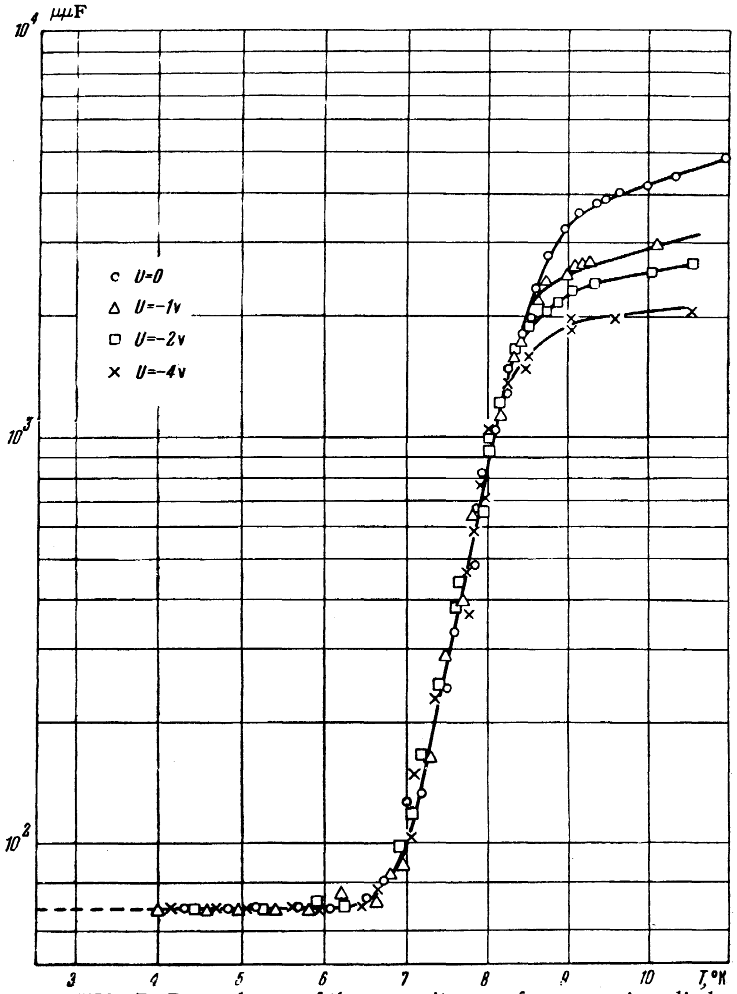


FIG. 7. Dependence of the capacitance of a germanium diode (specimen no. 1) on temperature in the "transitional" region for various bias voltages.

$$Q \approx \omega R_1 C_1 \sim 1/n.$$

The results given in Figs. 2 and 6 confirm that Q ~ 1/n to the right of the minimum, for small alternating signals.

For large signal amplitudes this dependence is destroyed because of the effect of the signal electric field on the quantity n (see reference 4).

As follows from (3), the capacitance C measured in the "transitional" region does not depend on C_2. Whence it follows that in this temperature interval it should not depend on the bias voltage; this is confirmed by the results given in Fig. 7 (common portion of the curves). On increasing the temperature R_1 gets smaller and then it is no longer possible to consider that $\omega^2 R_1^2 C_1 C_2 \gg 1$. In these conditions the measured capacity C becomes a function of C_2, and consequently depends on the applied bias voltage (divergence of the curves in Fig. 7). In some temperature intervals R is still sufficiently large, and thus relaxation effects appear. It is apparent that the nonlinear properties of p-n junction capacitances are fully displayed under the condition $\omega^2 R_1^2 C_1 C_2 \ll 1$ or

$$\nu\rho\varepsilon(h/\delta)^{1/2} \ll 1.8\cdot 10^{12}, \qquad (6)$$

where ρ is the specific resistance of the semiconductor.

3. VARIATION OF CAPACITY WITH BIAS VOLTAGE

For a step p-n junction

$$1/C^2 = 8\pi(U_0 - U)/S^2 q \varepsilon N_d \qquad (7)$$

where S is the area of the diode, U_0 is the additional potential difference, U is the applied voltage, ϵ is the dielectric permittivity, N_d the surplus concentration of ionized impurity.

If the concentration N_d remains unchanged during changes of temperature, then, as follows from (7), the straight lines $1/C^2 = \varphi(U)$ should be parallel and the intersections they make on the abscissa axis should be U_0. As is seen from the

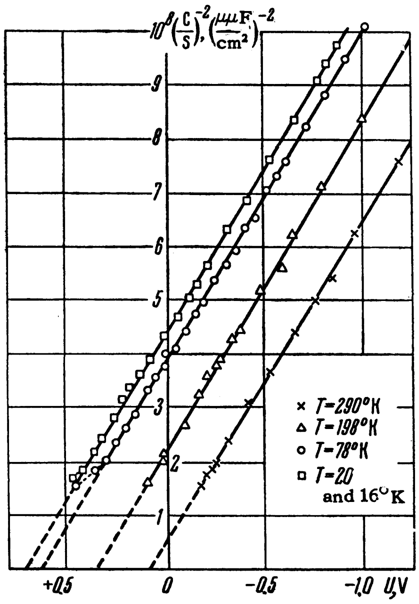


FIG. 8. Dependence of the inverse square of the capacitance of a germanium diode (specimen no. 1) on bias voltage for various temperatures; $\nu = 1.05 \times 10^5$ cps.

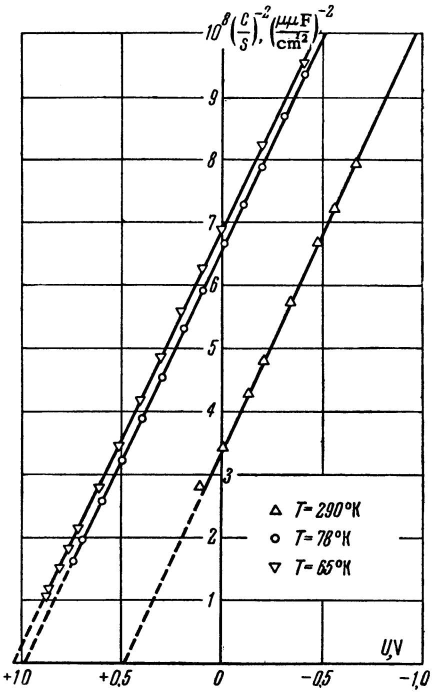


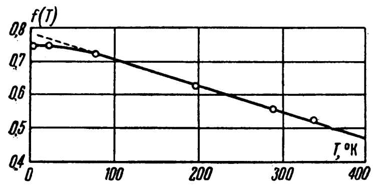
FIG. 9. Dependence of the inverse square of the capacitance of a silicon diode (specimen no. 6) on the bias voltage for various temperatures; $\nu = 1.05 \times 10^6$ ps.

results in Figs. 8 and 9, the slope of the straight lines remains constant. The linear variation of $1/C^2$ on voltage is retained almost to breakdown. If the measurements are made at lower frequencies, the slope is preserved to lower temperatures. This proves that the quantity N_d in (7) can be taken to be independent of temperature. For our germanium specimens $N_d = 8 \times 10^{13}$.

It is apparent that in a specimen of homogeneous n-type germanium, the concentration of surplus ionized impurities decreases sharply on going to low temperatures. However, as follows from the results quoted, in the immediate neighborhood of the boundary between the p-type and n-type parts of the semiconductor, the donor and acceptor impurities remain almost completely ionized, as they are at room temperature. This is also confirmed by the fact that at helium temperatures p-n junctions sustain large reverse voltages while germanium itself breaks down even in field strengths of several v/cm, owing to impurity ionization. Since the impurities in the region of the p-n junction are already ionized, breakdown can only occur when the germanium atoms are ionized, for which field strengths of the order of hundreds of thousands of v/cm are required.

The values of U_0 obtained by extrapolating the straight lines in Figs. 8 and 9 do not, as is well known,[5] agree with U_K—the contact or diffusion potential difference—but at low temperatures the difference between them is insignificant. As follows from (1), at low temperatures U_K is almost the difference between the energy levels of acceptor and donor impurities, and depends little on temperature. This is confirmed by the results given in Fig. 10.

FIG. 10. Dependence of $f(T) = U_0 + (3kT/q) \ln T$ for a germanium diode (specimen no. 1).



It is impossible to measure directly the variation of p-n junction capacity on voltage at helium temperatures. Therefore, by this means [using formula (7)] it is impossible to determine the value of the additional potential U_0. But it can be evaluated by measuring the forward branch of the voltage-current characteristic. The forward current through a p-n junction at low temperatures is

$$J \approx \exp\left\{-\frac{1}{kT}(G - Uq - W_i)\right\},$$

where G is the width of the forbidden gap. As is seen, a sharp increase of current occurs at the voltage $U \approx G/q$ (for germanium $U \approx 0.8$ v), when the external voltage exceeds the contact potential difference at the p-n junction. For our specimen at T = 4.2° K, the current increased from 10^{-8} to 10^{-4} amp on changing the voltage from 0.82 to 0.89 v. Under these conditions the base resistance plays an insignificant role, since for its breakdown 0.1 — 0.2 v is required in all.[6]

The difference between U_0 and U_K is known to be explained by the inversion layer in the space-charge region.[7] Since the potential drop in the inversion layer has no influence on the field on the p-n junction, it can be considered that the additional voltage is:

$$U_0 = U_K - U_i. \tag{8}$$

To evaluate U_i, the inversion layer can be taken to be of finite thickness in which the concentration of holes $p_n = \beta N_d$, where $\beta > 1$.[8] Then

$$U_i = (kT/q) \ln (N_a' / \beta N_d). \tag{9}$$

The contact potential difference is

$$U_K = (kT/q) \ln (N_a' N_d / n_i^2), \tag{10}$$

where

$$n_i^2 = AT^3 e^{-G/kT} \tag{11}$$

For germanium $A = 3.1 \times 10^{32}$ cm^{-6}/(deg K)^3, G = 0.785 ev; for silicon $A = 1.5 \times 10^{33}$ cm^{-6}/(deg K)^3, G = 1.21 ev.[9] Using (8) and (11) we obtain

$$U_0 + \frac{3kT}{q} \ln T = \frac{G}{q} - \frac{kT}{q} \ln \frac{A}{\beta N_d^2}. \tag{12}$$

In Fig. 10 the quantity $U_0 + (3kT/q) \ln T$ is plotted as a function of temperature; the experimental values U_0 were determined by extrapolating the straight lines $1/C^2 = \varphi(U)$ in Fig. 8. The agreement of the data given in Fig. 10 with (12) is good for $\beta = 4$.

A consideration of data quoted in the literature[5] for p-n junctions fabricated from germanium with surplus donor impurity concentrations from 10^{14} to 10^{17} gives values of β between 4 and 5. The capacity and potential distribution in a p-n junction at low temperatures has been calculated by Vul.[10]

In conclusion, the authors consider it a pleasant duty to express their deep gratitude to Academician P. L. Kapitza for permission to carry out the investigation in the Institute of Physical Problems, and to Prof. V. P. Peshkov for constant interest in the work.

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