POLARIZATION OF GAMMA-RAY QUANTA FROM THE INTERNAL COMPTON EFFECT

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Submitted to JETP editor July 4, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 180-183 (January, 1960)

The circular polarization of the γ-ray quanta from the internal Compton effect is considered. The forms of the angular correlation between the direction of emergence of the β-ray electron and that of a circularly polarized γ-ray quantum are given for transitions of both the magnetic and the electric types.

BECAUSE of parity nonconservation in the weak interactions the γ-ray quanta emitted by a nucleus after β decay are circularly polarized, and there is a correlation between the β-ray electron and the γ-ray quantum.[1] If after β decay there is internal conversion of the γ-ray quanta, the conversion electrons that are emitted will be longitudinally polarized.[2,3] A similar situation will exist also in the case of the so-called internal Compton effect occurring after β decay of a nucleus. Since the internal Compton effect is bremsstrahlung from the conversion electrons, it is clear that in the case of longitudinally polarized electrons the γ-ray quanta will be circularly polarized. It is obvious that there will also be an angular correlation of the β-ray electron and the circularly polarized γ-ray quantum.

Studies of this correlation and of the degree of circular polarization can be useful in some cases for the determination of the values of the nuclear matrix elements or even those of the β-decay constants.

In the present note we consider the circular polarization of γ-ray quanta emitted as the result of internal Compton effect occurring after an allowed β decay, and find the form of the angular correlation between the emission of the β-ray electron and the circularly polarized γ-ray quantum. The calculations have been made in the Born approximation, like the other calculations devoted to this question.[4,5] It is natural to expect that strictly speaking these calculations can apply only for light nuclei, although the results of work by Lindqvist and others[6] on the experimental observation of the internal Compton effect in Ba^{137} are in fair agreement with the theoretical results of Spruch and Goertzel,[4] which were also obtained with the Born approximation.

Suppose that before the β decay the nucleus was in a state with angular momentum j_1, that after the allowed β decay (we consider the V-A interaction) it was in a state with angular momentum j_2, and that after the γ-ray transition it was in a state characterized by the angular momentum j_3. Let the multipole character of the (virtual) γ-ray quantum be l, and let $\Delta \equiv E_{j_1} - E_{j_2}$ be the energy of the γ-ray transition of the nucleus. Then the angular correlation of the β-ray electron with the circularly polarized γ-ray quantum has the usual form

$$w(\theta_\beta) = 1 + \mu \frac{v}{c} \delta \frac{P}{N} \cos\theta_\beta, \tag{1}$$

where θ_β is the angle between the directions of emission of the β-ray electron and the circularly polarized γ-ray quantum, μ is the sign of the circular polarization ($\mu = \pm 1$), and v is the speed of the β-ray electron. For allowed β transitions we have

$$\delta = \frac{2B\sqrt{j_2(j_2+1)}\,\delta_{j_2 j_1} + D[2 + j_2(j_2+1) - j_1(j_1+1)]}{2\sqrt{j_2(j_2+1)}\,(A\delta_{j_2 j_1} + C)} \times \frac{l(l+1) + j_2(j_2+1) - j_3(j_3+1)}{2\sqrt{j_2(j_2+1)\,l(l+1)}}, \tag{2}$$

where

$$A = |\langle 1\rangle|^2 (|C_V|^2 + |C'_V|^2),$$
$$B = 2\mathrm{Re}\,\langle 1\rangle \langle\sigma\rangle^* (C_V C'^*_A + C'_V C^*_A),$$
$$C = |\langle\sigma\rangle|^2 (|C_A|^2 + |C'_A|^2), \quad D = |\langle\sigma\rangle|^2\, 2\mathrm{Re}\, C_A C'^*_A. \tag{3}$$

Here as usual we have denoted by $\langle 1\rangle$ and $\langle\sigma\rangle$ the matrix elements of the Fermi and Gamow-Teller types. The quantities N and P depend on the multipole character and type of the transition. For transitions of magnetic type with multipole index l we have

$$N^{(M)} = \frac{1}{p'^2}\{k^2 p' + p^2 \Delta \sin^2\theta_{kp}\} + \frac{k^2}{m^2}\left\{p' - \frac{p^2 k}{q^2}\sin^2\theta_{kp}\right\} - \frac{kp^2}{mp'}\frac{\sin^2\theta_{kp}}{q^2}\{kp\cos\theta_{kp} + p^2 + mk\},$$

$$P^{(M)} = \frac{1}{p'^2}\{k^2 p' c_1 + p^2 \Delta c_2\} + \frac{k^2}{m^2}\{p' c_1 - pc_3\} - \frac{m - \varepsilon_p}{mp'} pkc_3, \tag{4}$$

where the quantities appearing in $P^{(M)}$ are

$$c_1 = (-1)^l 2^{1/2} (2l+1)^{3/2} \cdot 4\pi \sum_K \frac{1}{(2K+1)^{1/2}} C^{K0}_{l0l0} X$$
$$\times (1ll \mid 1ll \mid 11K)\, h_{1K}(\theta_{kp}),$$
$$c_2 = (-1)^l 5^{-1/2} (2l+1)^{3/2} \cdot 4\pi C^{20}_{l0l0} X$$
$$\times (1ll \mid 1ll \mid 112)\, P_{1,1}(\theta_{kp})\, P_{2,-1}(\theta_{pq}),$$
$$c_3 = (-1)^{1+l} 2^{-1/2} 3^{1/2} (2l+1)^{3/2} \cdot 4\pi \sum_{K,F} \frac{4 - F(F+1)}{(2F+1)^{1/2}(2K+1)^{1/2}}$$
$$\times C^{K0}_{l0l0} C^{F0}_{1010}\, X(1ll \mid 1ll \mid 11K)\, h_{FK}(\theta_{kp}). \quad (5)$$

Somewhat more cumbersome formulas are obtained for transitions of the electric type. In this case

$$N^{(e)} = A_1 + A_2 + A_{12}, \qquad P^{(e)} = B_1 + B_2 + B_{12}, \quad (6)$$

with

$$A_1 = \frac{1}{k^2 p'^2}\Big\{(k^2 p' + p^2 \Delta \sin^2\theta_{kp})\Big[(2l+1)\frac{\Delta^2}{q^2} - l\Big]$$
$$+ 2lkp'\Big(m - \frac{p^2\Delta}{q^2}\sin^2\theta_{kp} + a_1\Big)\Big\},$$
$$A_2 = \frac{1}{m^2 k}\Big\{kp'\Big[(2l+1)\frac{\Delta^2}{q^2} - l\Big] - 2l(ma_1 + a_2)\Big\},$$
$$A_{12} = \frac{1}{mk^2 p'}\Big\{(2k^2 p' + kp^2 \sin^2\theta_{kp})\Big[(2l+1)\frac{\Delta^2}{q^2} - l\Big] \quad (7)$$
$$- 2k^2 p' a_3 + 2l\,[(\varepsilon_p - m)a_4 - \Delta a_2 - (m\Delta - kp')a_1$$
$$+ m(m(m - \varepsilon_p) + kp')]\Big\},$$

where

$$a_1 = -\varepsilon_p + \frac{p\Delta}{q}\cos\theta_{pq},$$
$$a_2 = (m-k)\varepsilon_p - \frac{\Delta}{q}[(m-k)p\cos\theta_{pq} - \varepsilon_p k\cos\theta_{kq}]$$
$$- \frac{1}{2l}\frac{\Delta^2}{q^2} kp[(l+1)\cos\theta_{kp} + (l-1)\cos\theta_{pq}\cos\theta_{kq}],$$
$$a_3 = \frac{2}{3}\frac{\Delta^2}{q^2}\Big[(2l+1) - \frac{(l-1)}{4}(3\cos^2\theta_{kp} - 1)\Big],$$
$$a_4 = \varepsilon_q(m-k) + \frac{\Delta}{q}(\varepsilon_q k\cos\theta_{kp} - \varepsilon_q q) - \frac{\Delta^2 k}{2lq}[(l+1)\cos\theta_{kp}$$
$$+ (l-1)\cos\theta_{kp}]; \quad (8)$$
$$B_1 = \frac{1}{k^2 p'^2}\Big\{k^2 p' b_2 + 2(\varepsilon_p - m)b_3$$
$$+ l^{1/2}(l+1)^{1/2}\frac{pk\Delta}{q^2}\sin^2\theta_{kp}(m\Delta + m(\varepsilon_p - m))\Big\},$$
$$B_2 = \frac{1}{m^2 k}\Big\{kp' b_2 + 2b_1 + l^{1/2}(l+1)^{1/2}\frac{pk^2\Delta}{q^2}\sin^2\theta_{kp}\Big\},$$
$$B_{12} = \frac{2}{mk^2 p'}\Big\{(m - \varepsilon_p)b_1 + l^{1/2}(l+1)^{1/2}\frac{pk\Delta}{2q^2}\sin^2\theta_{kp}$$
$$\times((m-k)(\varepsilon_p - m) + m\Delta - kp')\Big\}. \quad (9)$$

Here b_i denote the quantities

$$b_1 = (-1)^{1+l} 3^{1/2} 2^{-3/2} (2l-1)(2l+1)^{3/2} 4\pi \frac{pk\Delta^2}{q^2}$$
$$\times \sum_{K,F} \frac{4 - F(F+1)}{(2F+1)^{1/2}(2K+1)^{1/2}} C^{F0}_{10\,10} C^{K0}_{l-10\,l-10} X$$
$$\times (1ll-1 \mid 1ll-1 \mid 11K)\, h_{FK}(\theta_{kp}),$$
$$b_2 = (-1)^l 2^{1/2} (2l-1)(2l+1)^{3/2} 4\pi \frac{\Delta^2}{q^2} \sum_F (2F+1)^{-1/2}$$
$$\times C^{F0}_{l-10\,l-10} X(1ll-1 \mid 1ll-1 \mid 11F)\, h_{1F}(\theta_{kp}), \quad (10)$$
$$b_3 = (-1)^l 3^{-1/2} 5^{-1/2} (2l-1)(2l+1)^{3/2} 4\pi \frac{p^2\Delta^2}{q^2}$$
$$\times C^{20}_{l-10\,l-10} X(1ll-1 \mid 1ll-1 \mid 112)\, P_{1,1}(\theta_{kp})\, P_{2,-1}(\theta_{pq}).$$

The quantities that appear in Eqs. (5) and (10) are given by

$$h_{1K}(\theta_{kp}) = C^{K0}_{10\,10} P_{1,0}(\theta_{kp}) P_{K,0}(\theta_{pq})$$
$$- 2C^{K1}_{11\,10} P_{1,1}(\theta_{kp}) P_{K,-1}(\theta_{pq}),$$
$$h_{FK}(\theta_{kp}) = C^{F0}_{10\,10} C^{K0}_{1010} P_{F,0}(\theta_{kp}) P_{K,0}(\theta_{pq})$$
$$- 2C^{F1}_{11\,10} C^{K1}_{11\,10} P_{F,-1}(\theta_{kp}) P_{K,+1}(\theta_{pq}). \quad (11)$$

In the expressions (4) — (11) the following notations are used: ε_p, $\mathbf{p}$ are the energy and momentum of the electron ejected from the K shell; m is the mass of the electron; k, $\mathbf{k}$ are the energy and momentum of the real γ-ray quantum;

$$\mathbf{q} = \mathbf{p} + \mathbf{k}, \qquad \varepsilon_q = \varepsilon_p + k, \qquad p' = \varepsilon_p - p\cos\theta_{kp};$$

θ_{kp} denotes the angle between the directions of emergence of the conversion electron and the γ-ray quantum; θ_{pq} and θ_{kp} denote respectively the angles between the vectors $\mathbf{p}$ and $\mathbf{q}$ and between $\mathbf{k}$ and $\mathbf{q}$. (It is easy to see that in the final analysis all of these angles can be expressed in terms of the single angle θ_{kp}). The $P_{l,m}$ denote associated Legendre polynomials with the normalization given by Edmonds,[7] and $C^{a\alpha}_{b\beta c\gamma}$ and X(abc | efg | hik) are Clebsch-Gordan and Fano coefficients; their properties and numerical values can be found, for example, in reference 7.

To get an intuitive idea of the value of the degree of circular polarization as a function of the two variables θ_{kp} and k, we have tabulated this function for the γ-ray transition in Co^{60} (transition of type E = 2, Δ = 1.170 Mev). The results of calculations of the dependence of P/N on the energy k of the γ-ray quantum (in units mc^2) and on the angle θ_{kp} are shown in Figs. 1 and 2.

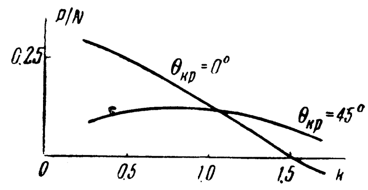

FIG. 1

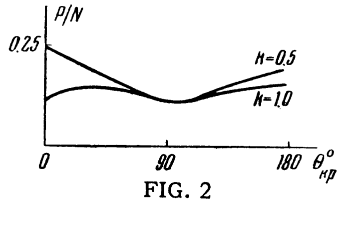

FIG. 2

We note that the degree of circular polarization of the γ-ray quanta for transitions with the same multipole index and transition energy is larger for the magnetic than for the electric transition. This result can be understood easily if we take into account the fact that in the case of the electric transition there is both longitudinal and also transverse polarization of the conversion electrons, and for small energies of the conversion electron the transverse polarization can even exceed the longitudinal.[2]

In conclusion the writer regards it as his pleasant duty to thank Professor I. S. Shapiro for suggesting the topic and for his interest in the work. The writer is also grateful to E. Naletova for assistance in the numerical calculations.

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Translated by W. H. Furry
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