PERIPHERAL INTERACTION OF 9 Bev NUCLEONS

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The peripheral interaction of two nucleons (E_L = 9 Bev) arising in the exchange of one pion is considered. The cross section for such processes is estimated. It is found that excited nucleons in such peripheral interactions are in states with isospin $^3/_2$.

THE following features were observed[1] in the interaction of 9 Bev nucleons: 1) in the case of collision of two protons (to be referred to as p-p interaction), the distribution of secondary particles was anisotropic in the center-of-mass system (c.m.s.); 2) in the case of neutron-proton collisions, the distribution of secondary charged particles was asymmetrical; namely, in the majority of cases the secondary proton is emitted in the initial direction ("the proton conserves its charge").

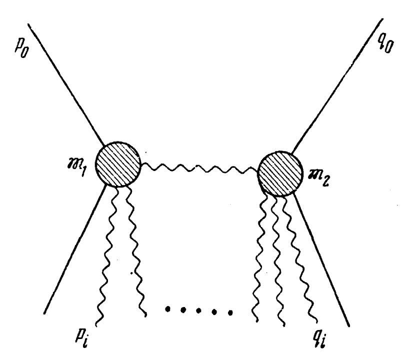
Tamm has shown[2] that all of these facts can be explained by assuming that in peripheral interactions of the two nucleons, both nucleons are simultaneously excited to the isobaric state with isospin $^3/_2$ and mass $\mathfrak{M} = 1.3\,m$ (m is the mass of the nucleon).

We shall investigate here the peripheral interaction in the exchange of one meson by two nucleons of energy E_L = 9 Bev. We shall concentrate mainly on the question as to how well justified the assumption (made by Tamm in reference 2) about simultaneous isobaric excitation of both nucleons is, and what the cross section for such a process is. It should be noted, that the calculation of peripheral collisions of two nucleons where both nucleons are excited should be carried out in perturbation theory (see reference 3); the Weizsäcker-Williams method gives incorrect results in this case.

The calculation was carried out, starting from Feynman diagrams (see the figure). Using the general rules, the expression for the probability of the process can be written in the form

$$dw = (2\pi)^4 \sum_{n,n'} \left| \frac{M_1(p_0, -k)\, M_2(q_0, k)}{k^2+\mu^2} - \frac{M_1(p_0, -k')M_2(q_0, k')}{k'^2+\mu^2} \right|^2 \times \delta\Big(\sum_i^n p_i + \sum_j^{n'} q_j - p_0 - q_0\Big) \prod_i^n \frac{d^3p_i}{(2\pi)^3} \prod_j^{n'} \frac{d^3 q_j}{(2\pi)^3}, \qquad (1)$$

where p_0 and q_0 are the 4-momenta of the colliding nucleons, p_i and q_j are the momenta of the secondary particles, $k = p_0 - \sum p_i = p_0 - P_1$;



$k' = p_0 - \sum q_j$; $k^2 = \mathbf{k}^2 - k_0^2$; M_1 and M_2 are the matrix elements corresponding to the first and second vertices of the diagram.

It should be noted that the denominator of the first term is small only in the region of small angles $\vartheta \sim \mu/p_0$ (ϑ is the angle between $\mathbf{p}_0$ and $\mathbf{P}_1$); the denominator of the second term is small in the region $\vartheta \sim \pi - \mu/p_0$. It can be shown that the square of the first term in the region $\vartheta \sim \mu/p_0$ has the same absolute value as the square of the second term in the region $\vartheta \sim \pi - \mu/p_0$. The interference term is small compared with these terms and is of comparable size only in the region $\vartheta \sim \pi/2$, where all three terms are small.* Because of this, the interference term can be neglected, and the interaction probability can be written as

$$dw = \sum_{n,n'} \frac{2(2\pi)^4}{(k^2+\mu^2)^2} |M_1|^2 |M_2|^2 \delta \times \Big(\sum_i^n p_i + \sum_j^{n'} q_j - p_0 - q_0\Big) \prod_i^n \frac{d^3 p_i}{(2\pi)^3} \prod_j^{n'} \frac{d^3 q_j}{(2\pi)^3}.$$

Calculation of the matrix elements M_1 and M_2 requires knowledge of operators for the angular

*In integrating over the angles ϑ, the contribution from the interference term is smaller by a factor of p_0^2/μ^2 than the contributions from the squares of the first or second term, if the angular dependence is assumed to be determined by the denominator.

parts, which is not known at these energies. Therefore, it is more convenient to use known results about the magnitude of the pion-nucleon interaction cross section. Noting that the probability of interaction of a π meson (with 4-momentum k) with a nucleon (of momentum q_0) is equal to

$$w_\pi(k) = \sum_{n'} \int \frac{(2\pi)^4}{2\omega} |M(q_0, k)|^2 \delta\Big(\sum_j^{n'} q_j - q_0 - k\Big) \prod_j^{n'} \frac{d^3 q_j}{(2\pi)^3},$$

$$\omega = \sqrt{\mathbf{k}^2 + \mu^2}, \tag{2}$$

it is possible to rewrite the expression for the cross section for peripheral collision of nucleons $\sigma(q_0, p_0)$ as

$$E_1 E_2 J \sigma(p_0, q_0) = \frac{8}{(2\pi)^4} \int \frac{d^4 P_1}{(k^2+\mu^2)^2} E_1 \omega J_\pi^{(1)} \sigma_1(\omega) E_2 \omega J_\pi^{(2)} \sigma_2(\omega), \tag{3}$$

where E_1 and E_2 are the energies of the colliding nucleons (they are equal in the c.m.s., $E_1 = E_2 = E_0$); ω is the energy of the π meson, $\sigma(\omega)$ is the cross section for pion-nucleon interaction; J_π is the relative current of π mesons, so that $J_\pi \sigma(\omega) = \omega_\pi$.

The quantity $\omega E J_\pi \sigma(\omega)$ is invariant with respect to Lorentz transformations and can depend only on invariant combinations of the momenta p_0 and k. It should be noted that the momentum of the virtual π meson enters into these invariant combinations; however, the degree of its "virtualness" in peripheral interactions is small: $k^2 + \mu^2 \sim \mu^2$.

In the future we neglect any differences in these expressions from those for the interaction of a real π meson with a nucleon. This neglect is equivalent to the assumption that $\omega E J_\pi \sigma(\omega)$ depends less on the angle ϑ between $\mathbf{p}_0$ and $\mathbf{P}_1$ than does the denominator $(k^2 + \mu^2)$. In fact, it is easy to see that for small imaginary values $\vartheta \sim i\mu/E_0$, the "virtualness" $k^2 + \mu^2 = 0$ and the quantity $\omega E J_\pi \sigma(\omega)$ coincide with the expression for real π mesons. Extrapolating this value into the region of real small angles (considering the dependence of $\omega E J_\pi \sigma(\omega)$ on angle to be both analytical and small), we find that in the region of real angles this expression is near to that for real π mesons. In this case, the quantities $\omega E_1 J_\pi^{(1)} \sigma_1(\omega)$ and $\omega E_2 J_\pi^{(2)} \sigma_2(\omega)$ depend only on the quantities m^2 and μ^2 (μ is the meson mass), and also on the quantities $\mathfrak{M}_1^2$ and $\mathfrak{M}_2^2$ ($\mathfrak{M}_1^2 = -P_1^2 = -|\sum_i^n p_i|^2$ and $\mathfrak{M}_2^2 = -|\sum_j^{n'} q_j|^2$) which can conveniently be introduced since they represent masses of excited nucleons which then rapidly decay into secondary particles.* The differential d^4P_1 can be expressed as

$$d^4 P_1 = \frac{P_1}{2E_0} \mathfrak{M}_1 d\mathfrak{M}_1 \mathfrak{M}_2 d\mathfrak{M}_2 \, 2\pi d(\cos\vartheta). \tag{4}$$

The cross section can then be put into the form

$$\sigma(E_0) = \frac{2}{(2\pi)^3 E_0^2} \int dz \int dy \int d(\cos\vartheta)$$
$$\times \frac{\sqrt{z^2 - m^2\mu^2}\sqrt{y^2 - m^2\mu^2}}{[\mu^2 + \varkappa^2 + 2p_0^2(1-\cos\vartheta)]^2} \sigma_\pi(z)\sigma_\pi(y), \tag{5}$$

where $z = (\mathfrak{M}_1^2 - m^2 - \mu^2)/2$, $y = (\mathfrak{M}_2^2 - m^2 - \mu^2)/2$

$$\varkappa^2 = zy/E_0^2 + (z+y)[2zy + m^2(z+y)]/4E_0^4.$$

From this expression it can be seen that the angular distribution of excited particles is anisotropic in the c.m.s. and concentrated in the region of small angles $\vartheta^2 \sim (\mu^2 + \kappa^2)/p_0^2$. As will be seen from the following, $\kappa^2 \sim \mu^2$ and, consequently, the angles ϑ are $\sim \mu/p_0$, in agreement with the generally accepted picture[4] that $p_\perp \sim \vartheta p_0 \sim \mu$.

Integrating over angles, we obtain the total cross section as

$$\sigma(E_0) = \frac{1}{(2\pi)^3 E_0^2} \int_{m\mu}^{z_{max}} dz \int_{m\mu}^{y_{max}} dy \frac{\sqrt{z^2 - m^2\mu^2}\sqrt{y^2 - m^2\mu^2}}{(\mu^2+\varkappa^2)[(\mu^2+\varkappa^2)\vartheta_{max}^{-2} + p_0^2]}$$
$$\times \begin{cases} \frac{10}{9}\sigma_{3/2}\sigma_{3/2} + \frac{16}{9}\sigma_{3/2}\sigma_{1/2} + \frac{1}{9}\sigma_{1/2}\sigma_{1/2} & \text{for p-p interactions} \\ \frac{14}{9}\sigma_{3/2}\sigma_{3/2} + \frac{8}{9}\sigma_{3/2}\sigma_{1/2} + \frac{5}{9}\sigma_{1/2}\sigma_{1/2} & \text{for p-n interactions} \end{cases} \tag{6}$$

Here $\sigma_{3/2}$ and $\sigma_{1/2}$ enter separately; they are the cross sections for pion-nucleon interaction in states with isospin $3/2$ and $1/2$.

In accordance with the above, we used experimental values for the pion-nucleon cross section for $\sigma(z/m)$ and $\sigma(y/m)$.

In order to evaluate the cross section, it is necessary to integrate (6) numerically, since the values of $\sigma(z/m)$ and $\sigma(y/m)$ are given numerically. The main contribution to the integral comes from the region $\mathfrak{M}_1^2 - m^2 \sim \mathfrak{M}_2^2 - m^2 \sim 2E_0\mu$ ($\kappa^2 \sim \mu^2$), since the denominator increases rapidly with larger values of $\mathfrak{M}_1$ and $\mathfrak{M}_2$.

It should be noted that in our case, where $E_L = 9$ Bev and $E_0 = 2.3$ Bev, this region coincides with the region of maximum cross section $\sigma_{3/2}(z/m)$, so that $\sigma_{3/2}$ makes the overwhelming contribution in all expressions.

We give now the results of calculations.†

*We note that here and in the following calculations, the masses $\mathfrak{M}_1$ and $\mathfrak{M}_2$ are considered to be variables which can take on arbitrary values and are not yet set equal to the mass of the isobar.

†Experimental data on the cross section for pion-nucleon interactions up to π-meson energy $\omega = 1.9$ Bev[5] was used in the calculations. The maximum angle was taken equal to $\vartheta_{max} = 3\mu/p_0$.

The quantity $\sigma_{3/2,3/2}(E_0)$ — the cross section for the process in which both excited nucleons are in the state with isospin $3/2$ — is equal to

$$\sigma_{3/2,\,3/2}(E_0) \approx \begin{cases} 2.9 \text{ mb for p-p interactions} \\ 4 \text{ mb for p-n interactions} \end{cases} \tag{7}$$

The quantity $\sigma_{3/2,1/2}(E_0)$ is the cross section for the process in which one of the excited nucleons has isospin $3/2$ and the other $1/2$:

$$\sigma_{3/2,\,1/2}(E_0) \approx \sigma_{3/2,\,3/2}(E_0) \times \begin{cases} 0.48 \text{ mb for p-p interactions} \\ 0.17 \text{ mb for p-n interactions.} \end{cases} \tag{8}$$

Further,

$$\sigma_{1/2,\,1/2}(E_0) \approx \sigma_{3/2,\,3/2}(E_0) \times \begin{cases} 0.01 \text{ mb for p-p interactions} \\ 0.05 \text{ mb for p-n interactions.} \end{cases}$$

From this it follows that the main process is that with two isobars.

It should be noted that Eq. (6) does not include processes in which at least one of the mesons remains unexcited. The contribution from these processes can be estimated, assuming that the angular operator in this process is equal to γ_5. This contribution turns out to be equal to

$$\sigma_0(E_0) \sim 10^{-2}\sigma_{3/2,\,3/2}(E_0). \tag{9}$$

From this it can be seen that it is negligible in comparison with (7) and (8) and has no effect on the conclusions drawn above.

In conclusion we note that (6) can be employed at higher energies and in cases where the nucleons are excited differently.

In the high-energy region it gives more complete information than either the Weizsäcker-Williams method or the expression in reference 3, obtained under rougher approximations, for peripheral interactions.

In conclusion, the authors would like to thank Academician I. E. Tamm, Academician V. I. Veksler, and Professor E. L. Feĭnberg for fruitful discussions.

[1] Gramenitskiĭ, Podgoretskiĭ, et al. See V. I. Veksler, Nucleon-Nucleon and Pion-Nucleon Interactions. Report to the Conference on High-Energy Physics, Kiev, 1959 (preprint, p. 16).

[2] I. E. Tamm, Report to the Conference on High-Energy Physics, Kiev, 1959.

[3] Yu. A. Romanov and D. S. Chernavskiĭ, JETP, in press.

[4] O. Minakawa and Y. Nischimura, Observation of High-Energy Jets with Emulsion Chambers, Univ. of Tokyo, Preprint, 1958.

[5] O. Piccioni, Proc. of Conference on High-Energy Physics at CERN, 1958, p. 65.

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