GREEN'S FUNCTION IN THE FIXED-SOURCE MODEL OF CHARGED SCALAR MESONS

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The calculation of the Green's function for a static nucleon, interacting with charged scalar mesons is given as an example of a new method of solution which is different from the perturbation method.

We consider a system with a Hamiltonian of the form

$$H = m(\psi^+\psi) + \frac{1}{2}\sum_{i=1}^{2}\int dx : [\pi_i^2(x) + (\nabla\varphi_i(x))^2 + \mu^2\varphi_i^2(x)] : + g\sum_{i=1}^{2}\int dx\,(\psi^+\tau_i\psi)\,\varphi_i(x)\,\rho(x). \quad (1)$$

Here ψ^+ and ψ are nucleon field operators, $\pi_i(\mathbf{x})$ and $\varphi_i(\mathbf{x})$ are meson field operators, $\rho(\mathbf{x}) = \sum v(k)\,e^{i\mathbf{k}\cdot\mathbf{x}}$ is the nucleon form factor, and the τ_i are the isotopic spin-$\frac{1}{2}$ matrices.

On the basis of results from references 1 — 3, it can be shown that the nucleon Green's function in our case can be represented as a functional integral of the following form:

$$G(t-t_0) = \langle 0|T\{\psi(t)\psi^+(t_0)S\}|0\rangle / \langle 0|S|0\rangle = \frac{1}{C}\iint \delta\Lambda_1\delta\Lambda_2\widetilde{G}(t-t_0;\ \Lambda_1,\ \Lambda_2) \times\exp\left\{\frac{i}{2}\int_{t_0}^{t}\int_{t_0}^{t} ds_1 ds_2\Delta^{-1}(s_1-s_2)\,\Lambda_j(s_1)\,\Lambda_j(s_2)\right\}. \quad (2)$$

Here $\Delta^{-1}(s_1 - s_2)$ is determined by the relation

$$\int_{t_0}^{t}\Delta^{-1}(s_1-s_2)\,\Delta(s_2-s_3)\,ds_2 = \delta(s_1-s_3),$$

where

$$i\delta_{kl}\Delta(s_1-s_2) = \left\langle 0\left|T\left\{\hat{\varphi}_k(s_1)\hat{\varphi}_l(s_2)\right\}\right|0\right\rangle = \delta_{kl}\sum\frac{v^2(k)}{2\omega_k}\exp\left\{-i\omega_k|s_1-s_2|\right\},$$

$$\hat{\varphi}_l(s) = \sum\frac{v(k)}{\sqrt{2\omega_k}}(a_{lk}e^{-i\omega_k s} + a_{lk}e^{+i\omega_k s}),$$

C is the normalization constant, and $\Lambda_j(s)$ are real scalar functions. $\widetilde{G}(t-t_0;\ \Lambda_1,\ \Lambda_2)$ is a nucleon Green's function in an external classical field $\Lambda_j(s)$ and obeys the equation

$$[i\partial/\partial t - m - g(\tau_1\Lambda_1(t) + \tau_2\Lambda_2(t))]\,\widetilde{G}(t-t_0;\ \Lambda_1,\ \Lambda_2) = i\delta(t-t_0),$$

$$\widetilde{G}(t-t_0;\ \Lambda_1,\ \Lambda_2)|_{t<t_0} = 0. \quad (3)$$

Therefore the problem of finding the nucleon Green's function reduces to the solution of (3) and to the functional integration of the solution found with the weight function

$$\exp\left\{\frac{i}{2}\int_{t_0}^{t}\int_{t_0}^{t} ds_1 ds_2\Delta^{-1}(s_1-s_2)\,\Lambda_j(s_1)\,\Lambda_j(s_2)\right\}.$$

To solve (3) we write $\widetilde{G}$ in the form

$$\widetilde{G}(t-t_0;\ \Lambda_1,\ \Lambda_2) = \theta(t-t_0)\,e^{-im(t-t_0)}\,Y(t-t_0;\ \Lambda_1,\ \Lambda_2).$$

Then $Y(t-t_0;\ \Lambda_1,\ \Lambda_2)$ will obey the following equation:

$$i\frac{\partial}{\partial t}Y(t-t_0;\ \Lambda_1,\ \Lambda_2) = g(\tau_1\Lambda_1(t) + \tau_2\Lambda_2(t))\,Y(t-t_0;\ \Lambda_1,\ \Lambda_2),$$

$$Y(t-t_0;\ \Lambda_1,\ \Lambda_2)|_{t=t_0} = I. \quad (4)$$

Methods of solving matrix equations like (4) were developed by Lappo-Danilevskiĭ.[4] Making use of them, one can find the integral matrix of (4) as an entire function of the matrices I and τ_i.

$$Y(t-t_0;\ \Lambda_1,\ \Lambda_2) = \Phi_0(t-t_0;\ \Lambda_1,\ \Lambda_2)\,I + \sum_{i=1}^{3}\Phi_i(t-t_0;\ \Lambda_1,\ \Lambda_2)\,\tau_i. \quad (5)$$

If we introduce the notation

$$p^{(m)}(t-t_0;\Lambda_1,\Lambda_2) = (i\sqrt{2}g)^m\int_{t_0}^{t}d\xi_1\int_{t_0}^{\xi_1}d\xi_2 \ldots\int_{t_0}^{\xi_{m-1}}d\xi_m\Lambda_1(\xi_1)\,\Lambda_2(\xi_2)\,\Lambda_1(\xi_3) \ldots\exp\left\{ig\int_{t_0}^{t}\rho^{(m)}_{\xi_1\xi_2\ldots\xi_m}(s)\,[\Lambda_1(s)-\Lambda_2(s)]\,ds\right\},$$

where

$$\rho^{(m)}_{\xi_1\xi_2\ldots\xi_m}(s) = \begin{cases} 1 & \xi_{2k} < s < \xi_{2k+1}, \\ -1 & \xi_{2k+1} < s < \xi_{2k}, \end{cases}$$

the functions Φ_0 and Φ_i can be written in the form

$$\Phi_0(t-t_0;\ \Lambda_1,\Lambda_2) = \frac{1}{2}\sum_{n=0}^{\infty}\{p^{(2n)}(t-t_0;\ \Lambda_1,\Lambda_2)+p^{(2n)}(t-t_0;\ \Lambda_2,\Lambda_1)\};$$

$$\Phi_1(t-t_0;\ \Lambda_1,\Lambda_2)=\Phi_2(t-t_0;\ \Lambda_2,\Lambda_1) = \frac{1}{2}\sum_{n=0}^{\infty}\{p^{(2n)}(t-t_0;\ \Lambda_1,\Lambda_2)-p^{(2n)}(t-t_0;\ \Lambda_2,\Lambda_1) - \sqrt{2}\,p^{(2n+1)}(t-t_0;\ \Lambda_1,\Lambda_2)\},$$

$$\Phi_3(t-t_0;\ \Lambda_1,\Lambda_2)=\frac{1}{2}\sum_{n=0}^{\infty}\{-p^{(2n)}(t-t_0;\ \Lambda_1,\Lambda_2) + p^{(2n)}(t-t_0;\ \Lambda_2,\Lambda_1)-\sqrt{2}\,p^{(2n+1)}(t-t_0;\ \Lambda_1,\Lambda_2) + \sqrt{2}\,p^{(2n+1)}(t-t_0;\ \Lambda_2,\Lambda_1)\}.$$

Further on, it is shown that having solution (5) one can carry out the functional integration (2), since there arise integrals of the Gaussian type, which can be calculated by a method given by Feynman.[1] Omitting a long computation, we write down the final expression for the Green's function

$$G(t-t_0)=\theta(t-t_0)\,e^{-im(t-t_0)}\Big[\exp\Big\{-\frac{ig^2}{2}\int_{t_0}^{t}\int_{t_0}^{t}\Delta(s_1-s_2)\,ds_1ds_2\Big\} + (-ig^2)\int_{t_0}^{t}dt_1\int_{t_0}^{t_1}dt_2\,\Delta(t_1-t_2)\exp\Big\{-\frac{ig^2}{2}\int_{t_0}^{t}\int_{t_0}^{t}\rho^{(2)}_{t_1t_2}(s_1)$$
$$\times\ \Delta(s_1-s_2)\,\rho^{(2)}_{t_1t_2}(s_2)\,ds_1ds_2\Big\}+\ldots + (-ig^2)^n\int_{t_0}^{t}dt_1\int_{t_0}^{t_1}dt_2\ldots\int_{t_0}^{t_{2n-1}}dt_{2n}$$
$$\times\,[P\Delta(t_1-t_2)\,\Delta(t_3-t_4)\ldots\Delta(t_{2n-1}-t_{2n})]$$
$$\times\exp\Big\{-\frac{ig^2}{2}\int_{t_0}^{t}\int_{t_0}^{t}\rho^{(2n)}_{t_1,t_2,\ldots t_{2n}}(s_1)\,\Delta(s_1-s_2)\,\rho^{(2n)}{}_{t_1,t_2,\ldots t_{2n}}$$
$$\times(s_2)\,ds_1,ds_2\Big\}+\ldots\Big], \tag{6}$$

where P is a symmetrization operator on the variables t_1, t_2, $\ldots t_{2n}$, for example

$$P\Delta(t_1-t_2)\,\Delta(t_3-t_4)=\Delta(t_1-t_2)\,\Delta(t_3-t_4) + \Delta(t_1-t_3)\,\Delta(t_2-t_4)+\Delta(t_1-t_4)\,\Delta(t_2-t_3).$$

We note that the method used allows us to write down the n-th term in the series (6), in contrast to perturbation theory. The first term in the series (6) is the exact Green's function for a nucleon interacting with scalar neutral mesons.[3] On expanding in terms of g^2, our series goes over to the perturbation theory result. For series (6) there exists the bounding function

$$\exp\Big\{-\frac{ig^2}{2}\int_{t_0}^{t}\int_{t_0}^{t}\Delta(s_1-s_2)\,ds_1ds_2\Big\}\Big(1+\exp\Big\{\frac{g^2t^2}{2}\sum\frac{v^2(k)}{2\omega_k}\Big\} + \exp\Big\{\frac{g^2t^2}{4}\sum\frac{v^2(k)}{2\omega_k}\Big\}\Big).$$

In that way, with $v(k)$ for which the sum $\sum v^2(k)/2\omega_k$ is finite, the series (6) converges absolutely and uniformly for arbitrary finite values of t and g^2.

Questions of renormalizing the Green's functions obtained require further investigation.

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