ON THE THEORY OF ULTRASONIC ABSORPTION BY METALS IN A STRONG MAGNETIC FIELD

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The ultrasonic absorption coefficient α is calculated for metals in strong magnetic fields, where $r \ll \lambda \ll l$ (r and l are the characteristic orbital radius and the free path of the electrons, and λ is the length of the acoustic wave in the metal). Closed electron orbits are considered, with an arbitrary law of electron dispersion. It is shown that in a strong field the quantity α becomes saturated regardless of whether $n_1 = n_2$ or $n_1 \neq n_2$. For $\mathbf{k} \perp \mathbf{H}$ ($\mathbf{k}$ being the wave vector, $\mathbf{H}$ the magnetic field, and n_1 and n_2 the number of "electrons" and "holes" respectively) the saturation value is $kl \gg 1$ times larger than its value α_0 for $H = 0$; otherwise $\alpha \sim \alpha_0$. Comparison of theory with experiment[3] shows good agreement.

1. INTRODUCTION

EXPERIMENTAL study of ultrasonic absorption in metals located in a constant magnetic field at low temperatures[1-3] has led to the observation of fluctuations in the coefficient of sound absorption as a function of the magnetic field $\mathbf{H}$. Pippard's qualitative theory of the phenomenon[4] and the more detailed calculations of V. Gurevich[5] have shown that the magnetic fluctuations of the absorption coefficient are related to an unusual "spatial resonance" when an extremal diameter of the electron orbit in the direction perpendicular to the vectors $\mathbf{k}$ and $\mathbf{H}$ ($\mathbf{k}$ being the wave vector of the sound wave and $\mathbf{k} \perp \mathbf{H}$) becomes an integral multiple of the acoustic wavelength λ. From an experimental study of the anisotropy of the fluctuations it is possible to determine the extremal diameters of the Fermi surface for the electrons in the metal, and in certain simple cases to establish its shape completely.

The fluctuations in the absorption coefficient occur in a range of relatively weak magnetic fields, where $1 \lesssim kr \ll kl$. It is of some interest to discover which of the properties of the Fermi surface can be determined by a study of the absorption in a strong magnetic field. The present paper is devoted to this question.

2. STATEMENT OF THE PROBLEM. THE BASIC EQUATIONS

In this paper we limit ourselves basically to the consideration of closed Fermi surfaces only. More precisely, we shall determine the contribution made to the absorption coefficient by the closed electron trajectories. We shall assume that the vectors $\mathbf{k}$ and $\mathbf{H}$ are mutually perpendicular, and that the magnetic field is sufficiently strong that $kr \ll 1$ and $\gamma = r/l \ll 1$. At the same time we consider that the magnetic field is not so large that it is necessary to take into account the "skin effect," i.e., the non-uniformity of the alternating electromagnetic field. The latter condition reduces to the requirement that the "skin depth," i.e., the length δ of the corresponding electromagnetic wave in the metal, with the same frequency ω as the acoustic wave, should be the smallest parameter with the dimensions of a length in the problem.*

The absorption coefficient α is found from the quotient

$$\alpha = |Q|/\mathcal{E}, \tag{2.1}$$

where Q is the dissipative function and $\mathcal{E} = \frac{1}{4}\rho\omega^2 |\mathbf{u}_0^2|$ is the energy density in the sound wave. ρ is the density of the metal, and $\mathbf{u}(\mathbf{r}, t) = \mathbf{u}_0 \exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})$ is the displacement vector in the sound wave.

$$Q = -T\frac{d}{dt}\int \frac{2}{h^3} d\tau_p \{(1-f)\ln(1-f) + f\ln f\}, \tag{2.2}$$

where T is the electron temperature in energy units, the integral represents the entropy of the

*$\delta = c/(2\pi\omega\sigma)^{1/2}$, where, in calculating the electrical conductivity σ, the effect of the magnetic field and the spatial dispersion are taken into account.

electrons in the metal, $f(\mathbf{r}, \mathbf{p}, t)$ is the electron distribution function, $d\tau_p$ is the volume element in momentum space, and h is Planck's constant.

The function f satisfies the kinetic equation

$$\frac{\partial f}{\partial t}+\left(\frac{\partial \varepsilon}{\partial \mathbf{p}}+\dot{\mathbf{u}}\right)\nabla f-\left(e\mathbf{E}+\frac{e}{c}\left[\frac{\partial \varepsilon}{\partial \mathbf{p}}+\dot{\mathbf{u}}\right]\times\mathbf{H}+\nabla\varepsilon\right)\frac{\partial f}{\partial \mathbf{p}}+\left(\frac{\partial f}{\partial t}\right)_{st}=0, \quad (2.3)$$

where e, ϵ, and $\partial\epsilon/\partial\mathbf{p}$ are the charge, energy and velocity of the electron and c is the speed of light. Following the work of Akhiezer, Kaganov, and Lyubarskiĭ[6] we consider that the energy of an electron in the field of the acoustic wave is equal to

$$\varepsilon(\mathbf{r}, \mathbf{p}, t)=\varepsilon_0(\mathbf{p})+\lambda_{ik}\partial u_i/\partial x_k, \quad (2.4)$$

where $\lambda_{ik}(\mathbf{p})$ is some tensor of the second rank which will, generally speaking, not be symmetrical in the indices i and k.

The second term in (2.4) is connected with the assumption that the frequency ω is much smaller than the collision frequency ν, so that at every instant of time there exists a thermal equilibrium, in which the electrons acquire some additional energy proportional to the tensor $\partial u_i/\partial x_k$. ϵ_0 is the energy of the electron in the absence of the sound. A change in electron energy leads to a change in the chemical potential $\mu=\mu_0+\mu'$ and in the temperature. The latter, as has been shown in reference 6, can be neglected because $T \ll \mu_0$.

In order to determine the rotational part of the electric field it is necessary to use Maxwell's equations, which upon elimination of the alternating magnetic field take the form

$$\nabla^2 E_\beta=(4\pi i\omega/c^2)j_\beta, \quad (2.5)$$

where the index β refers to the y and z axes (the Ox axis is chosen along $\mathbf{k}$, and Oz along $\mathbf{H}$). In order to find the longitudinal components of the field it is necessary to make use of the vanishing of the space charge density, which by virtue of Maxwell's equations is equivalent to the equation

$$j_x=0. \quad (2.6)$$

In the case where the length δ of the electromagnetic wave in the metal is small compared with all the other dimensions, equation (2.5) reduces to $j_\beta=0$, which, together with (2.6), leads to the equation

$$\mathbf{j}=0, \quad (2.7)$$

from which the electric field vector is determined.

Let us now linearize the kinetic equation (2.3):

$$f=f_0(\varepsilon-\mu)-\chi\partial f_0/\partial\varepsilon,$$

$$f_0(\varepsilon-\mu)=[\exp\{(\varepsilon-\mu)/T\}+1]^{-1}, \quad (2.8)$$

where χ is a new unknown function. After some simple calculations we obtain

$$Q=-\mathrm{Re}\,\frac{1}{h^3}\int\frac{\partial f_0}{\partial\varepsilon}\chi^*\left(\frac{\partial\chi}{\partial t}\right)_{st}d\tau_p; \quad (2.9)$$

$$\frac{\partial\chi}{\partial t}+(\mathbf{v}\nabla)\chi+\Omega\frac{\partial\chi}{\partial\tau}+\left(\frac{\partial\chi}{\partial t}\right)_{st}=\lambda_{ik}\frac{\partial\dot{u}_i}{\partial x_k}-\mu'-e\mathbf{E}^*\mathbf{v}. \quad (2.10)$$

where τ is the dimensionless time of the orbital motion of the electrons in the magnetic field;[7] $\Omega=eH/mc$ is the frequency of revolution of an electron in its orbit; $m=(2\pi)^{-1}\partial S(\epsilon_0, p_z)/\partial\epsilon_0$ is the effective mass, $S(\epsilon_0, p_z)$ is the area of the intersection of the surface $\epsilon_0(\mathbf{p})=\mathrm{const}$ with the plane $p_z=\mathrm{const}$, and

$$\mathbf{v}=\partial\varepsilon_0/\partial\mathbf{p}; \quad e\mathbf{E}^*=e\mathbf{E}+\frac{e}{c}[\dot{\mathbf{u}}\times\mathbf{H}]+\nabla\mu'.$$

Averaging (2.9) over the angle in momentum space, and making use of the conservation law for the total number of particles, we obtain

$$\mu'=\langle\lambda_{ik}\rangle\frac{\partial u_i}{\partial x_k}-\left(\int f_0 d\tau_p\Big/\int\limits_{\varepsilon_0=\mu_0}\frac{dS}{v}\right)\mathrm{div}\,\mathbf{u},$$

$$\langle\lambda_{ik}\rangle=\int\limits_{\varepsilon_0=\mu_0}\frac{dS}{v}\lambda_{ik}\Big/\int\limits_{\varepsilon_0=\mu_0}\frac{dS}{v}. \quad (2.11)$$

The current density is

$$\mathbf{j}=\frac{2e}{h^3}\int d\tau_p\frac{\partial f_0}{\partial\varepsilon}\chi\mathbf{v}. \quad (2.12)$$

Noting that all the quantities in (2.10) depend on time and the coordinates in the manner $\exp(i\omega t-i\mathbf{k}\cdot\mathbf{r})$, and assuming that it is possible to introduce a relaxation time* $t_0(\mathbf{p})=1/\nu(\mathbf{p})$, we shall re-write equation (2.10) in the following final form:

$$(i\omega-i\mathbf{k}\mathbf{v}+\nu)\chi+\Omega\partial\chi/\partial\tau=\Delta\lambda_{ik}\partial\dot{u}_i/\partial x_k-e\mathbf{E}^*\mathbf{v}, \quad (2.13)$$

where

$$\Delta\lambda_{ik}=\lambda_{ik}-\langle\lambda_{ik}\rangle+\delta_{ik}\int f_0 d\tau_p\Big/\int\frac{ds}{v}\quad(\langle\chi\rangle=0).$$

The solution of equation (2.13) which is periodic in τ has the form

$$\chi(\tau)=\frac{1}{\Omega}\int\limits_{-\infty}^{\tau}d\tau_1\left[\Delta\lambda_{ik}(\tau_1)\frac{\partial\dot{u}_i}{\partial x_k}-e\mathbf{E}^*\mathbf{v}(\tau_1)\right]\exp\left(\int\limits_{\tau}^{\tau_1}\frac{\nu-i\mathbf{k}\mathbf{v}+i\omega}{\Omega}d\tau_2\right) \quad (2.14)$$

and gives the expression for χ which is valid for

*This assumption does not affect the qualitative nature of the results.

arbitrary magnetic fields. Let us consider the behavior of χ in the strong-field region. Making use of the fact that the function preceding the exponent in the integral (2.14) is periodic in τ, it can easily be shown[8] that $\chi(\tau)$ is equal to

$$\chi(\tau) = \left\{\Omega\left[1 - \exp\left(-\frac{2\pi\bar{\nu} - 2\pi i\overline{\mathbf{kv}}}{\Omega}\right)\right]\right\}^{-1} \times \int_{\tau-2\pi}^{\tau} d\tau_1 \left(\Delta\lambda_{ik}(\tau_1)\frac{\partial \dot{u}_i}{\partial x_k} - e\mathbf{E}^*\mathbf{v}(\tau_1)\right) \exp\left(\int_{\tau}^{\tau_1} \frac{\nu - i\mathbf{kv}}{\Omega} d\tau_2\right) \quad (2.15)$$

(we have neglected the small quantity ω in comparison with ν in all the exponents). In equation (2.15) the bar denotes the average over a period 2π in τ:

$$\bar{\varphi} = (1/2\pi)\oint \varphi(\tau)\, d\tau.$$

In the case which we are considering (a closed Fermi surface, and $\mathbf{k} \perp \mathbf{H}$), we have $\mathbf{k}\cdot\bar{\mathbf{v}} = 0$ because[7]

$$v_x = \frac{1}{2\pi}\oint \frac{1}{m}\frac{dp_y}{d\tau} d\tau \equiv 0.$$

Noting that the quantities ν and $|\mathbf{k}\cdot\mathbf{v}|$ are $\ll \Omega$, we may expand all the exponentials in series, obtaining as a result

$$\chi = \frac{1}{2\pi\bar{\nu}} \int_{\tau-2\pi}^{\tau} d\tau_1 \left(\Delta\lambda_{ik}(\tau_1)\,\partial\dot{u}_i/\partial x_k - e\mathbf{E}^*\mathbf{v}\right) \times \left(1 + \int_{\tau}^{\tau_1} \frac{\nu - i\mathbf{kv}}{\Omega} d\tau_2 + \frac{1}{2}\left[\int_{\tau}^{\tau_1} \frac{\nu - i\mathbf{kv}}{\Omega} d\tau_2\right]^2\right). \quad (2.16)$$

Finally, in order to find the non-equilibrium contribution to the distribution function it is necessary to calculate the electric field $\mathbf{E}^*$ from equation (2.7).

3. DETERMINATION OF THE ELECTRIC FIELD

There is a fundamental difference in the forms of the function χ in the two cases $n_1 \neq n_2$ and $n_1 = n_2$, where n_1 (or n_2) is the number of electrons (or holes) in the definite volume bounded by the surface $\epsilon(\mathbf{p}) = \mu_0$ within which the energy is less (greater) than μ_0 (see reference 7).

Let us first consider the case $n_1 \neq n_2$. Substituting (2.16) into formula (2.12) for the current density, we obtain

$$j_i = \sigma_{ik}E_k^* - j_i', \quad (3.1)$$

where the electrical conductivity tensor is equal to

$$\sigma_{ik} = \frac{2e^2}{h^3}\int dp_z \frac{m}{2\pi\bar{\nu}}\oint d\tau v_i(\tau) \int_{\tau-2\pi}^{\tau} d\tau_1 v_k(\tau_1) \times \left\{1 + \int_{\tau}^{\tau_1}(\gamma - iq)\, d\tau_2 + \frac{1}{2}\left[\int_{\tau}^{\tau_1}(\gamma - iq)\, d\tau_2\right]^2\right\}, \quad \gamma = \nu/\Omega, \quad q = kv_x/\Omega, \quad (3.2)$$

and the expression for j_i' is obtained from $\sigma_{ik}E_k^*$ by replacing $eE_k^*v_k(\tau_1)$ in (3.2) by the quantity $\Delta\lambda_{ik}(\tau_1)\,\partial\dot{u}_i/\partial x_k$.

The field $\mathbf{E}^*$ is found from (2.7):

$$E_i^* = \rho_{ik}j_k', \quad (3.3)$$

where $\rho_{ik} = (\sigma^{-1})_{ik}$ is the electrical resistivity tensor.

When the resistivity tensor is evaluated, it leads to the following results:

$$\sigma_{ik} = \begin{pmatrix} \gamma_0^2 a_{xx} & \gamma_0 a_{xy} & \gamma_0 a_{xz} \\ \gamma_0 a_{yx} & q_0^2 a_{yy} & \gamma_0 a_{yz} + q_0^2 a'_{yz} \\ \gamma_0 a_{zx} & \gamma_0 a_{zy} + q_0^2 a'_{zy} & a_{zz} \end{pmatrix}. \quad (3.4)$$

Here $\gamma_0 = H_0/H \ll 1$; H_0 is the field for which the characteristic radius r_0 of the orbit is equal to the characteristic free path; $q_0 = kr_0 \ll 1$ although $q_0 \gg \gamma_0$, but q_0^2, generally speaking, may be comparable with γ_0 (this is why two terms have been retained in the σ_{yz} and σ_{zy} components).

The expansion of the quantities a_{ik} in γ_0 and q_0 begins with terms which do not depend on the magnetic field, and whose form, generally speaking, depends on the nature of the collisions. The exceptions to this are the components $\sigma_{yx} = -\sigma_{xy} = ecH^{-1}(n_1 - n_2)$, which do not depend upon the collision integral because they are due to the drift of particles across the magnetic field (Hall current). The quantities a_{ik} are of the order of σ_0, the static conductivity of the metal when $H = 0$.

The "deformation" current $\mathbf{j}'$ is equal to

$$j_x' = \frac{ikc^2}{eH^2}\frac{2}{h^3}\int \frac{dS}{v}\left\{\frac{1}{2}p_y^2 - \frac{1}{\bar{\nu}}\overline{p_y\nu p_y} + \frac{1}{2}\frac{1}{\bar{\nu}}\overline{p_y^2\nu}\right\}\Delta\lambda_{ik}\frac{\partial\dot{u}_i}{\partial x_k},$$

$$j_y' = \frac{ikc}{H}\frac{2}{h^3}\int \frac{dS}{v}\frac{1}{\bar{\nu}}\overline{v_x p_x}\Delta\lambda_{ik}\frac{\partial\dot{u}_i}{\partial x_k},$$

$$j_z' = -\frac{ikc}{H}\frac{2}{h^3}\int \frac{dS}{v}\frac{1}{\bar{\nu}}(p_y\bar{v}_z - \overline{p_y v_z})\,\Delta\lambda_{ik}\frac{\partial\dot{u}^i}{\partial x_k}. \quad (3.5)$$

The matrix of the resistivity tensor σ_{ik} has the form

$$\rho_{ik} = \begin{pmatrix} q_0^2\gamma_0^{-2}b_{xx} & \gamma_0^{-1}b_{xy} & b_{xz} + q_0^2\gamma_0^{-1}b'_{xz} \\ \gamma_0^{-1}b_{yx} & b_{yy} & b_{yz} \\ b_{zx} + q_0^2\gamma_0^{-1}b'_{zx} & b_{zy} & b_{zz} \end{pmatrix}, \quad (3.6)$$

where all components of the tensor b_{ik}, with the exception of $\rho_{yx} = -\rho_{xy} = H/ec(n_1 - n_2)$, depend on the collision integral and are of the order of σ_0^{-1}.

Substituting ρ_{ik} from (3.6) into (3.3) for the electric field, we obtain

$$E_x^* = \rho_{xy}j_y' \sim (q_0/\gamma_0 ev_0)\,\Delta\lambda_{ik}\partial\dot{u}_i/\partial x_k,$$
$$E_y^* = \rho_{yk}j_k' \sim (q_0/ev_0)\,\Delta\lambda_{ik}\partial\dot{u}_i/\partial x_k,$$
$$E_z^* = \rho_{z\beta}j_\beta' \sim (q_0/ev_0)\,\Delta\lambda_{ik}\partial\dot{u}_i/\partial x_k. \quad (3.7)$$

Upon substituting E^* into the expression (2.16) for χ, it is easy to verify that the terms containing the electric field can be neglected, since at most they are $1/q_0 \gg 1$ times smaller than the terms containing $\Delta\lambda_{ik}\partial\dot{u}_i/\partial x_k$.

Thus, in the case where $n_1 \neq n_2$,

$$\chi = (1/\bar{\nu})\,\overline{\Delta\lambda_{ik}}\,(\partial\dot{u}_i/\partial x_k). \tag{3.8}$$

In the case where $n_1 = n_2$ an analogous treatment, taking into account the electric field (to which the component E_y^* makes the chief contribution) leads to the following result:

$$\chi = \frac{1}{\bar{\nu}}\left\{\overline{\Delta\lambda_{ik}} - \frac{S}{m}\sum\langle S\overline{\Delta\lambda_{ik}}\bar{\nu}^{-1}/m\rangle\left(\sum\langle S^2\bar{\nu}^{-1}/m^2\rangle\right)^{-1}\right\}\frac{\partial\dot{u}_i}{\partial x_k}, \tag{3.9}$$

where

$$S_j(\mu_0,\ p_z) = \oint p_x dp_y,\quad m = (2\pi)^{-1}\frac{\partial S_j}{\partial\mu_0}$$

is the area of the section of the j-th surface $\epsilon_j(p) = \mu_0$ by the plane $p_z = \text{const}$. The sign of the area depends on the direction of the contour path:[7] for surfaces bounded by states of lower energy ("electrons"), $S > 0$; for "holes," $S < 0$. The summation extends over the entire surface $\epsilon_j = \mu_0$, and m is the effective mass.

4. THE ABSORPTION COEFFICIENT. SOME REMARKS

When the function χ is known, it is easy to find the dissipation function, and with it the coefficient of ultrasonic absorption also. In the case $n_1 \neq n_2$,

$$|Q| = \frac{1}{h^3}\sum\int_{\epsilon_j=\mu_0}\frac{dS}{v}\,|\overline{\Delta\lambda_{ik}}(\partial\dot{u}_i/\partial x_k)|^2\frac{1}{\bar{\nu}}. \tag{4.1}$$

When $n_1 = n_2$,

$$|Q| = \frac{1}{h^3}\sum\int_{\epsilon_j=\mu_0}\frac{dS}{v}\frac{1}{\bar{\nu}}\left|\overline{\Delta\lambda_{ik}}\frac{\partial\dot{u}_i}{\partial x_k} - \frac{S}{m}\sum\langle S\overline{\Delta\lambda_{ik}}\bar{\nu}^{-1}/m\rangle\left(\sum\langle S^2\bar{\nu}^{-1}/m^2\rangle\right)^{-1}\frac{\partial\dot{u}_i}{\partial x_k}\right|^2. \tag{4.2}$$

In both cases the absorption coefficient α is of the order of magnitude

$$\alpha \sim N\mu_0\omega^2 t_0/\rho s^2, \tag{4.3}$$

where s is the speed of sound in the metal and $N \sim n_1$.

The expression (4.3) for α agrees in form with the absorption coefficient in the absence of a magnetic field, in the frequency and temperature range where $kl \ll 1$ (cf. reference 6). When $kl \gg 1$ the absorption coefficient is kl times smaller in the absence of the magnetic field. Consequently the ultrasonic absorption coefficient saturates in strong magnetic fields, while at the same time the magnitude of the absorption under saturation conditions is considerably larger than the absorption in the absence of a field. This conclusion agrees qualitatively with the experiments of Galkin and Korolyuk[3] on ultrasonic absorption by single crystals of lead in strong magnetic fields.

In this connection, however, a few qualifying remarks are necessary. The fact is that the experiments mentioned above showed a strong anisotropy in the absorption in strong fields. The absorption coefficient α varied by a factor of 5 or 6 for different orientations of **H** with respect to the crystal axes. This difference may be due to the existence of open electron trajectories, over which the mean value of the velocity component v_x does not reduce to zero.

This would lead to the appearance of $\bar{\nu} - (ik\bar{v}_x/\Omega)$ instead of $\bar{\nu}$ in the denominator of the expression for χ, the second term being much larger than the first.

In the case where the direction of an open trajectory* is perpendicular to the vectors **k** and **H** (or inclined at a small angle $\vartheta \ll \gamma_0$), the contribution to the absorption coefficient from the open (or extremely elongated) trajectories turns out to be considerably smaller than the contribution from the closed trajectories. For example, in the case of a Fermi surface in the shape of a smooth cylinder, the absorption coefficient is kl times smaller than when the trajectories are closed, and its order of magnitude is the same as that of the zero-field absorption coefficient ($H = 0$). The asymptotic value of the absorption coefficient depends greatly on the nature of the open surface and on the angle between the magnetic field and the direction of the open trajectory. A study of the peculiarities of strong-field ultrasonic absorption by metals with open Fermi surfaces will be published in a separate paper.

For the same reason, an analogous reduction in the acoustic absorption coefficient should also take place for closed trajectories, when the wave vector **k** is not perpendicular to **H** ($\cos\vartheta \gg 1$, where ϑ is the angle between **k** and **H**).

In conclusion I should like to thank M. I. Kaganov for valuable advice and comments.

*For example, in the case of a Fermi surface of the "fluted cylinder" type, the direction of the cylinder axis; in the case of a "three-dimensional grid" type of surface, the line of intersection of the plane perpendicular to **H** with the nearest crystallographic plane.

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34