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CERTAIN SPECIAL FEATURES OF OHMIC HEATING OF ELECTRON GAS IN A PLASMA

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An investigation is made of the heating of the electron gas in a plasma in a constant electric field, taking into account inelastic electron collisions. It is shown that the electron temperature may be in a steady state only for small values of the intensity of the electric field $E < E_k$; in the case $E \geq E_k$ this condition is no longer stable. An investigation is made of the dependence of the field E_k on the degree of ionization of the plasma. A comparison is made with the results of experimental papers.[9,10] An explanation is given of the two modes of heating observed in those papers; good quantitative agreement with their results is obtained.

PREVIOUSLY[1] some peculiarities were pointed out in the behavior of the electron temperature in a strongly ionized plasma, arising as a result of the fact that the frequency of electron-ion collision falls off sharply as the electron velocity increases. It turned out that in a constant electric field the electron temperature may be in a stationary state only for low values of the intensity of the electric field $E < E_k^{el}$. For $E \geq E_k^{el}$ no stationary state exists anymore; in this case the electron temperature increases continuously with time.* The whole investigation is carried out in reference 1 on the assumption that only elastic electron collisions occur. However, it is well known that inelastic collisions usually also play a significant, and even a dominant, role. The object of the present article is to take them into account.

We consider an infinite plasma in an atomic gas situated in a spatially homogeneous constant electric field. When the following conditions are satisfied

$$dT_e/dt \ll \nu_e T_e, \quad Q(T_e) \ll \tfrac{3}{2} kT_e \nu_{\text{eff}}, \quad Q(T_e) \ll \tfrac{3}{2} kT_e \nu_e \tag{1}$$

the main part of the electron distribution function which depends only on the absolute value of the velocity is Maxwellian;† in this case the electron temperature T_e is determined by the following equation

$$\frac{dT_e}{dt} + \frac{2}{3k} Q(T_e) = \frac{2}{3k} \frac{e^2 E^2}{m} \frac{K_\sigma}{\nu_{\text{eff}}(T_e)}. \tag{2}$$

Here, as usual, k is the Boltzmann constant, e and m are the charge and the mass of the electron, E is the intensity of the electric field, ν_e is the electron-electron collision frequency. $\nu_{\text{eff}} = \nu_{\text{eff}\,i} + \nu_{\text{eff}\,n}$, where $\nu_{\text{eff}\,i}$ is the effective electron-ion collision frequency and $\nu_{\text{eff}\,n}$ is the electron-neutral particle collision frequency; the expressions for $\nu_{\text{eff}}(T_e)$ are given, for example, in the book by Al'pert, Ginzburg, and Feĭnberg,[4] and also in reference 5. K_σ is a numerical coefficient whose value depends on the ratio between $\nu_{\text{eff}\,i}$ and $\nu_{\text{eff}\,n}$; it varies from 1.95 (for $\nu_{\text{eff}\,n} \ll \nu_{\text{eff}\,i}$) to 1.05 – 1.13 (for $\nu_{\text{eff}\,n} \gg \nu_{\text{eff}\,i}$).[1,5] Finally, $Q(T_e)$ is the energy lost by the electrons per unit time in collisions with heavy particles:

$$Q(T_e) = \frac{3}{2} k(T_e - T)\,\delta_{el}\,\nu_{\text{eff}} + \frac{16}{3}\left(\frac{2\pi kT_e}{3m}\right)^{1/2} \frac{e^6}{m\hbar c^3} \times \left(\sum_p Z_p N_{+p}\right) + \sum_l \nu_l \varepsilon_l + \nu_i\left(\varepsilon_i + \frac{3}{2} kT_e\right). \tag{3}$$

The first term in this expression describes the energy losses by the electron in elastic collisions with heavy particles ($\delta_{el} = 2m/M$), the second term describes bremsstrahlung losses, the third term describes excitation losses, the fourth term describes ionization losses (here ϵ_l is the energy of the l-th level, ν_l is the frequency of its excitation; similarly ϵ_i and ν_i are the ionization energy and frequency).*

*In an alternating electric field this effect depends significantly on its frequency ω; thus, for $\omega \gg \omega_k \approx 0.2\nu_{\text{eff}}$ no instability of the electronic apparatus occurs at all. A magnetic field directed at right angles to the electric field also has a similar effect; for example, no instability arises[1] if $eH/mc = \omega_H \gg \omega_k$.

†If the first of conditions (1) is not satisfied, i.e., if the losses are small and the field $E \gg \sqrt{kT_e m \nu_{\text{eff}} \nu_e}/e$ [cf. (2)], the velocity distribution of the electrons becomes sharply directed (this case in a fully ionized plasma was investigated by Dreicer[2]). If the second of conditions (1) is not satisfied this also leads to the appearance of a pronounced directed part in the distribution function. When the third condition is not satisfied, the distribution function remains symmetric, but its shape may be significantly altered (cf., for example, reference 3).

*Equation (2) was utilized, in particular, in references 6 and 7 for numerical calculations of plasma heating in the stellarator.

The stationary electron temperature is determined, naturally, by the relation

$$Q(T_e) = e^2E^2K_0/m\nu_{\text{eff}}(T_e). \quad (4)$$

Moreover, it is necessary that the energy given to the electrons by the field should increase with increasing T_e more slowly than the energy loss by collisions. However, it may be easily seen that these conditions are by no means always satisfied. Indeed, the energy given to the electrons by the field increases, as is well known, in the case of a sufficiently high degree of plasma ionization, proportionally to $T_e^{3/2}$ (since $\nu_{\text{eff}\,i} \sim T_e^{-3/2}$). Only the energy lost by the electron through excitation and ionization increases equally rapidly. However, at temperatures of the order of ϵ_i/k the rate of growth of $\nu_i(T_e)$ and $\nu_l(T_e)$ is reduced, while at higher temperatures even a decrease of $\nu_i(T_e)$ and $\nu_l(T_e)$ begins (cf. reference 8). The role played by the remaining terms, which describe the energy lost by the electrons in elastic collisions and by bremsstrahlung, is not very significant under these conditions. Consequently, the electron temperature may be stationary only for $T_e \lesssim \epsilon_i/k$ and at low values of the intensity of the electric field.

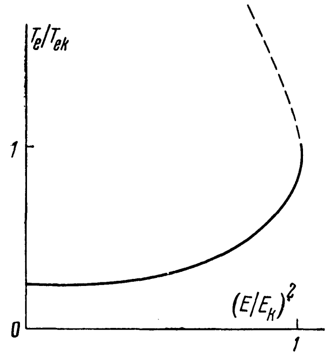

FIG. 1. The dependence of T_e/T_{ek} on E/E_k in a strongly ionized helium plasma.

We now calculate the stationary electron temperature in a helium plasma.* We assume initially that the plasma is strongly ionized ($\nu_{\text{eff}\,i} \gg \nu_{\text{eff}\,n}$), and we take into account in $Q(T_e)$ only the main part of the electron energy losses — the losses due to excitation and ionization of helium atoms, i.e., we neglect initially the effect of losses due to bremsstrahlung, due to excitation and to further ionization of singly ionized helium, and we neglect losses in elastic collisions (they all lead only to a small correction, cf. below). As may be easily seen, in this case the stationary temperature T_e depends on only one parameter. It is shown in Fig. 1; along the vertical axis we have plotted the ratio T_e/T_{ek}, where $T_{ek} \approx 1.03\,\epsilon_i/k$ is the maximum stationary electron temperature; along the horizontal axis we have plotted the ratio $(E/E_k)^2$, where

$$E_k \approx 2.8\left[\frac{e^2N_i\nu_i(\varepsilon_i/k)}{\sqrt{\varepsilon_i/m}}\ln\frac{\varepsilon_i D}{e^2}\right]^{1/2} \approx 7.0\cdot10^{-15}N_{n0}\sqrt{q_i(1-q_i)}\left(1+\frac{1}{45}\ln\frac{kT_i}{2}\right) \quad (5)$$

is the critical field (in v/cm). Here ϵ_i is the ionization energy, D is the Debye radius, $\nu_i(\epsilon_i/k)$ is the frequency of ionization at $T_e = \epsilon_i/k$, $q_i = N_i/N_{n0}$ is the degree of plasma ionization, T_i is the ion temperature (in electron volts), $N_{n0} = 3.52 \times 10^{16}\,p_0$ is the total gas density (p_0 is the initial pressure).*

It is seen from the diagram that the electron temperature can be stationary (solid curve) only in fields lower than the critical field; however, if $E \geq E_k$, then there exists no stationary state. In this case the electrons have no time to lose the energy communicated to them by the field, and their temperature increases continuously with time.†

The value of the critical field (5) depends significantly on the degree of plasma ionization q_i. Moreover, if $q_i < 0.5$, then E_k increases with increasing q_i, while if $q_i > 0.5$, then E_k decreases. The critical field reaches the maximum value

$$E_{k\,max} \approx 3.5\cdot10^{-15}N_{n0}\ \text{v/cm}$$

when $q_i = 0.5$. Naturally, in the case when the electric field intensity is greater than $E_{k\,max}$, a stationary condition cannot occur irrespectively of the degree of plasma ionization (since $E > E_{k\,max} \geq E_k$). However, if E is less than $E_{k\,max}$, a stationary state may be realized, if q_i takes on some value in the interval

$$0.5\left(1-\sqrt{1-(E/E_{k\,max})^2}\right) < q_i < 0.5 \times \left(1+\sqrt{1-(E/E_{k\,max})^2}\right). \quad (6)$$

*For the calculation of $\nu_i(T_e)$, $\nu_l(T_e)$ use was made of effective excitation and ionization cross sections in helium given in reference 8. It should be noted that the corresponding experimental data are not sufficiently complete; moreover, the measured cross sections differ noticeably from values calculated theoretically. As a result of this, the accuracy in the calculation of electron temperature apparently does not exceed 20%.

*The same expression (5) for the critical field E_k will also be obtained in the case of any other atomic gas: only the constant in front of the square root will be altered (it depends on the ratio between the excitation and ionization losses in any given gas).

†When sufficiently high values of T_e are attained the first of conditions (1) is violated, and the electron velocity distribution becomes sharply directed (cf. footnote † on p. 85). We also note that the electron temperature may likewise increase with time in the case of fields $E < E_k$; for this it is necessary that at the time the field is switched on T_e should exceed T_{ek}, with the point T_e/T_{ek} lying above the dotted curve in Fig. 1.

In this case, as the degree of plasma ionization q_i increases, the electron temperature in the stationary state diminishes for $q_i < 0.5$ [since for $q_i < 0.5$ the ratio E/E_k diminishes as q_i increases, and consequently $T_e(E/E_k)$ also diminishes (cf., Fig. 1)], while it increases when $q_i > 0.5$. The minimum value of T_e in the stationary state is given by $T_{e\,min} = T_e(E/E_{k\,max})$. From formula (6) it may also be seen that in the case of fields $E < E_{k\,max}$, the electron temperature likewise becomes nonstationary both in the case of high and low degrees of plasma ionization.

In the case of low degrees of plasma ionization, however, we cannot neglect collisions with neutral particles as was done earlier. Therefore, a suitable calculation was carried out taking into account $\nu_{eff\,n}$, and we also took into account terms describing the energy lost by the electrons in elastic collisions, by bremsstrahlung, and by excitation and ionization of singly ionized helium, which were all neglected previously. The dependence of the electron temperature on the degree of plasma ionization for different values of $(E/E_{k\,max})^2$ obtained as a result of this calculation is given in Fig. 2. It may be seen from the diagram that the instability of electron temperature arises only for high degrees of ionization; in the case of small q_i no such instability occurs. Consequently, it is very important to take into account collisions between electrons and neutral particles at low degrees of ionization, as should be the case.

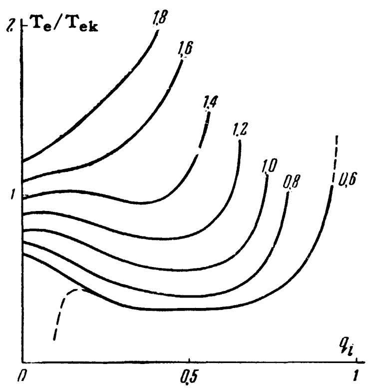

FIG. 2. The dependence of T_e/T_{ek} on q_i. The numbers labelling the curves correspond to different values of $(E/E_{k\,max})^2$.

We now investigate qualitatively the process of plasma heating in a constant electric field E which is smaller than $E_{k\,max}$ or is of the same order of magnitude. In doing this we assume that at the instant of switching on the heating field the electron temperature is lower than T_{ek}, and the degree of plasma ionization is not great. Then in the initial period of heating the electron temperature will increase up to the corresponding stationary value [as is shown by the dotted line in Fig. 2 for the case $(E/E_{k\,max})^2 = 0.6$]. We now take into account the fact that the degree of plasma ionization also increases with time. This means that the electron temperature will subsequently vary in accordance with the stationary state curve, but only up to the point where q_i reaches its critical value, at which point instability occurs. After this the electron temperature again begins to grow sharply (dotted line in Fig. 2). Consequently, in the case of plasma heating in the field $E \lesssim E_{k\,max}$ a "plateau" having a characteristic maximum and minimum may appear in the curve which shows the variation with time of the electron temperature (or of the electron current); this "plateau" corresponds to the region of the stationary state. As may be clearly seen in Fig. 2, the extent of the "plateau" decreases as the intensity of the heating field increases. When $(E/E_{k\,max})^2 > 1.5$ it disappears completely; in this case the electron temperature grows continuously with time.

Thus, when plasma is heated in a constant electric field two modes of heating may occur, depending on the value of the field: with a "plateau" and without a "plateau." The field E_s, which separates these two modes is equal to (in v/cm)

$$E_s \approx \sqrt{1.5}\, E_{k\,max} \approx 4.3 \cdot 10^{-15} N_{n0}. \qquad (7)$$

The ohmic heating of a helium plasma has been investigated experimentally in the case of the stellarator.[9,10] In the experimental paper[9] characteristic curves are given which show the variation with time of the current in the stellarator for various values of the heating field. We note first the good qualitative agreement with the special features of plasma heating described earlier. From the curves shown in the figure given above we may easily determine the field which separates the two modes of heating:

$$E_s \approx (0.083 \pm 0.006) \text{ v/cm}. \qquad (8)$$

Formula (7) gives for this case ($N_{n0} = 1.76 \times 10^{13}$) $E_s = 0.076$ v/cm; it is seen that the agreement of the theoretical value with the experimental one (8) is sufficiently good.

Further, in reference 9 the dependence of the maximum current at the current "plateau" (I_{max}) on the electric field intensity has also been measured; these results are shown in Fig. 3, where along the horizontal axis we have plotted the ratio of E/E_s, while along the vertical axis we have plotted the ratio $I_{max}(E)/I_m$ (for $E = E_s$ the maximum and the minimum values of the current, naturally, coincide: $I_{max}(E_s) = I_{min}(E_s) = I_m$). The solid and the dotted curves in the same diagram indicate the possible limits of variation of the current at the current plateau obtained as a

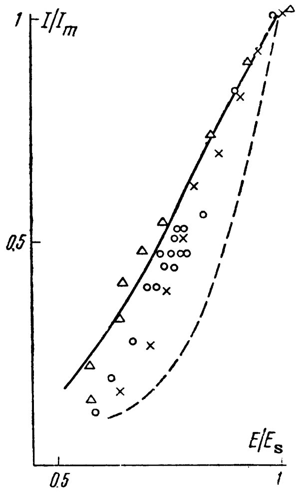

FIG. 3. The dependence of I_{max}/I_m (solid curve) and I_{min}/I_m (dotted curve) on E/E_s; o, Δ, × are the experimental results[9] of measuring $I_{max}(E)/I_m$.

result of a theoretical calculation according to the following formula

$$I(E)/I(E_s) = (E/E_p)\nu_{eff}[T_e(E_p)]/\nu_{eff}[T_e(E)];$$

it may be seen from the diagram that they are in good agreement with experiment.*

One might also note a number of qualitative remarks made by the authors of the experimental papers[9,10] which are also in agreement with the results of the present investigation. In particular, the fact that there exists a current "plateau" for a long time (up to 3 millisec) shows that during all this period there exists a considerable number of neutral particles in the plasma (cf. Fig. 2), which is in accord with experimental observations on the luminosity of neutral helium.

*The slope of the curve $I_{max}(E)/I_m$, shown in Fig. 3, may vary somewhat depending on the initial degree of plasma ionization, and also on the initial electron temperature, etc. The minimum value of the current depends on these parameters to a smaller extent.

Consequently, on adding neutral gas to the discharge it is possible to maintain the current and the electron temperature at constant values for long periods of time. By lowering the electric field in a heated and singly ionized gas it is possible, in principle, to achieve similar stationary conditions in the domain of second (or higher) ionization, i.e., at a higher stationary electron temperature.

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